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THE SILVER-BURDETT ARITHMETICS

COMPLETE BOOK

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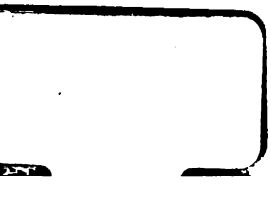

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THE SILVER-BURDETT ARITHMETICS

COMPLETE BOOK

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SILVER–BURDETT ARITHMETICS

Two Book Series

BOOK ONE
COMPLETE BOOK

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BOOK THREE

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PREFACE

THIS book completes a two-book course in arithmetic. In closely graded schools it will be found best to introduce it in the fifth grade. In ungraded and loosely graded schools it may be introduced whenever the elementary book of this series has been satisfactorily completed or an equivalent amount of work done. Few pupils will need more advanced work in arithmetic than this book offers; for, although it is intended for grammar grades, nevertheless it is suited to the needs of the many high schools which find a review of arithmetic profitable.

The authors believe that in teaching arithmetic nothing can take the place of abundant and constant drill and practice in the intelligent solution of problems. They believe that pupils should be given a large number of simple, practical problems, rather than a few difficult ones. They are convinced that accuracy is the first essential, and then speed: and that while the pupil should be taught the best method of solving a problem, he should understand the *how* and *why* of it, instead of following blindly a rule or a model solution.

Attention is invited to the following features of the book:

1. *The thorough review and extension* of Fundamental Processes, Fractions, Decimals, and Denominate Amounts, designed to perfect the work done in the lower grades.

18646

2. *The abundance of well-graded up-to-date problems* based on facts derived first hand from the most reliable sources, and closely associated with the everyday life of the home, the farm, the market, and a variety of other activities.

3. The treatment of such subjects as *Parcel Post*, *Postal Savings Banks*, etc.

4. *The explanations of illustrative examples*, which set forth in correct form the logical steps leading to results, and which are designed to induce correct habits of thought and orderly arrangement of work.

5. *The treatment of Negotiable Paper and Banking*, in which are found photographic forms of business papers in everyday use and explanations of them in language free from technicalities.

6. *The extensive treatment of Mensuration* and the method employed of deriving most of the important principles of the subject.

7. *The topical arrangement* and the numerous reviews.

8. *The placing of the less important subjects* in a supplement.

9. *The omission* of subjects and problems which are *obsolete, overcomplex, and impracticable.*

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COMPLETE ARITHMETIC

WHOLE NUMBERS

NOTATION AND NUMERATION

Definitions

1. A **number** is the direct answer to the question, *How many?*

2. **Notation** is the method of expressing numbers by means of characters called **numerals**. **Numeration** is the method of expressing numbers in words.

3. The **Arabic notation**, the system in general use, expresses numbers by the figures 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9.

4. The **Roman notation**, which is employed only for special purposes, expresses numbers by the letters, I, V, X, L, C, D, and M.

Arabic System

5. For convenience in reading large numbers, the figures used to express them are generally separated, as far as possible, into **periods** of three figures each, beginning at

the right. Counting from the right the first five periods are named **units, thousands, millions, billions,** and **trillions.** Each period contains units, tens, and hundreds of that period.

6. The following table shows the names of the first five periods and of the three **orders** or **places** in each period.

Table of Notation

Hundred-trillions	Ten-trillions	Trillions	Hundred-billions	Ten-billions	Billions	Hundred-millions	Ten-millions	Millions	Hundred-thousands	Ten-thousands	Thousands	Hundreds	Tens	Units
		1	4	2	8	0	1	6	3	5	0	3	0	7
		Trillions 5th period			Billions 4th period			Millions 3d period			Thousands 2d period			Units 1st period

The number in the table is read *one trillion four hundred twenty-eight billion sixteen million three hundred fifty thousand three hundred seven.*

7. Units, tens, hundreds, thousands, ten-thousands, hundred-thousands, etc., are called respectively units of the *first, second, third, fourth, fifth, sixth, etc.,* orders.

8. The system of grouping and representing numbers which is in general use and which is based on the number *ten,* is called the **decimal system.** *In the decimal system of notation ten units of any order make one unit of the next higher order.*

Exercise 1

1. Name in order, beginning at one, the numbers that can be expressed each with one figure.

2. Ten ones make what number?

3. Beginning at ten, name in order the nine tens.

4. Name the number made up of six tens and seven units.

5. Ten tens make what number?

6. Beginning at one hundred, name the first nine hundreds.

7. Name the number made up of six hundreds, four tens, and three units.

8. Name the number made up of three hundreds and four tens.

9. Name the number made up of eight hundreds and two units.

10. In 782 for what number does the 7 stand? the 8? the 2?

11. Name the number consisting of five units of the first order, two of the second, and seven of the third.

Read aloud each of the following:

12. 6591	13. 5028	14. 2608
15. 76,187	16. 60,325	17. 76,032
18. 275,832	19. 907,428	20. 320,810
21. 2,932,418	22. 7,600,302	23. 6,803,001
24. 76,353,489	25. 563,819,306	26. 897,315,724

27. The salary of the President of the United States is $ 75,000; the salary of the Vice-President is $ 12,000.

28. The greatest depth of the Pacific Ocean is 30,000 feet.

29. In a recent year 4,002,000 cords of wood were used in the manufacture of paper.

30. We have used in a single year 90,000,000 cords of firewood and 44,000,000,000 board feet of lumber.

31. In 1910 the population of the earth was estimated at 1,522,700,000.

32. In a recent year the United States produced 3,125,713,000 bushels of Indian corn; 1,126,765,000 bushels of oats; 162,227,000 bushels of barley; 33,039,000 bushels of rye; 17,239,000 bushels of buckwheat; and 695,443,000 bushels of wheat.

Exercise 2

Copy the following, writing the numbers in figures:

1. The English language contains about six hundred thousand words.

2. Yellowstone National Park contains two million one hundred forty-two thousand seven hundred twenty acres, and Yosemite National Park contains seven hundred nineteen thousand six hundred twenty-two acres.

3. The Post Office Department of the United States employs more than three hundred thousand persons, who handle more than fifteen billion pieces of mail each year. They handle more than eight hundred thousand letters every hour of the twenty-four hours of each day.

4. The Dead Letter Office receives about twelve million "dead" letters and parcels a year, and about eight

million post cards. Ten million letters and parcels are opened, and of these about six million are forwarded to the persons to whom they are addressed or are returned to the writers. More than eighty-five thousand photographs come to the dead letter office each year.

5. Washington is the most beautiful city in the United States. It has ten large parks, twenty-six others more than an acre each in size, and two hundred seventy-five little parks of less than an acre each. It has more than ninety-two thousand trees, while Paris, one of the most beautiful cities in the world, has but eighty-five thousand. Yet there are more than three million people living in Paris, and only about three hundred fifty thousand living in Washington.

6. The English language is spoken by more than one hundred fifty million people; the German, by more than one hundred twenty million people; the Russian, by more than ninety million; the French, by more than sixty million; the Spanish, by more than fifty million; the Italian, by more than forty million: and the Portuguese, by more than forty million.

7. The United States produced, in a recent year, three hundred twenty-eight million one hundred ten thousand seven hundred forty-nine pounds of wool, and Argentina three hundred twenty-eight million seven hundred thirty-one thousand pounds. The total production of wool in North America in that year was three hundred forty million three hundred twenty thousand seven hundred forty-nine pounds; the total in Europe was eight hundred four

million nine hundred five thousand seventy-four pounds; in South America four hundred thirty-six million seven hundred sixteen thousand pounds.

Roman System

9. The Romans represented all numbers by seven letters, and a systematic combination of them. These letters are I, representing *one;* V, *five;* X, *ten;* L, *fifty;* C, *one hundred;* D, *five hundred;* and M, *one thousand.*

10. The combinations used to represent the nine simple numbers, the tens, the hundreds, and the thousands are formed according to the following principles:

1. *Every time a letter is repeated, its value is repeated.*

2. *When a letter or a combination of letters of less value is written after a letter or a combination of letters of greater value, the sum of the values is expressed.*

3. *When a letter of less value is written before one of greater value, the difference of their values is expressed.*

4. *A dash placed over a letter or a combination of letters increases the value a thousand fold.*

Table

Units	Tens	Hundreds	Thousands
I = 1	X = 10	C = 100	M = 1000
II = 2	XX = 20	CC = 200	MM = 2000
III = 3	XXX = 30	CCC = 300	MMM = 3000
IV = 4	XL = 40	CD = 400	$\overline{\text{IV}}$ = 4000
V = 5	L = 50	D = 500	$\overline{\text{V}}$ = 5000
VI = 6	LX = 60	DC = 600	$\overline{\text{VI}}$ = 6000
VII = 7	LXX = 70	DCC = 700	$\overline{\text{VII}}$ = 7000
VIII = 8	LXXX = 80	DCCC = 800	$\overline{\text{VIII}}$ = 8000
IX = 9	XC = 90	CM = 900	$\overline{\text{IX}}$ = 9000

11. In writing numbers between those in the above table, hundreds are written after thousands, tens after hundreds, and units after tens.

Thus, 1567 is written MDLXVII; 2053, MMLIII; 4508, $\overline{\text{IV}}$DVIII; 3600, MMMDC.

NOTE 1. IIII = 4; XXXX = 40; CCCC = 400, are also in use.

NOTE 2. The use of the Roman system of notation is confined chiefly to the numbering of chapters and sections of books, pages of prefaces and introductions, medical prescriptions, public documents, divisions on clock dials, etc.

Exercise 3

Write by the Roman system the following:

1. The nine simple numbers. 2. The nine tens.
3. The nine hundreds.
4. The numbers between 10 and 30.
5. The numbers between 50 and 70.
6. The numbers between 80 and 100.

7. 490	8. 940	9. 1150	10. 1664
11. 550	12. 1876	13. 1861	14. 1812
15. 1914	16. 1919	17. 1926	18. 1899

Write in figures the following:

19. CIX	20. MIV	21. LXI
22. XIX	23. XLVIII	24. MCMIX
25. XXV	26. XCIV	27. MDCLXVI
28. LXVII	29. CXIV	30. CDLIII
31. XLIV	32. CCXIX	33. MCCCXXXV
34. XCIX	35. CDXLIV	36. MCMIV
37. XXXIX	38. CMXCIV	39. MDCCXCIX
40. LXXXIX	41. DCCCIV	42. MDCCCX

General Definitions

12. **Quantity** is anything that can be measured; as, *weight, length, value, time.*

13. A **unit** is any standard of reference employed in counting a collection or in measuring a quantity; as, *a pound, a yard, a dollar, a day.*

14. An **amount** is an expression denoting the number and kind of units in a quantity; as, *2 pounds, 5 yards, 6 dollars, 7 days.*

15. **Similar amounts** are such amounts as are composed of the same kind of units; as, *3 bushels* and *5 bushels.*

16. **Arithmetic** is the science of numbers and the art of computing with them.

Reading and Writing Dollars and Cents

17. The sign $ stands for dollar or dollars; *c*, ¢ or *ct.* stands for cent or cents.

Thus, *7 dollars* may be written, $7; *5 cents* may be written *5 c*, *5* ¢, or *5 ct.*

18. If a sum of money consists of dollars and cents, a point called the **decimal point** is written between the number of dollars and the number of cents. If the number of cents is less than ten, a cipher is written between the decimal point and the number of cents.

Thus, 4 dollars 65 cents is written $4.65, and 3 dollars 8 cents is written $3.08. 35 cents may be written $0.35.

Exercise 4

1. Write, using the dollar sign:

4 dollars;	22 dollars;	281 dollars;	1365 dollars;
9 dollars;	35 dollars;	450 dollars;	2565 dollars;
7 dollars;	46 dollars;	475 dollars;	5680 dollars.

2. Write, using ¢ for cents:

3 cents;	12 cents;	21 cents;	42 cents;
6 cents;	19 cents;	36 cents;	53 cents;
1 cent;	25 cents;	50 cents;	75 cents.

3. Write, using the dollar sign:

5 dollars;	3 dollars 18 cents;	6 cents;
35 cents;	5 dollars 1 cent;	25 cents;
50 cents;	7 dollars 75 cents;	60 cents;
20 cents;	9 dollars 25 cents;	45 cents.

4. Read:

$2	$39	$48	$5.65	$ 0.74
$7	$26	$52	$7.05	$ 0.59
$5	$50	$75	$7.01	$ 0.75
$3	$10	$99	$8.30	$10.10
$4	$28	$62	$5.80	$20.50
$9	$25	$48	$0.01	$10.05
$6	$18	$80	$1.50	$12.65
$8	$42	$69	$2.98	$13.35

FUNDAMENTAL PROCESSES

Addition

19. Addition is the process of uniting two or more numbers into one number called their **sum.**

NOTE. Similar amounts, and only such, may be united (added); the result (sum) is similar to the amounts added.

20. The sign of addition is +, read **plus.**

Exercise 5

In the following look for groups and add the groups:

1.	2.	3.	4.	5.	6.
7	1	4	3	6	7
3	5	2	7	4	8
1 } 9	2	6	3	5	9
5	6	3	5	8	6
4	3	7	6	3	7
3 } 9	4	5	2	6	9
2	5	1	4	2	5
5	6	4	7	8	9
3 } 10	2	6	1	4	8
2	1	3	2	6	7

Thus, in Example 1, look for groups, as indicated, and say aloud: *10, 19, 28, 35.*

Add at sight the following:

7. $54+37$ Thus, say aloud: *54, 84, 91.*

8. $12+20$	9. $37+40$	10. $58+30$
11. $21+17$	12. $35+16$	13. $44+23$
14. $48+34$	15. $55+36$	16. $55+38$

21. Find the sum of 372, 565, 356, and 489.

372
565
356
489
1782

Add from the bottom upward.

Add first the units, then the tens, then the hundreds.

What is the sum of the units?

The sum of the units is equal to how many tens and how many units over? How many tens are carried to the tens' column from the sum of the units?

Counting in the number of tens carried, what is the sum of the tens? The sum of the tens is equal to how many hundreds and how many tens over? How many hundreds are carried to the hundreds' column from the sum of the tens? Counting in the number of hundreds carried, what is the sum of the hundreds? What is the sum of the numbers added?

See if the sum found agrees with the sum obtained when the columns are added downward.

22. Testing the correctness of a result by finding it in different ways or by using it in making comparisons is called **checking** the result.

Exercise 6

(Accuracy and speed test. Practice until you can add each of the first 15 examples in this exercise and check the result in less than 2 minutes.)

1.	2.	3.	4.	5.
251847	6	438761	641072	125637
6538	78	54721	334078	946183
78	534	6310	840149	627854
54310	6429	278	648732	249836
5	96823	13	357986	135792
621	681024	7	177162	357546
426	65237	333	987634	143657

6.	7.	8.	9.	10.
2078	6290	10785	112817	197643
1213	1816	7521	9154	6528
4398	4336	4695	2736	8045
338746	547236	163187	975313	468102
135709	129735	153846	197663	165482
253748	268943	216480	206489	310647
497481	421087	436459	465291	508467
22078	24362	34689	39720	47611
31107	36794	48936	46723	50897
8741	1638	2367	1489	7629
56778	767831	10892	26783	237469

11.	12.	13.	14.	15.
183645	256387	48756	12345	7536781
163789	267348	78439	12647	6372846
278564	263746	54236	14526	1362735
263845	163703	17373	10843	2738465
345271	263809	25748	10839	4527384
327846	352748	34253	26645	3647586
436485	462748	36489	35471	4094611
501625	352810	33947	63109	3284764
410987	346784	26738	63923	5981006
329706	352189	50723	69800	2073392

Add upward; add across; check the results by seeing that the sum of the results found by adding the columns is equal to the sum of the results found by adding the rows.

16. $3278 + 6017 + 5923 + 7409 = ?$
$6724 + 3984 + 7018 + 6934 = ?$
$5027 + 5673 + 4216 + 8451 = ?$
$5374 + 6709 + 3980 + 4879 = ?$
$? + ? + ? + ? = ?$

17. $5609 + 6322 + 7804 + 3219 = ?$
$6371 + 6123 + 4076 + 5346 = ?$
$7432 + 8095 + 4367 + 7236 = ?$
$2367 + 7809 + 9762 + 5879 = ?$
$? + ? + ? + ? = ?$

Illustrative Examples

1. A cattle dealer sold 2565 head of stock in January, 3856 head in February, 4357 head in March, 5210 head in April, and 6432 head in May; how many head did he sell in the 5 months?

2565 = the number of head he sold in January.
3856 = the number of head he sold in February.
4357 = the number of head he sold in March.
5210 = the number of head he sold in April.
6432 = the number of head he sold in May.
22420 = the number of head he sold in the five months.

2. A man bought a lot for $600, paid $65 to have it graded, $132 for paving, and $4250 to have a house built on it. What did his property cost him?

$ 600 = the cost of the lot.
$ 65 = the cost of the grading.
$ 132 = the cost of the paving.
$4250 = the cost of the house.
$5047 = the cost of the property.

Exercise 7

1. A man's crop of grain consisted of 248 bu. of wheat, 678 bu. of oats, and 124 bu. of rye; how many bushels of grain were in his crop?

2. Five loads of wheat weighed as follows: 3256 lb., 3478 lb., 3981 lb., 2108 lb., and 3456 lb.; how many pounds were in the five loads?

3. A farmer's barn cost him as follows: for lumber $476; for masons' work $148; for hardware $62; for painting $85; and for grading $12; what did the barn cost him?

4. Copy and add:

A dealer bought in one week:

	675 chickens weighing	3275 lb. for	$493
	375 turkeys weighing	3300 lb. for	495
	245 ducks weighing	980 lb. for	98
He bought	fowls weighing	lb. for	$

5. Add 2 million 65 thousand 4 hundred, 14 million 860 thousand 16, 17 million 4 thousand 220, and 175 million 24 thousand 3 hundred 6.

6. The following are recent estimates of the wealth of the nations named; find their total wealth.

United States 130 billion dollars; Great Britain and Ireland 82 billion dollars; France 65 billion dollars; Germany 60 billion 500 million dollars; Russia 40 billion dollars; Austria-Hungary 25 billion dollars; Italy 20 billion dollars; Belgium 9 billion dollars; Spain 5 billion 400 million dollars; Netherlands 5 billion dollars; Portugal 2 billion 5 hundred million dollars; Switzerland 2 billion 4 hundred million dollars.

7. A dealer owns two tracts of land, the first containing 476 acres and the second 95 acres more than the first. How many acres do both tracts contain?

8. In a recent census the total population of the United States was classed thus; fill in the totals:

Nativity and Color	Males	Females	Total
Native White:			
Native Parents	25,229,294	24,259,147	
Foreign Parents	9,427,053	9,473,610	
Foreign White	7,522,445	5,821,138	
Negro	4,886,358	4,941,936	
All other	266,972	144,313	
Total			

Subtraction

23. Subtraction is the process of finding by how many units one number exceeds another or how many units are left after taking part of a number from it. The greater number is called the **minuend**, the smaller the **subtrahend**, and the result the **difference** or the **remainder**.

Note. Similar amounts, and only such, may be subtracted; the result is similar to the amounts subtracted.

Exercise 8

Starting with the number above the line, subtract at sight the numbers below the line, one after another, in the following examples:

1.	2.	3.	4.	5.	6.	7.	8.
57	76	83	91	86	115	123	164
2	9	4	7	8	6	5	9
9	2	5	4	5	7	4	5
6	9	9	6	7	4	8	6
5	3	7	5	9	9	6	7
8	5	6	2	2	8	9	3
3	4	1	8	3	3	1	2
4	7	2	3	1	2	7	8
1	9	8	9	4	8	9	7
9	8	6	5	8	7	5	6

Thus, in Example 1 say aloud: *57, 55, 46, 40, 35, 27, 24, 20, 19, 10.*

Give the results at sight:

9. 49 20.

Thus, say instantly: *29.*

10. 78 − 32.

Thus, think 78 minus 30 minus 2; say instantly: *46*

11. 82 − 25.

Thus, think 82 minus 20 minus 5; say instantly: *57.*

12. $65-20$	13. $86-30$	14. $58-16$	15. $96-42$
16. $78-25$	17. $97-56$	18. $41-28$	19. $52-48$
20. $60-18$	21. $54-36$	22. $85-47$	23. $91-26$

24. What is the difference between 625 and 378?

625 **Minuend**
378 **Subtrahend**
247 **Difference**

Subtract first the units, then the tens, then the hundreds.

Since 8 units cannot be subtracted from 5 units, how are the 7 units in the difference found?

After the units are subtracted, how many tens are there in the minuend? how many in the subtrahend?

Since there are more tens in the subtrahend than in the minuend, how are the 4 tens in the difference found?

After the tens are subtracted, how many hundreds are there in the minuend? how many in the subtrahend?

How are the 2 hundreds in the difference found?

What is the difference between 625 and 378?

Check: $247 + 378 = 625$.

NOTE. *In practice, think:* 8 from 15 leaves 7; 7 from 11 leaves 4; 3 from 5 leaves 2.

25. To check the result in subtraction see that the sum of the difference (or the remainder) and the subtrahend is equal to the minuend.

Exercise 9

(Accuracy and speed test. Practice until you can subtract the first 32 examples in this exercise in less than 10 minutes. Check all results. In checking do not rewrite the difference and the subtrahend.)

1.	2.	3.	4.
3754	8471	4732	5663
2363	8374	3645	2349

5.	6320 5468	6.	6534 3985	7.	6905 2907	8.	8408 4686
9.	7074 5385	10.	7136 3367	11.	32964 13079	12.	42580 28374
13.	78013 59634	14.	93486 67099	15.	97523 78642	16.	60413 29865
17.	55463 17085	18.	76054 52706	19.	39064 19375	20.	87531 48719
21.	874312 486153	22.	520016 240167	23.	401435 398561	24.	751946 520975
25.	329648 168739	26.	420653 30968	27.	619862 59906	28.	367452 174863
29.	437109 248318	30.	617542 49635	31.	86146 9907	32.	48007 8099

Add upwards; subtract across; check by seeing that the sum of the differences is equal to the difference of the sums of the columns.

33.
$7865 - 3476 = ?$
$5982 - 4219 = ?$
$6809 - 5189 = ?$
$9385 - 6357 = ?$
$8906 - 5314 = ?$
$? - ? = ?$

34.
$57643 - 32608 = ?$
$69075 - 45238 = ?$
$83341 - 57649 = ?$
$75643 - 34986 = ?$
$95014 - 57463 = ?$
$? - ? = ?$

Illustrative Examples

1. There were 380,546 people living in Maryland in 1810 and 1,295,346 in 1910; what was the increase for the hundred years?

1,295,346 = the population of Maryland in 1910.
380,546 = the population of Maryland in 1810.
914,800 = the increase in the 100 years.

2. The Mississippi River is 3160 miles long and the Arkansas 2170 miles long; what is their difference in length?

3160 mi. = the length of the Mississippi River.
2170 mi. = the length of the Arkansas River.
990 mi. = the difference in length.

Exercise 10

1. A man deposited, during a certain week, $ 2275. He drew out at one time $ 280, at another $ 340, and at another $ 1286; how much had he then in the bank?

2. Copy and find the missing remainders:

	Bu. of Wheat	Bu. of Oats	Bu. of Rye
A farmer's crop consisted of	873	621	430
He sold	424	321	220
There then remained			

3. In 1910 the United States produced 695,443,000 bushels of wheat; all the countries of the world produced 3,626,336,000. How much was produced outside of the United States?

4. In 1910 Iowa produced the greatest yield of oats in the United States, 181,440,000 bushels; Illinois produced the next largest quantity, 171,000,000 bushels. What was the difference in yield in the two states?

5. In 1910 Illinois produced more corn than any other state, 414,812,000 bushels; Iowa produced the next greatest quantity, 343,900,000 bushels. What was the difference in yield in these two states?

6. $7500 is to be collected for a poor fund; at the beginning of a certain day $5225 had already been collected. During the day there were made contributions, as follows: $25, $15, $125, and $18. How much more is needed?

7. A farmer sold 4 loads of corn. The wagon and corn were first weighed and afterwards the wagon; find the weight of the corn in each load and fill in the totals:

Loads	Gross Weight	Weight of Wagon	Weight of Corn
First . . .	4370 lb.	1273 lb.	
Second . .	4558 lb.	1559 lb.	
Third . . .	4757 lb.	1268 lb.	
Fourth . .	4223 lb.	1236 lb.	
Total . .			

8. How many years are there between 365 years and 472 years? How many years from 365 years to 472 years (which includes the latter)? How many years from 365 years to 472 years inclusive (which includes both)?

Making Change

26. In making change it is customary to add to the amount of the sales enough change to make the sum of the sales and the change equal the money given in payment.

Thus, a man bought a basket for 35 cents and gave the merchant a dollar bill; the merchant, using the least number of pieces of money possible, gave the customer a 5-cent piece, a dime, and a half-dollar, and as he did so said, "*thirty-five, forty, fifty, one dollar.*"

Exercise 11

Count aloud the change in each of the following, using in each case as few pieces of money as possible:

	Sales	Money Paid	Change
1.	\$0.16	\$ 0.25	?
2.	\$0.39	\$ 0.50	?
3.	\$0.34	\$ 0.50	?
4.	\$0.45	\$ 1.00	?
5.	\$0.32	\$ 1.00	?
6.	\$1.18	\$ 1.50	?
7.	\$1.21	\$ 2.00	?
8.	\$2.85	\$ 4.00	?
9.	\$2.10	\$ 5.00	?
10.	\$2.25	\$10.00	?
11.	\$7.18	\$10.00	?
12.	\$2.56	\$10.00	?
13.	\$9.62	\$20.00	?

Multiplication

27. **Multiplication** is the process of taking one number or amount, called the **multiplicand**, additively as many times as there are units in another number, called the **multiplier**. The result of multiplication is called the **product**.

28. The sign of multiplication ($\times$) is read **times** when the multiplier is written before it, and **multiplied by** when the multiplier is written after it.

Thus, 4×7 days is read *4 times 7 days*, 7 days $\times$ 4 is read *7 days multiplied by 4*.

29. The multiplier is always a *number*, and if the multiplicand is an amount, the product must be a similar amount.

REMARK. It is evident from Sec. 29 that such expressions as "2 ft. $\times$ 3 ft. = 6 sq. ft.," "2 ft. $\times$ 5 sq. ft. = 10 cu. ft.," and "15° $\times$ 2 hr. = 30°," cannot be justified and should not be employed.

Exercise 12

Read the following; name the multiplicand, the multiplier, and give at sight the product:

1. 7×6 pk.	2. 7 bu. $\times$ 8
3. 8×12 doz.	4. 9 sq. ft. $\times$ 8
5. 11×12 units	6. 10×7 da.
7. 6×9 ft.	8. 12 doz. $\times$ 12
9. $10 \times \$12$	10. 8×8 qt.

Special Multipliers

30. 1. Multiply 48 by 10.

10×8 units $= 80$ units, or 8 tens.
10×4 tens $= 40$ tens, or 4 hundreds.
The product is 4 hundreds + 8 tens, or 480.

Annexing one cipher to a number multiplies it by 10.

2. Multiply 48 by 100.

$$100 \times 48 = 10 \times 10 \times 48$$
$$= 10 \times 480$$
$$= 4800$$

Annexing two ciphers to a number multiplies it by 100.

3. Multiply 48 by 1000.

$$1000 \times 48 = 10 \times 100 \times 48$$
$$= 10 \times 4800$$
$$= 48000$$

Annexing three ciphers to a number multiplies it by 1000.

4. Multiply 7 by 30.

$$30 \times 7 = 10 \times 3 \times 7$$
$$= 10 \times 21$$
$$= 210$$

In practice, multiply 7 by 3 and annex a cipher.

5. Multiply \$3 by 32.

$$32 \times \$3 = 3 \times \$32$$
$$= \$96$$

Exercise 13

Name at sight the product:

1. 10×7	2. 10×8	3. 10×6
4. 10×25	5. 10×48	6. 10×52

7. 10×216	8. 10×346	9. 10×428
10. 10×90	11. 10×150	12. 10×360
13. 100×5	14. 100×7	15. 100×4
16. 100×35	17. 100×82	18. 100×26
19. 1000×4	20. 1000×5	21. 1000×8
22. 1000×7	23. 1000×2	24. 1000×6
25. 40×6	26. 50×8	27. 70×5
28. 60×9	29. 90×4	30. 80×7
31. 200×8	32. 300×6	33. 200×9
34. $24 \times \$2$	35. $48 \times \$2$	36. $32 \times \$3$
37. $46 \times \$3$	38. $27 \times \$3$	39. $21 \times \$4$

31. Multiply 3582 by 346.

3582	**Multiplicand**	
346	**Multiplier**	The multiplier is $300 + 40 + 6$.
21492		$6 \times 3582 =$ what? (1st partial product)
14328		$40 \times 3582 =$ what? (2d partial product)
10746		$300 \times 3582 =$ what? (3d partial product)
1239372	**Product**	

In the form, what figure is omitted from the 2d partial product? In the form, what figures are omitted from the 3d partial product?

Special Forms

(*A*)	(*B*)	(*C*)	(*D*)
\$ 75.25	3286	4586	7600
37	304	3009	270
526 75	13144	41274	532
2257 5	9858	13758	152
\$ 2784.25	998944	13799274	2052000

Exercise 14

Find the product of:

1. 348×269	2. 287×430	3. 408×563
4. 456×607	5. 398×490	6. 563×489
7. 675×564	8. 578×731	9. 570×604
10. $513 \times \$21.34$	11. $465 \times \$30.25$	12. $324 \times \$42.50$
13. 639×599	14. 667×708	15. 723×405
16. 704×598	17. 780×509	18. 820×564
19. 865×697	20. 996×670	21. 878×703
22. 569×4508	23. 640×6892	24. 753×7084
25. 4306×2398	26. 3097×5680	27. 4670×6709
28. 5073×4987	29. 6708×4538	30. 5007×6832
31. $8135 \times \$60.08$	32. $7830 \times \$65.09$	33. $6998 \times \$72.15$

Multiply across; add upward; check by seeing that the sum of the products above the line is equal to the product under it:

34. $45 \times 233 = ?$	35. $67 \times 312 = ?$	36. $89 \times 219 = ?$
$45 \times 418 = ?$	$67 \times 183 = ?$	$89 \times 326 = ?$
$45 \times 165 = ?$	$67 \times 287 = ?$	$89 \times 193 = ?$
$45 \times 128 = ?$	$67 \times 368 = ?$	$89 \times 297 = ?$
$45 \times \ ? \ = ?$	$67 \times \ ? \ = ?$	$89 \times \ ? \ = ?$

Illustrative Examples

1. A printing press that prints 295 copies of a paper in a minute will print how many copies in 9 minutes?

295 = the number of copies printed in 1 minute.

9

2655 = the number of copies printed in 9 minutes.

2. Find the cost of 375 tons of coal at $6 a ton.

SUGGESTION. $375 \times \$6 = 6 \times \$375$.

Exercise 15

1. At $175 an acre, what will a farm of 250 acres cost?

2. How much tuition is paid in a year by 900 students, if each pays $250?

3. How many pounds are there in 35 bales of cotton, each weighing 500 lb.?

4. What is the cost of 275 tons of coal at $5 a ton?

5. Light moves at the rate of 186,000 miles a second; how far will it move in 12 seconds?

6. A contractor built 24 houses and sold them for $3250 each; what did he receive for them?

7. If sound travels 1090 feet per second and you see the flash of a gun 6 seconds before you hear the sound, how far are you from the gun?

8. If one pipe discharges 40 gallons of water per hour, and another 86 gallons per hour; how much more will the second discharge than the first in 24 hours?

9. A man bought a farm of 80 acres at $58 an acre and built a house on it at a cost of $3250; he then sold the farm for $9000. What was the profit?

10. The following is a good ration for a cow:

	Ration
Corn silage	30 lb. per day
Red clover hay	16 lb. per day
Corn	6 lb. per day
Cottonseed meal	1 lb. per day

How many pounds of each of the food products given in the ration are required to keep 10 cows one week?

11. If I gave in payment for the following purchases a $2 bill, a quarter, a dime, and two nickels, how much change should I receive?

4 lb. of butter at $0.32 = $
2 lb. of cheese at $0.16 = $
3 doz. eggs at $0.23 = $
1 qt. of molasses at $0.12 = $
$

Division

32. **Division** is the process of finding one of the equal parts of a number or an amount, or how many times one number or amount is contained in another.

33. The **dividend** is the number or amount divided. The **divisor** is the number or amount by which we divide. The **quotient** is the result of the division.

34. The sign of division is ÷, read *divided by*.

Thus, $24 \div 4 = 6$, is read *24 divided by 4 = 6*.

35. If in the division there is an undivided part of the dividend less than the divisor, this undivided part is called the **remainder**.

Thus, $27 \div 4 = 6$, with a *remainder* of 3 undivided. If the division were completed, the quotient would be $6\frac{3}{4}$ with *no remainder*.

36. Finding one of the equal parts of a number is sometimes called **partitive division**, or **partition**. In this kind of division the divisor is always a number, and if the dividend is an amount, the quotient and the remainder must be amounts similar to the dividend.

Thus, $\$60 \div 5 = \$12$.

37. Finding how many times one number or amount is contained in another is sometimes called **comparative division.** In this kind of division the quotient is always a number, and if the dividend is an amount, the divisor and the remainder must be similar amounts.

Thus, 20 yd. ÷ 2 yd. = 10. *In practice* divide 20 by 2.

REMARK. It is evident from Secs. 36 and 37 that such expressions as "45° ÷ 15 = 3 hr.," "6 sq. ft ÷ by 2 ft. = 3 ft.," and "12 cu. ft. ÷ 4 sq. ft. = 3 ft." cannot be justified and should not be employed.

Exercise 16

Name at sight the quotient and the remainder:

1. 49 ÷ 6	2. 57 ÷ 7	3. 60 ÷ 9
4. 58 ÷ 7	5. 52 ÷ 8	6. 70 ÷ 8
7. 83 ÷ 9	8. 67 ÷ 8	9. 54 ÷ 7
10. 74 ÷ 8	11. 59 ÷ 9	12. 73 ÷ 8
13. 82 ÷ 9	14. 75 ÷ 8	15. 68 ÷ 7
16. 86 ÷ 9	17. 77 ÷ 8	18. 89 ÷ 9
19. 71 ÷ 9	20. 85 ÷ 9	21. 78 ÷ 8

22. $50 ÷ 7	23. 52 bu. ÷ 5
24. 61 qt. ÷ 8 qt.	25. 55 ft. ÷ 6
26. 60 yd. ÷ 8 yd.	27. 53 da. ÷ 7 da.
28. 68 men ÷ 8	29. $46 ÷ $8
30. 78 men ÷ 9 men	31. 62 trees ÷ 9 trees
32. 59 in. ÷ 6 in.	33. 87¢ ÷ 9

Special Divisors

38. 1. Since 10 × 46 = 460, 460 ÷ 10 = 46, or 46Ø.

Cutting one cipher from the right of a number divides the number by 10.

2. Since $100 \times 46 = 4600$, $4600 \div 100 = 46$, or $46\emptyset\emptyset$.

Cutting two ciphers from the right of a number divides the number by 100.

3. Since $1000 \times 46 = 46000$, $46000 \div 1000 = 46$, or $46\emptyset\emptyset\emptyset$

Cutting three ciphers from the right of a number divides the number by 1000.

4. Since $30 \times 7 = 210$, $210 \div 30 = 7$; that is, $21\emptyset \div 3\emptyset = 7$.

In practice, cut off the same number of ciphers from the right of both dividend and divisor before dividing.

Exercise 17

Name at sight the quotient:

1. $230 \div 10$	2. $350 \div 10$	3. $280 \div 10$
4. $390 \div 10$	5. $560 \div 10$	6. $470 \div 10$
7. $420 \div 10$	8. $670 \div 10$	9. $580 \div 10$
10. $300 \div 100$	11. $600 \div 100$	12. $900 \div 100$
13. $2400 \div 100$	14. $3500 \div 100$	15. $4300 \div 100$
16. $6700 \div 100$	17. $5800 \div 100$	18. $8900 \div 100$
19. $4000 \div 1000$	20. $7000 \div 1000$	21. $8000 \div 1000$
22. $36{,}000 \div 1000$	23. $42{,}000 \div 100$	24. $76{,}000 \div 1000$
25. $40 \div 20$	26. $60 \div 30$	27. $80 \div 20$
28. $240 \div 20$	29. $630 \div 30$	30. $840 \div 40$
31. $3600 \div 300$	32. $1800 \div 200$	33. $1500 \div 300$
34. $2700 \div 300$	35. $2400 \div 400$	36. $3500 \div 500$
37. $3000 \div 30$	38. $3600 \div 600$	39. $4900 \div 700$
40. $5600 \div 800$	41. $7200 \div 800$	42. $6300 \div 900$
43. $5400 \div 600$	44. $7200 \div 900$	45. $8100 \div 900$

Long Division

39. 1. Divide 89,429 by 324.

	276	**Quotient**
Divisor 324	89429	**Dividend**
	648	
	2462	
	2268	
	1949	
	1944	
	5	**Remainder**

8 ten thousands ÷ 324 = how many ten-thousands?

89 thousands ÷ 324 = how many thousands?

894 hundreds ÷ 324 = how many hundreds?

324 × 2 hundreds = what?

894 hundreds − 648 hundreds = what?

246 hundreds + 2 tens = how many tens?

2462 tens ÷ 324 = how many tens?

324 × 7 tens = what?

2462 tens − 2268 tens = what?

194 tens + 9 units = how many units?

1949 units ÷ 324 = how many units?

324 × 6 units = how many units?

1949 units − 1944 units = what?

Check: 324 × 276 + 5 = 89429.

NOTE. The quotient may be placed to the right of the dividend thus: 324) 89429 (276

2. Divide 654,282 by 326.

2007	
326)654282	
652	
2282	
2282	

654 thousands ÷ 326 = what?

654 thousands − 652 thousands = what?

2 thousands + 2 hundreds = how many hundreds?

22 hundreds ÷ 326 = how many hundreds?

22 hundreds + 8 tens = how many tens?

228 tens ÷ 326 = how many tens?

228 tens + 2 units = how many units?

2282 units ÷ 326 = how many units?

Check: 326 × 2007 = 654282

40. To check the result in division, find the product of the divisor and quotient and to it add the remainder (if any); the result should be the dividend.

NOTE. The method of dividing by expressing the successive partial dividends is called **long division**. The method of dividing without expressing the partial dividends is called **short division**.

Special Form

41. Divide 726 by 200.

This shows the complete work:

$$200\overline{)726} = 200\overline{)600+126} \qquad 3+\tfrac{126}{200}$$

In practice write only this:

$$200\overline{)726} \qquad 3\tfrac{126}{200}$$

Exercise 18

Divide and check:

1. 2176 by 32
2. 4462 by 46
3. 4984 by 56
4. 5378 by 89
5. 1005 by 67
6. 7569 by 87
7. 7560 by 35
8. 17,325 by 25
9. 16,555 by 43
10. 33,761 by 49
11. 65,142 by 77
12. 34,970 by 26
13. 142,196 by 38
14. 383,614 by 47
15. 182,790 by 45
16. 13,212 by 48
17. 20,145 by 56
18. 37,604 by 63
19. 48,325 by 95
20. 40,106 by 87
21. 41,001 by 97
22. 142,272 by 416
23. 111,616 by 218
24. 211,932 by 609
25. 39,552 by 309
26. 477,006 by 642
27. 586,704 by 816
28. 480,198 by 489
29. 34,968 by 215
30. 236,418 by 327

31. 485 by 10
32. 2675 by 100
33. 7568 by 1000
34. 756 by 20
35. 789 by 30
36. 896 by 80
37. 7568 by 200
38. 986 by 300
39. 10,096 by 300
40. 8642 by 400
41. 9587 by 5000
42. 76,832 by 800

Divide across; add upward; check by seeing that the sum of the quotients above the line is equal to the quotient obtained by dividing the sum of the dividends by the indicated divisor:

43.
$$\begin{array}{r} 4489 \div 67 = ? \\ 6767 \div 67 = ? \\ \underline{14070 \div 67 = ?} \\ ? \quad \div 67 = ? \end{array}$$

44.
$$\begin{array}{r} 5785 \div 89 = ? \\ 8099 \div 89 = ? \\ \underline{4183 \div 89 = ?} \\ ? \quad \div 89 = ? \end{array}$$

45.
$$\begin{array}{r} 11400 \div 456 = ? \\ 42408 \div 456 = ? \\ 35568 \div 456 = ? \\ \underline{29184 \div 456 = ?} \\ ? \quad \div 456 = ? \end{array}$$

46.
$$\begin{array}{r} 366768 \div 849 = ? \\ 722499 \div 849 = ? \\ 515343 \div 849 = ? \\ \underline{248757 \div 849 = ?} \\ ? \quad \div 849 = ? \end{array}$$

Illustrative Examples

1. If 45 beams of equal weight weigh 16,200 lb., how much does each beam weigh?

$$\begin{array}{r} 360 \text{ lb.} = \text{the weight of each.} \\ \hline 45)16200 \text{ lb.} = \text{the total weight.} \\ 135 \\ \hline 270 \\ 270 \\ \hline 0 \end{array}$$

2. A miller shipped 9800 lb. of flour in barrels; how many barrels did he ship, if one barrel weighs 196 lb.?

9800 lb. = the weight of the flour shipped.
196 lb. = the weight of each barrel.
50 = the number of barrels.

$$196\overline{)9800}$$
$$980$$
$$0$$

Exercise 19

1. If 12 acres of land produced 288 bu. of wheat, what was the yield per acre?

2. 84 town lots are valued at $44,184. If the lots are of equal value, what is the value of each?

3. A farmer sold his farm, consisting of 85 acres, for $5100; how much was that an acre?

4. A teamster hauled 1190 rails in 14 loads; how many was that to the load?

5. Wheat weighs 60 lb. to the bushel. A farmer's wheat crop weighed 10,600 lb. How many bushels had he and how many pounds over?

6. In how many months will a clerk who saves $25 a month save enough to pay for a $4000 house?

7. A mail carrier who travels 24 miles per day travels a distance of 12,000 miles in how many days?

8. What is the average weight of three steers whose weights are: 675 lb., 832 lb., and 902 lb.?

REMARK. The **average** of two or more numbers is found by dividing their sum by the number of numbers added.

Thus, the average of 8, 12, and 16 is the sum of 8, 12, and 16, or 36, divided by 3, or 12.

9. What is the average age of three men, whose ages are 60 yr., 75 yr., and 81 yr.?

10. What is the average age of four children if two of them are each 9 yr. old, one of them is 8 yr. old, and one of them 6 yr. old?

11. Three loads of corn weigh 3846 lb., 3978 lb., and 4011 lb.; what is the average weight?

12. A man invested $11,220 in land at $85 an acre. How many acres did he buy?

13. The distance around a certain wheel is 3 ft.; how many times will it turn in going 5280 ft.?

14. A drover bought young mules at an average of $85 a head and sold them at $100 a head; if he gained $900, how many mules did he sell?

15. If 75 acres of land cost $3000, what should 12 acres cost at the same price per acre?

16. If the quotient of two numbers is 125 and the divisor is 25, what is the dividend?

17. The number of square miles in Texas is 265,780, and the number in Ireland is 32,360. How many countries the size of Ireland would Texas make, and how many square miles over?

18. If 21 men earn $210 in 5 days, how much does each man earn per day?

19. When a certain number is divided by 12, the quotient is 144 and the remainder 5; what is the number?

20. A carload of coal weighed 67,500 lb.; how many tons (2000 lb.) will this make and how many pounds over?

Review of Fundamental Processes

Exercise 20

1. Find the sum of $1\frac{1}{2}$ million and 1 hundred thousand.

2. How many $1000 bills make 1 million dollars?

3. Counting at the rate of $100 per minute, how many minutes would be required, counting continuously, to count $1,000,000 in $1 bills?

4. How much change should be received from a $2 bill given in payment for 12 oranges at 3¢ apiece?

5. Five men have equal interest in a tract of 500 acres of land which is sold at $30 an acre; how much should each receive from the sale, after paying an agent $200 for selling it?

6. If a girl pastes 12 samples on a card and finishes 60 cards in a day, how many samples does she paste in a day? In how many days can she paste 1440 samples?

7. Divide 2 million 500 thousand by 25 thousand.

8. In the average coal train of the Pennsylvania Railroad Company there are 67 cars, each containing about 28 tons (2240 lb.) of coal. Estimate how many pounds of coal there are in one of these train loads.

9. Find the amount due a woman who sold and bought as follows:

Sold		Bought	
28 doz. eggs	@ 30¢	12 yd. of gingham	@ 18¢
18 lb. butter	@ 32¢	2 table covers	@ $2.50
25 lb. poultry	@ 14¢	12 yd. of toweling	@ 5¢
42 lb. lard	@ 7¢	20 yd. of muslin	@ 14¢

10. Two men commenced business with $500 each. During the year one gained $275 and the other lost $56. How much more had the one then than the other?

11. It requires 7260 plants for an acre of ground when they are set in rows three feet apart and set two feet apart in the rows; how many plants would it take for three acres, set in this way?

12. A man bought a farm for $7850. He spent $175 for improvements on it, and then sold it for $9000. What was his profit?

13. If the minuend is 2756 and the difference 596, what is the subtrahend?

14. If the subtrahend is 7450 and the difference is 1896, what is the minuend?

15. How many poor families can be supplied with 48 barrels of flour, each barrel weighing 196 lb., if 24 lb. are given to each family?

16. Five tons of coal, of 2000 lb. each, were distributed among some poor families, giving each 200 lb.; how many families received coal?

17. If a girl can paste 100 samples an hour, how much will she earn in 6 hours if she is paid 3¢ for each card of samples pasted, 24 samples to the card?

18. A bought a drove of cattle for $8150 and sold it for $10,750. There were 100 cattle in the drove. What was the average gain per head?

19. In a certain city the average age of the citizens above 70 years of age is 76 years. The sum of their ages is 3572 years. How many of these citizens are there?

20. Which is nearer a million dollars, and by how much, $1,100,000 or $900,009?

21. A merchant who spent $225, bought 65 lb. of butter at 30¢ per pound, 84 barrels of apples at $2.25 per barrel, and spent the remainder for coffee; how much did he spend for coffee?

22. During the month of August 450,000 bushels of wheat were shipped from a certain port. During September 87,960 more bushels were shipped than during August. What was the total number of bushels shipped during the two months?

23. $1,000,000 is how many times $100,000?

24. A laborer who saves $20 a month must labor how many years to pay for 60 acres of land at $40 an acre?

25. If the dividend is 1728 and the divisor 144, what is the quotient?

26. A bushel of loose lime weighs 70 lb.; how many bushels should a man load who wishes to haul 2800 lb.?

27. Powder weighs 25 lb. per keg; how many pounds are there in a shipment of 2500 kegs?

28. A gardener sold in one season 675 bunches of onions at an average price of 9¢ a bunch; how much did he receive for all?

29. A bushel of dried apples weighs 26 lb.; what is the weight of 75 bu.?

30. The average wages paid to men working on farms in Pennsylvania, in a recent year, were $17.60 per month with board, or $27.18 without board; how much is the board estimated at per year?

31. The average monthly wages paid to farm laborers in Maine, in a recent year, were $23.17 with board; in Wisconsin, $24.39; what was the difference in the yearly wages of two laborers, one working in each state?

32. There are 60 seconds in a minute, 60 minutes in an hour, and 24 hours in a day; how many seconds are there in a day?

33. A farmer's corn crib held 1026 bu. of corn; his corn crop filled the crib, and he had 126 bu. over. What was his corn crop?

34. Recently it was estimated that each person in Philadelphia used 210 gal. of water every 24 hr., but that each person should use not more than 150 gal. in that time, the additional amount being wasted. Estimating the population of Philadelphia at 1,600,000, how much water did they waste every 24 hr.?

35. A farmer was paid for a horse 9 ten-dollar bills, 6 five-dollar bills, 8 two-dollar bills, and 14 dollar bills. How much was he paid for the horse?

36. A merchant was given a 20-dollar bill in payment of a debt of $2. What bills did he give in change, if he used the least number possible?

37. A farm that cost $50 per acre was sold for $65 per acre. The gain being $1125, how many acres did the farm contain?

38. How long will 4 men, each husking an average of 40 shocks of corn a day, require to husk 4800 shocks?

39. A man owed $11.26. He gave in payment $15. Nine pieces of money were given him as correct change; what were they?

ARITHMETICAL EXPRESSIONS

42. An **arithmetical expression,** or simply an **expression,** is any number or combination of numbers expressed in arithmetical symbols.

Thus, *5*, $3+2+4$, $6-2+1$, 2×3, *and* $2\times 6\div 3$ are expressions.

43. Parts of an expression connected by + or − are called **terms** of the expression.

Thus, in the expression $4\times 2+7-6\div 3$ there are three terms; the first term is 4×2, the second 7, and the third $6\div 3$.

44. Before the terms of an expression are combined, the operations of multiplication and division indicated in any term must be performed; and if two or more such operations are indicated in the same term, they should be performed in the order of their occurrence from left to right.

Thus, in the expression $6\times 8\div 4+12\div 6\times 2-24\div 4\div 2$,
the first term, $6\times 8\div 4=48\div 4$, or 12,
the second term, $12\div 6\times 2=2\times 2$, or 4,
the third term, $24\div 4\div 2=6\div 2$, or 3,
the whole expression $=12+4-3$, or 13.

45. The **value** of an expression is the result obtained by performing the operations indicated.

Thus, the value of $12\div 4\times 3$ is 9.

46. The **parentheses** ( ) indicate that the value of the expression inclosed by them is to be found and used instead of the expression as a whole.

Thus, $5\times(7-3)$ denotes that the difference of 7 and 3, or 4, is to be multiplied by 5.

47. The brackets [ ], the braces { }, or the vinculum $\overline{\quad}$, may be used instead of the parentheses.

Exercise 21

Find the value of:

1. $4+3+5-1-3\times 2$
2. $6\times 2+5+4-2\times 3$
3. $9\div 3+6\times 2+3$
4. $24\div 8\times 3-6\div 2$
5. $35\div 7\times 5+5\times 2$
6. $32\div 4+4\times 6\div 2$
7. $\overline{3+2}\times 5+\overline{6-3}\times 2$
8. $\overline{6-4}\times\overline{3+2}\div\overline{5\times 2}$
9. $(12+4)\div(4\times 2)\times(5+1)$
10. $(20-15)\times(10-10\div 5)\times(6-6\div 2)$
11. $(100+44)\div 12+(20+44)\div 8-(25+5)\div 5$
12. $6\times(9+3)-(120-20)\div 10+6\times 2\times(3+2)$
13. $(320-160)\div 8-(324-248)\div 4+5\times(720-640)$
14. $12\times(350-280)\div 6-(540-480)\div 15$
    $+(412-216)\times(24-18)$
15. $(36+48)\times(48-36)-(120-84)\div(18-6)$
    $+(140-70)\div(70-35)$

FACTORS AND DIVISORS

48. The **factors** of a number are such numbers as multiplied together will produce the number.

Thus, the factors of 15 are 3 and 5.

49. A number that has no other factors than itself and 1 is a **prime number**; as, $5=5\times 1$.

50. A number composed of other factors than itself and 1 is a **composite number**; as, $15=5\times 3$.

51. The same factor may be used more than once in forming a composite number. Each time it is so used it is considered a separate factor.

Thus, $36 = 2 \times 2 \times 3 \times 3$; here the factors of 36 are shown to be 2, 2, 3, and 3.

52. When all the factors of a number are prime numbers, they are called the **prime factors** of the number.

Thus, in $12 = 2 \times 2 \times 3$, *2*, *2*, and *3* are the prime factors of 12.

53. An **even** number is a number that is divisible by 2.

54. An **odd** number is a number that is not divisible by 2.

Exercise 22

Name two factors that will produce:

1. 6; 10; 15; 18.
2. 8; 9; 16; 25.
3. 14; 24; 21; 36.
4. 28; 52; 49; 50.
5. 56; 48; 63; 72.
6. 81; 54; 90; 110.

Name three factors that will produce:

7. 12; 8; 27; 20.
8. 30; 28; 50; 42.

Tell which of the following numbers are prime:

9. 1; 2; 5; 8; 11; 14; 21.
10. 3; 6; 9; 12; 15; 18; 23.
11. 4; 7; 10; 13; 16; 20; 25.
12. Write all the prime numbers that are less than 20.
13. Write all the composite numbers that are less than 20.

14. Find the prime factors of 18.

Thus, $18 = 2 \times 3 \times 3$.

Find the prime factors of:

15. 10	16. 12	17. 15	18. 20
19. 25	20. 30	21. 16	22. 36
23. 48	24. 50	25. 54	26. 60
27. 45	28. 40	29. 24	30. 32

31. Name the even numbers to 24.

32. Name the odd numbers to 25.

55. A number that will divide a given number without a remainder is an **exact divisor,** or simply a **divisor,** of the given number.

Thus, 5 is a divisor of 10, but not of 11.

56. A divisor of each of two or more numbers is called a **common divisor** of them.

Thus, 5 is a common divisor of 10 and 20.

57. The greatest number that will divide each of two or more numbers is called their **greatest common divisor** (g. c. d.).

Thus, 5 is the greatest common divisor of 10, 15, and 20.

58. Some easy tests of divisibility:

1. A number is divisible by 2 if the units' figure is 0, 2, 4, 6, or 8.

Thus, 68 is divisible by 2, since the units' figure is 8.

2. A number is divisible by 5 if the units' figure is 0, or 5.

Thus, 105 is divisible by 5, since the units' figure is 5.

3. A number is divisible by 4 if the number expressed by the two right-hand figures is divisible by 4.

Thus, 124 is divisible by 4, since 24 is divisible by 4.

4. A number is divisible by 3 if the number expressed by the sum of its digits is divisible by 3.

Thus, 126 is divisible by 3, since $1+2+6$, or 9, is divisible by 3.

5. A number is divisible by 9 if the number expressed by the sum of its digits is divisible by 9.

Thus, 765 is divisible by 9 since $7+6+5$, or 18, is divisible by 9.

59. If a number is divided by one of its prime divisors, the quotient by one of its prime divisors, the next quotient by one of its prime divisors, and so on until the quotient becomes 1, these successive divisors are called the **continued prime divisors** of the number.

3	105
5	35
7	7
	1

Thus, 3, 5, and 7 are the continued prime divisors of 105.

60. The continued prime divisors of a number are the prime factors of the number.

3	105
5	35
7	7
	1

Thus, 3, 5, and 7, the continued prime divisors of 105, are the prime factors of 105.

For, $105 = 3 \times 35$; and $35 = 5 \times 7$.

Therefore, $105 = 3 \times 5 \times 7$.

61. When the prime factors of a number cannot readily be seen, they may be found thus:

2	1050
5	525
5	105
3	21
	7

That is, the prime factors of 1050 are 2, 5, 5, 3, and 7.

62. The continued common prime divisors of two or more numbers are their common prime factors.

2	90	120	210
5	45	60	105
3	9	12	21
	3	4	7

Thus, 2, 5, and 3, the continued common prime divisors of 90, 120, and 210, are the common prime factors of 90, 120, and 210.

63. Any of the common prime factors of two or more numbers, or the product of any two or more of such factors, is a common divisor of the numbers.

Thus, in Section 62, the common prime factors of 90, 120, and 210 are seen to be 2, 5, and 3. Hence 90, 120, and 210 are each divisible by 2, 5, 3, 2×5, 2×3, 5×3, and $2 \times 5 \times 3$.

64. The product of all the common prime factors of two or more numbers is their greatest common divisor.

Thus, the greatest common divisor of 90, 120, and 210 is $2 \times 5 \times 3$.

Illustrative Example

Find the greatest common divisor of 294, 378, and 462.

2	294	378	462
3	147	189	231
7	49	63	77
	7	9	11

2, 3, and 7 are the common prime factors of 294, 378, and 462, since they are the continued common prime divisors of them.

Therefore, $2 \times 3 \times 7$, or 42, is the greatest common divisor of 294, 378, and 462.

Exercise 23

Find the prime factors of:

1. 156	2. 182	3. 280	4. 630
5. 735	6. 765	7. 1001	8. 3150

Find all the common divisors of:

9. 30, 42, and 56. 10. 50, 70, and 110.
11. 60, 105, and 120. 12. 66, 88, and 110.
13. 60, 90, and 210. 14. 140, 210, and 350.

Find the greatest common divisor of:

15. 56 and 72. 16. 70 and 98. 17. 63 and 91.
18. 90 and 108. 19. 75 and 175. 20. 112 and 168.
21. 42, 49, and 56. 22. 40, 56, and 72.
23. 70, 80, and 90. 24. 52, 78, and 104.
25. 245, 280, and 315. 26. 288, 384, and 528.
27. 432, 540, and 648. 28. 384, 576, and 768.

29. Three pieces of lead, weighing respectively 24 oz., 32 oz., and 40 oz., were cast without waste into balls of equal weight and as heavy as possible. What was the weight of each ball?

30. What is the length of the longest board that can be used, without waste, to build a fence around a lot which is 60 ft. long and 48 ft. wide?

MULTIPLES

65. A number that a given number will exactly divide is called a **multiple** of the given number.

Thus, 6 is a multiple of 2, since 2 will exactly divide 6.

66. A number that each of two or more numbers will exactly divide is called a **common multiple** of them.

Thus, 12 is a common multiple of 2 and 3, since 2 and 3 will each exactly divide 12.

67. The least number that each of two or more numbers will exactly divide is called their **least common multiple** (l. c. m.).

Thus, 6 is the least common multiple of 2 and 3, since 6 is the least number that 2 and 3 will exactly divide.

Exercise 24

1. Name a multiple of 2; of 3; of 5; of 7.
2. Name three multiples of 2.
3. Name three multiples of 3.
4. Name two multiples of 7.
5. Name a common multiple of 3 and 4.
6. Name two common multiples of 2 and 3.
7. Name two common multiples of 4 and 5.
8. Name two common multiples of 2, 3, and 4.
9. Name a common multiple of 3, 4, and 6.

Name the least common multiple of:

10. 2 and 5.
11. 6 and 9.
12. 6 and 8.
13. 2, 3, and 4.
14. 2, 4, and 6.
15. 5, 10, and 15.
16. 4, 6, and 8.
17. 3, 6, and 9.
18. 2, 3, 4, and 6.

Illustrative Example

Find the least common multiple of 60, 90, and 210.

$60 = 2 \times 2 \times 3 \times 5$
$90 = 2 \times 3 \times 5 \times 3$
$210 = 2 \times 3 \times 5 \times 7$

The least common multiple of 60, 90, and 210 must contain the prime factors 2, 2, 3, and 5 in order to contain 60; it must contain the additional prime factor 3, in order to contain 90, and the additional prime factor 7, in order to contain 210.

Therefore, $2 \times 2 \times 3 \times 5 \times 3 \times 7$, or 1260 = l. c. m. of 60, 90, and 210.

The least common multiple of two or more numbers is the product of all their different prime factors each taken the greatest number of times it occurs in any one of them.

Exercise 25

Find the least common multiple of:

1. 28 and 42. 2. 84 and 96. 3. 63 and 84.
4. 66 and 99. 5. 56 and 120. 6. 75 and 125.
7. 52 and 130. 8. 105 and 135. 9. 154 and 242.
10. 45, 75, and 90. 11. 52, 78, and 104.
12. 30, 90, and 150. 13. 56, 70, and 84.
14. 110, 275, and 605. 15. 65, 91, and 143.

16. What is the least sum of money that can be put up in packages of either $ 20 bills or $ 50 bills?

17. What is the least weight of hay that can be put up in either 100 lb. bales or 125 lb. bales?

18. What is the least debt that can be paid in either $ 2 bills, $ 5 bills, or $ 10 bills?

19. What is the least number of children that can be formed in groups of 8, 12, or 20, and have none remaining?

CANCELLATION

68. Cancellation is the process of crossing out a number and writing the result obtained by multiplying or dividing it.

Important Principles

69. *Multiplying one number and dividing another by the same number does not change their product.*

Thus, $6 \times 10 = 60$, and $(6 \times 2) \times (10 \div 2)$, or $12 \times 5 = 60$.

70. *Multiplying both dividend and divisor by the same number does not change the quotient.*

Thus, $12 \div 3 = 4$, and $(5 \times 12) \div (5 \times 3)$, or $60 \div 15 = 4$.

71. *Dividing both dividend and divisor by the same number does not change the quotient.*

Thus, $18 \div 6 = 3$, and $(18 \div 3) \div (6 \div 3)$, or $6 \div 2 = 3$.

Illustrative Examples

1. Find the product of 25 and 48.

Thus, $\overset{100}{\cancel{25}} \times \overset{12}{\cancel{48}} = 1200$ — 25 has been multiplied and 48 divided by what number?

2. Divide 415 by 5.

Thus, $\overset{830}{\cancel{415}} \div \cancel{5} = 83$ — 415 and 5 have each been multiplied by what number?

3. Divide 144 by 24.

Thus, $\overset{36}{\cancel{144}} \div \cancel{24} = 6$ — 144 and 24 were each divided by what number?

4. Find the value of $\dfrac{6 \times 9 \times 25}{10 \times 3}$, read *6 times 9 times 25, divided by 10 times 3.*

Thus, $\dfrac{\overset{2}{\cancel{6}} \times 9 \times \overset{5}{\cancel{25}}}{\underset{5}{\cancel{10}} \times \cancel{3}} = 45$

We cancel the 3 in the divisor, and the 3 from the 6 in the dividend, writing the quotient 2. We next cancel the 2 in the dividend and the 2 from the 10 in the divisor, writing the quotient 5. Then we cancel 5 from the divisor and 5 from the 25 in the dividend, writing the quotient 5. There then remain the factors 9 and 5 in the dividend; their product, 45, is the value of the expression.

5. A laborer who saves $20 a month must save for how many years to have sufficient to pay for 40 acres of land at $60 an acre?

$12 \times \$20$ = the amount the laborer saves each year.
$40 \times \$60$ = the cost of the land.

$\frac{\cancel{40}^{2} \times \cancel{60}^{5}}{\cancel{12} \times \cancel{20}}$, or 10 = the number of years required.

Exercise 26

Find the result by cancellation:

1. 60×25
2. 35×500
3. 28×50
4. 16×250
5. $132 \div 22$
6. $144 \div 36$
7. $140 \div 28$
8. $180 \div 45$
9. $3 \times 5 \times 8$ divided by 30
10. $6 \times 9 \times 12$ divided by 36
11. $5 \times 9 \times 14$ divided by 15×7
12. $\frac{16 \times 10 \times 6}{24 \times 20}$
13. $\frac{3 \times 14 \times 18}{21 \times 9}$
14. $\frac{15 \times 6 \times 12}{27 \times 10}$
15. $\frac{4 \times 27 \times 28}{12 \times 7 \times 18}$
16. $\frac{6 \times 8 \times 24}{12 \times 16 \times 3}$
17. $\frac{5 \times 8 \times 18}{2 \times 15 \times 12}$
18. $\frac{7 \times 26 \times 18}{21 \times 39 \times 2}$
19. $\frac{9 \times 8 \times 12 \times 15}{3 \times 16 \times 10}$
20. $\frac{7 \times 35 \times 16 \times 8}{28 \times 5 \times 14}$
21. $\frac{9 \times 27 \times 24 \times 40}{36 \times 18 \times 15}$
22. $\frac{8 \times 14 \times 20 \times 9}{16 \times 35 \times 12}$
23. $\frac{5 \times 26 \times 18 \times 4}{39 \times 15 \times 8}$

24. A farmer exchanged 18 pounds of butter, at 20 cents a pound, for cloth at 12 cents a yard; how many yards did he obtain?

25. A quantity of corn consisting of 273 bushels, each bushel weighing 56 pounds, was hauled in 6 loads; what was the average weight of the loads?

26. How many years would it require a laborer who saves 15 dollars a month for 12 months in a year, to pay for 54 acres of land at 50 dollars an acre?

27. The yield of wheat from a 10-acre field is estimated at 30 bushels per acre; if a bushel of wheat weighs 60 pounds, how many tons of 2000 pounds each are there in the yield?

GENERAL REVIEW

Exercise 27

1. Beginning at billions, name the periods to units.
2. Name the orders in units' period.
3. Name the orders in millions' period.
4. Name the orders in thousands' period.
5. How many ciphers are required to write one billion?
6. Counting from the right, what is the third period?
7. Name the number made up of 267 units of the second period.
8. Name the number made up of seven units of the third order.
9. Name the number made up of 46 units of the third period.

10. Name the number made up of 5 units of the sixth order.

11. How many thousand make a million?

12. How many million make a billion?

13. In the number 276,853 tell what number each figure represents.

14. In the number 7,840,009,060 what does the 7 represent? the 840? the 009? the 060?

15. What is the name of 1000 billion?

16. Write the largest number possible that contains all of the letters I, V, X, C, each used but once in writing the number; write the smallest number possible.

17. Write the largest number possible that contains all of the letters L, C, D, M, each used but once in writing the number; write the smallest number possible.

18. Fill in the totals in the following sales of grain made by a farmer in one year:

176 bu. of wheat	@	$1.10	= ?
516 bu. of oats	@	.50	= ?
78 bu. of rye	@	.76	= ?
28 bu. of buckwheat	@	.85	= ?
Selling price of ? bu. of grain			= ?

19. If a man whose income is $175 a month spends $780 during the year, how much does he save that year?

20. What will it cost to construct a highway for a distance of 5280 ft. at $1.50 per foot?

21. How many days are there in 4 common years (365 da.) and 1 leap year (366 da.)?

22. Divide at sight:

2500 by 10; 7500 by 100; 25,000 by 1000.

23. How many tens are there in 9,000,000? how many hundreds? how many thousands? how many ten-thousands? how many hundred-thousands?

24. Divide at sight each of the following numbers by 1000, stating the quotient and the remainder:

1111; 2760; 4723; 6400; 7913; 8001.

25. After borrowing $175, a farmer had sufficient money to pay for 25 acres of land at $50 per acre. How much had he at first?

26. If two trains are running from a certain point, one due north at the rate of 25 miles an hour, and the other due south at the rate of 40 miles an hour, how far apart are they in 3 hours?

27. A laborer bought a house for $1850; he paid $800 cash and agreed to pay the balance in monthly payments of $15 each. How many monthly payments must he make?

28. A carload of 35 trolley rails, each 56 ft. long and weighing 45 lb. to the foot, weighs how much?

29. The farm land in the United States, exclusive of buildings, was valued at $13,058,008,000 in 1900 and $28,386,770,000 in 1910. What was the average increase in value for each of the 10 years?

30. Recently it was estimated that 40,744,200 gal. of water was furnished each 24 hr. to 373,800 persons in Milwaukee. How much was furnished each person?

31. The freight rate on electrical goods shipped first-class from Chicago to Portland, Ore., is $3 per 100 pounds.

Find the freight on a shipment of 2600 pounds of these goods.

32. Find the sum of all the numbers that are greater than 999,996 and less than 1,000,003.

33. The divisor is 46, the quotient 19, and the remainder 25. What is the dividend?

34. A load of corn weighed 5628 lb. Allowing 25 lb. for the weight of the bags, how many bushels (56 lb.) did the load contain, and how many pounds over?

35. The minuend is 274 and the subtrahend is 196. What is the difference?

36. Counting 52 weeks to a year, which is the more pay for a year, and how much, \$ 12 a week or \$ 50 a month?

37. A farmer's load of oats amounted to 67 bu. and 28 lb. over. Estimating 32 lb. to a bushel of oats, what was the weight of the load?

38. If it is desired to buy a coil of rope that may be cut into 12 ft. lengths, 16 ft. lengths, or 20 ft. lengths, and have no remnant, what is the least length the coil can have?

39. A clock that strikes the hours strikes how many times in a week?

40. The following are the new possessions of the United States and the areas thereof. Find the total of the areas.

Philippines	115,026 sq. mi.
Porto Rico	3,435 sq. mi.
Hawaii	6,449 sq. mi.
Tutuila and islets	77 sq. mi.
Guam	210 sq. mi.
Panama Canal Zone	474 sq. mi.

41. A man borrowed $5000. He paid back $2750 at one time, $300 at another, and the remainder in three equal payments. How much was each of these three payments?

42. Minnesota contains 58,915 sq. mi., and Delaware 2050 sq. mi. Minnesota would make how many states the size of Delaware and how many square miles over?

43. How many eggs are there in 24 crates each containing 36 doz.?

44. Simplify: $8 \div 4 \times 2 + 8 \times 4 \div 2 - 8 \div 4 \div 2$

45. Simplify:

$$4 \times \overline{125 + 250} - 6 \times (250 - 125) + \overline{500 - 250} \div 25$$

46. Simplify:

$$375 \times 40 \div 50 + 1200 \div 60 \times 40 - (1000 - 600)$$

47. Solve by cancellation:

$$\frac{10 \times 63 \times 40 \times 132}{8 \times 90 \times 55 \times 84}$$

48. Name all the odd numbers to 50 that are prime.

49. Solve by cancellation:

$$\frac{77 \times 26 \times 51 \times 75 \times 11}{51 \times 65 \times 22 \times 105}$$

50. Below are given the expenditures in a recent year in the United States for the articles named; find the total.

Intoxicating liquors	$2,000,000,000
Tobacco	1,200,000,000
Jewelry	800,000,000
Confectionery	200,000,000
Soft drinks	120,000,000
Tea and coffee	100,000,000
Millinery	90,000,000
Patent medicines	80,000,000

FRACTIONS

DEFINITIONS: INTRODUCTORY EXERCISES

72. Any one of the equal parts of a unit is called a **fractional unit.**

Thus, *one half* and *one third* are fractional units.

73. Any number of equal fractional units is called a **fraction.**

Thus, *one half, two halves, two thirds,* and *four thirds* are fractions.

74. The number of equal parts into which a unit is divided to form fractional units is called the **denominator** of the fraction.

Thus, in the fraction two thirds, *three* is the denominator.

75. The number of equal fractional units taken to make a fraction is called the **numerator** of the fraction.

Thus, in the fraction two thirds, *two* is the numerator.

76. The numerator and denominator are together called the **terms** of the fraction.

77. A fraction is usually written in figures by writing the denominator below and the numerator above a line.

Thus, two thirds is written $\frac{2}{3}$.

78. A fraction which has both terms expressed is a **common fraction.**

Thus, $\frac{3}{4}$, $\frac{1}{4}$, and $\frac{7}{4}$ are common fractions, but .7 (seven tenths) is not a common fraction.

79. A fraction whose numerator is less than its denominator is called a **proper fraction.**

Thus, $\frac{1}{3}$ and $\frac{3}{4}$ are proper fractions.

80. A fraction whose numerator is equal to, or greater than, its denominator is called an **improper fraction.**

Thus, $\frac{4}{4}$ and $\frac{5}{4}$ are improper fractions.

81. A number made up of a whole number and a fraction combined is called a **mixed number.**

Thus, $2\frac{1}{2}$, which means $2 + \frac{1}{2}$, is a mixed number.

NOTE. A whole number is sometimes called an **integer.**

82. A fraction may be regarded as an expression denoting that the numerator is to be divided by the denominator. The quotient is called the value of the fraction.

Thus, $\frac{3}{4}$ of a unit is equal to $\frac{1}{4}$ of 3 units; that is, $\frac{3}{4}$ is equal to $3 \div 4$.

83. When the terms of a fraction are interchanged, the fraction is said to be **inverted.**

Thus, $\frac{4}{3}$ is $\frac{3}{4}$ inverted; that is, $\frac{4}{3}$ is the *inverse* of $\frac{3}{4}$.

84. A fraction and its inverse are called **reciprocals** of each other.

Thus, $\frac{3}{4}$ and $\frac{4}{3}$ are reciprocals of each other

Exercise 28

Read aloud each of the following and tell what it expresses:

1. $\frac{3}{4}$

Thus, $\frac{3}{4}$ is read *three fourths;* it expresses three fractional units, each of which is one fourth.

2. $\frac{5}{4}$ of a gallon.

Thus, $\frac{5}{4}$ of a gallon is read *five fourths of a gallon;* it expresses five fractional units, each of which is one fourth of a gallon.

3. $2\frac{3}{4}$ bushels.

Thus, $2\frac{3}{4}$ bushels is read *two and three fourths bushels;* it expresses 2 bushels plus $\frac{3}{4}$ of a bushel.

4. $\frac{4}{5}$	5. $\frac{7}{8}$	6. $\frac{2}{3}$ of a yard
7. $\frac{5}{4}$	8. $\$ 2\frac{1}{2}$	9. $\frac{9}{8}$ of a peck
10. $\frac{3}{5}$	11. $\frac{10}{3}$	12. $\frac{5}{2}$ of a dollar
13. $7\frac{1}{2}$	14. $\$ 3\frac{3}{4}$	15. $6\frac{3}{4}$ pounds
16. $8\frac{1}{3}$	17. $\$ 12\frac{1}{2}$	18. $33\frac{1}{3}$ feet

Read aloud each of the following and tell whether it is equal to one, less than one, or greater than one:

19. $\frac{6}{7}$	20. $\frac{4}{2}$	21. $1\frac{1}{4}$
22. $\frac{3}{2}$	23. $\frac{5}{6}$	24. $\frac{8}{7}$
25. $2\frac{1}{3}$	26. $\frac{4}{3}$	27. $\frac{9}{10}$
28. $\frac{24}{25}$	29. $\frac{25}{24}$	30. $\frac{3}{3}$

31. In Examples 19–30 read the proper fractions; the improper fractions; the mixed numbers.

Exercise 29

Write in figures:

1. One half.
2. Two thirds.
3. Four fourths.
4. Two and one half.
5. Three and one third.
6. Ten and two thirds.
7. Five fourths.
8. Three halves.
9. Two sevenths of a week.
10. Three and three fourths pecks.
11. Three fourths of a mile.
12. Two and one half square yards.
13. Three twenty-fourths.
14. Five eighths of an inch.

Write in words:

15. $2\frac{1}{5}$ 16. $\frac{4}{3}$ 17. $\frac{1}{2}$

18. $1\frac{1}{7}$ 19. $\frac{1}{6}$ 20. $1\frac{7}{8}$

21. $\frac{1}{9}$ 22. $\frac{1}{4}$ 23. $\frac{6}{5}$

24. Write three proper fractions each with the denominator 4, and arrange them according to size, the least first.

25. If two unequal fractions have equal denominators, which has the greater numerator?

26. Write three improper fractions each with the numerator 4, and arrange them according to size, the least first.

27. If two unequal fractions have equal numerators, which has the greater denominator?

REDUCTION

85. Changing the form of an expression without changing its value is called **reduction.**

86. A fraction is said to be reduced to **higher terms** when its numerator and denominator are made larger numbers.

Thus, $\frac{1}{2}$ is said to be reduced to higher terms when it is changed to $\frac{2}{4}$.

87. A fraction is said to be reduced to **lower terms** when its numerator and denominator are made smaller numbers.

Thus, $\frac{2}{4}$ is said to be reduced to lower terms when it is changed to $\frac{1}{2}$.

88. A fraction is in its **lowest terms** when its numerator and denominator have no common divisor except 1.

Thus, $\frac{4}{9}$ is in its lowest terms, since 4 and 9 have no common divisor except 1.

Reducing Fractions to Higher Terms and to Lower Terms

89. Since a fraction may be regarded as an expression denoting that the numerator is to be divided by the denominator [Sec. 82], and since multiplying or dividing both dividend and divisor by the same number does not change the quotient [Sec. 70], therefore:

Multiplying or dividing both terms of a fraction by the same number does not change its value.

Exercise 30

Supply at sight the missing numerators in the following:

1. $\frac{2}{3} = \frac{\quad}{24}$

SUGGESTION. Multiply both terms of $\frac{2}{3}$ by 8; that is, $24 \div 3$.

2. $\frac{18}{24} = \frac{\quad}{4}$

SUGGESTION. Divide both terms of $\frac{18}{24}$ by 6; that is, $24 \div 4$.

3. $1 = \frac{\quad}{2}$	4. $\frac{2}{2} = \frac{\quad}{4}$	5. $1 = \frac{\quad}{8}$	6. $\frac{3}{4} = \frac{\quad}{8}$
7. $\frac{1}{2} = \frac{\quad}{4}$	8. $\frac{3}{2} = \frac{\quad}{4}$	9. $1\frac{1}{2} = \frac{\quad}{8}$	10. $\frac{2}{8} = \frac{\quad}{4}$
11. $\frac{14}{28} = \frac{\quad}{2}$	12. $\frac{1}{2} = \frac{\quad}{8}$	13. $\frac{1}{4} = \frac{\quad}{8}$	14. $\frac{8}{8} = \frac{\quad}{2}$
15. $2 = \frac{\quad}{2}$	16. $2 = \frac{\quad}{4}$	17. $2 = \frac{\quad}{8}$	18. $\frac{4}{8} = \frac{\quad}{2}$
19. $\frac{4}{4} = \frac{\quad}{2}$	20. $\frac{4}{4} = \frac{\quad}{8}$	21. $\frac{4}{8} = \frac{\quad}{4}$	22. $\frac{3}{8} = \frac{\quad}{4}$
23. $\frac{2}{3} = \frac{\quad}{6}$	24. $\frac{3}{6} = \frac{\quad}{2}$	25. $\frac{6}{8} = \frac{\quad}{4}$	26. $\frac{3}{4} = \frac{\quad}{12}$
27. $\frac{3}{4} = \frac{\quad}{20}$	28. $\frac{9}{12} = \frac{\quad}{4}$	29. $\frac{2}{3} = \frac{\quad}{15}$	30. $\frac{10}{12} = \frac{\quad}{6}$
31. $\frac{12}{15} = \frac{\quad}{5}$	32. $\frac{5}{6} = \frac{\quad}{18}$	33. $\frac{16}{24} = \frac{\quad}{3}$	34. $\frac{15}{25} = \frac{\quad}{5}$
35. $\frac{3}{4} = \frac{\quad}{24}$	36. $\frac{21}{24} = \frac{\quad}{8}$	37. $\frac{7}{8} = \frac{\quad}{24}$	38. $\frac{20}{30} = \frac{\quad}{3}$
39. $\frac{24}{32} = \frac{\quad}{4}$	40. $\frac{5}{8} = \frac{\quad}{40}$	41. $\frac{3}{8} = \frac{\quad}{32}$	42. $\frac{9}{10} = \frac{\quad}{50}$
43. $\frac{4}{5} = \frac{\quad}{20}$	44. $\frac{7}{10} = \frac{\quad}{100}$	45. $\frac{12}{16} = \frac{\quad}{4}$	46. $\frac{60}{50} = \frac{\quad}{5}$

Reducing Fractions to Lowest Terms

Illustrative Example

Reduce $\frac{210}{336}$ to lowest terms.

$\frac{2)210}{2)336} = \frac{3)105}{3)168} = \frac{7)35}{7)56} = \frac{5}{8}$

Divide both terms by common divisors that are easily observed. Shorten the work by using as great common divisors as possible.

Exercise 31

(Solve as many as possible at sight.)

Reduce each of the following to its lowest terms:

1. $\frac{8}{10}$	2. $\frac{12}{15}$	3. $\frac{16}{20}$	4. $\frac{18}{24}$	5. $\frac{24}{36}$
6. $\frac{12}{16}$	7. $\frac{4}{12}$	8. $\frac{15}{20}$	9. $\frac{9}{12}$	10. $\frac{14}{21}$
11. $\frac{9}{15}$	12. $\frac{6}{24}$	13. $\frac{36}{48}$	14. $\frac{25}{40}$	15. $\frac{56}{64}$
16. $\frac{36}{60}$	17. $\frac{35}{49}$	18. $\frac{42}{63}$	19. $\frac{132}{144}$	20. $\frac{32}{56}$
21. $\frac{45}{60}$	22. $\frac{98}{168}$	23. $\frac{72}{288}$	24. $\frac{125}{175}$	25. $\frac{36}{288}$
26. $\frac{100}{120}$	27. $\frac{180}{195}$	28. $\frac{168}{175}$	29. $\frac{144}{288}$	30. $\frac{105}{150}$
31. $\frac{84}{288}$	32. $\frac{108}{240}$	33. $\frac{100}{250}$	34. $\frac{275}{300}$	35. $\frac{120}{128}$

Reducing Integers and Mixed Numbers to Improper Fractions

Illustrative Example

Reduce $18\frac{3}{4}$ to an improper fraction.

This shows the complete work:

$1 = \frac{4}{4}$

$18 = 18 \times \frac{4}{4}$, or $\frac{72}{4}$

$18\frac{3}{4} = \frac{72}{4} + \frac{3}{4}$, or $\frac{75}{4}$

Explanation:

In one there are four fourths, and in eighteen there are eighteen times four fourths, or seventy-two fourths; and in eighteen and three fourths there are seventy-two fourths plus three fourths, or seventy-five fourths.

In practice take (4 × 18) plus 3 for the numerator, and 4 for the denominator.

Exercise 32

(Solve as many as possible at sight.)

Reduce:

1. 3 to halves. 2. 4 to thirds. 3. 5 to tenths.
4. 3 to eighths. 5. 7 to fourths. 6. 9 to fifths.
7. 5 to eighths. 8. 3 to ninths.

Reduce to improper fractions.

9. $3\frac{1}{3}$	10. $3\frac{1}{4}$	11. $2\frac{1}{3}$	12. $1\frac{2}{3}$
13. $2\frac{2}{5}$	14. $3\frac{1}{2}$	15. $3\frac{3}{4}$	16. $5\frac{1}{2}$
17. $2\frac{4}{5}$	18. $5\frac{1}{3}$	19. $7\frac{1}{2}$	20. $2\frac{7}{8}$
21. $24\frac{1}{3}$	22. $33\frac{1}{3}$	23. $66\frac{2}{3}$	24. $18\frac{3}{4}$
25. $16\frac{1}{4}$	26. $37\frac{1}{2}$	27. $62\frac{1}{2}$	28. $87\frac{1}{2}$
29. $15\frac{5}{6}$	30. $22\frac{2}{9}$	31. $44\frac{4}{9}$	32. $55\frac{5}{9}$
33. $14\frac{2}{7}$	34. $77\frac{7}{9}$	35. $28\frac{4}{7}$	36. $18\frac{5}{12}$
37. $56\frac{3}{8}$	38. $87\frac{3}{4}$	39. $86\frac{7}{16}$	40. $105\frac{7}{12}$

Read each and give the answer:

41. A 2-foot rope will make how many half-foot pieces?

42. A $5 note is worth how many quarter dollars?

43. A piece of cloth 2 yd. long can be cut into how many pieces each $\frac{1}{8}$ of a yard long?

44. A $2\frac{1}{2}$ gold piece is worth how many half dollars?

45. How many one eighth pound packages can be made from $1\frac{7}{8}$ pounds of drugs?

46. A line $5\frac{1}{2}$ yd. long will make how many half-yard lines?

47. How many times will $4\frac{1}{2}$ bu. of potatoes fill a half-bushel measure?

48. $\$12\frac{1}{2}$ in half dollars make how many half dollars? $\$5\frac{1}{4}$ in quarter dollars make how many quarter dollars?

49. $\$2\frac{1}{2}$ in quarter dollars make how many quarter dollars?

Reducing Improper Fractions to Whole or Mixed Numbers

Illustrative Example

Reduce $\frac{100}{3}$ to a mixed number.

This shows the complete work: $\frac{100}{3} = 3\overline{)100}$, $33\frac{1}{3}$

In practice write only this: $3\overline{)100}$, $33\frac{1}{3}$

In practice divide the numerator by the denominator.

Exercise 33

(Solve as many as possible at sight.)

Reduce to whole or mixed numbers:

1. $\frac{8}{2}$	2. $\frac{9}{3}$	3. $\frac{31}{5}$	4. $\frac{18}{5}$
5. $\frac{20}{8}$	6. $\frac{15}{7}$	7. $\frac{25}{6}$	8. $\frac{14}{4}$
9. $\frac{17}{3}$	10. $\frac{17}{4}$	11. $\frac{21}{4}$	12. $\frac{25}{5}$
13. $\frac{89}{4}$	14. $\frac{100}{8}$	15. $\frac{96}{6}$	16. $\frac{242}{5}$
17. $\frac{312}{4}$	18. $\frac{581}{3}$	19. $\frac{348}{10}$	20. $\frac{673}{12}$
21. $\frac{592}{4}$	22. $\frac{1001}{11}$	23. $\frac{250}{8}$	24. $\frac{375}{4}$
25. $\frac{908}{6}$	26. $\frac{425}{5}$	27. $\frac{654}{3}$	28. $\frac{576}{4}$
29. $\frac{275}{10}$	30. $\frac{326}{7}$	31. $\frac{976}{5}$	32. $\frac{787}{9}$

Read each and give the answer:

33. 8 half bushels of oats will fill how many bushel sacks?

34. 7 quarter days are how many days?

35. 18 silver half dollars can be exchanged for how many dollar bills?

36. How many quarts of cream are needed to fill 16 one eighth quart bottles?

37. 15 quarter pecks of seed are how many pecks?

38. How many tons of coal will be required to give 21 families each a quarter of a ton?

39. 39 quarter dollars are worth how many dollars?

40. How many yards of ribbon are required to make 40 pieces, each one eighth of a yard long?

41. How many pints of seed are there in 50 packages each containing $\frac{1}{4}$ pint?

Reducing Fractions to a Common Denominator

90. Fractions that have the same number for a denominator are said to have a **common denominator.**

Thus, $\frac{1}{4}$ and $\frac{3}{4}$ have a common denominator.

91. Fractions that have a common denominator are called **similar fractions.**

Thus, $\frac{2}{5}$ and $\frac{4}{5}$ are similar fractions.

Illustrative Example

Reduce $\frac{2}{3}$, $\frac{1}{4}$, and $\frac{5}{6}$ to 12ths.

$\frac{2}{3}=\frac{4\times 2}{4\times 3}=\frac{8}{12}$ Multiply both terms of $\frac{2}{3}$ by 4; that is, $12\div 3$.

$\frac{1}{4}=\frac{3\times 1}{3\times 4}=\frac{3}{12}$ Multiply both terms of $\frac{1}{4}$ by 3; that is, $12\div 4$.

$\frac{5}{6}=\frac{2\times 5}{2\times 6}=\frac{10}{12}$ Multiply both terms of $\frac{5}{6}$ by 2; that is, $12\div 6$.

Exercise 34

Reduce:

1. $\frac{3}{5}$ and $\frac{2}{3}$ to 15ths.
2. $\frac{4}{5}$ and $\frac{3}{4}$ to 20ths.
3. $\frac{1}{2}$ and $\frac{3}{4}$ to 8ths.
4. $\frac{5}{9}$ and $\frac{5}{6}$ to 18ths.
5. $\frac{3}{10}$ and $\frac{3}{20}$ to 100ths.
6. $\frac{3}{4}$ and $\frac{1}{6}$ to 36ths.
7. $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{1}{2}$ to 12ths.
8. $\frac{4}{5}$, $\frac{2}{3}$, and $\frac{9}{10}$ to 30ths.
9. $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{3}{5}$ to 20ths.
10. $\frac{2}{3}$, $\frac{1}{4}$, and $\frac{3}{8}$ to 24ths.
11. $\frac{1}{3}$, $\frac{3}{4}$, and $\frac{4}{5}$ to 60ths.
12. $\frac{5}{6}$, $\frac{1}{9}$, and $\frac{3}{8}$ to 72ds.

Reducing Fractions to their Least Common Denominator

92. The **least common denominator** (l. c. d.) of two or more fractions in their lowest terms is the least number that their denominators will divide.

Thus, the least common denominator of $\frac{2}{3}$, $\frac{1}{4}$, and $\frac{5}{6}$ is 12; for 12 is the least number that 3, 4, and 6 will divide.

93. The least common multiple of the denominators of two or more fractions in their lowest terms is the least common denominator of the fractions.

Illustrative Example

Reduce $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{5}{8}$ to fractions having the least common denominator.

The least common multiple of 3, 4, and 8 is 24.

Therefore, the l. c. d. of $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{5}{8}$ is 24.

$$\frac{2}{3}=\frac{8\times 2}{8\times 3}=\frac{16}{24}$$

$$\frac{3}{4}=\frac{6\times 3}{6\times 4}=\frac{18}{24}$$

$$\frac{5}{8}=\frac{3\times 5}{3\times 8}=\frac{15}{24}$$

Exercise 35

Reduce to fractions having the least common denominator:

1. $\frac{2}{3}, \frac{3}{4}$
2. $\frac{3}{4}, \frac{4}{5}$
3. $\frac{4}{5}, \frac{5}{6}$
4. $\frac{2}{3}, \frac{3}{5}$
5. $\frac{4}{5}, \frac{7}{10}$
6. $\frac{5}{6}, \frac{8}{9}$
7. $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$
8. $\frac{1}{3}, \frac{1}{4}, \frac{1}{6}$
9. $\frac{2}{3}, \frac{1}{6}, \frac{2}{9}$
10. $\frac{3}{5}, \frac{1}{2}, \frac{3}{4}$
11. $\frac{7}{8}, \frac{5}{12}, \frac{2}{3}$
12. $\frac{5}{6}, \frac{1}{2}, \frac{2}{9}$
13. $\frac{2}{5}, \frac{7}{10}, \frac{1}{2}$
14. $\frac{3}{5}, \frac{3}{10}, \frac{2}{15}$
15. $\frac{7}{8}, \frac{5}{16}, \frac{3}{4}$
16. $\frac{1}{6}, \frac{1}{2}, \frac{2}{5}$
17. $\frac{7}{12}, \frac{3}{8}, \frac{5}{6}$
18. $\frac{2}{5}, \frac{1}{25}, \frac{3}{50}$
19. $\frac{2}{9}, \frac{3}{4}, \frac{5}{6}$
20. $\frac{7}{8}, \frac{5}{12}, \frac{5}{9}$
21. $\frac{3}{10}, \frac{5}{6}, \frac{7}{12}$
22. $\frac{3}{7}, \frac{7}{8}, \frac{3}{4}$
23. $\frac{5}{12}, \frac{7}{8}, \frac{3}{16}$
24. $\frac{3}{25}, \frac{3}{50}, \frac{3}{20}$
25. $\frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{8}$
26. $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{5}{6}$
27. $\frac{1}{2}, \frac{2}{5}, \frac{3}{4}, \frac{7}{10}$

ADDITION AND SUBTRACTION

94. Fractions that do not have a common denominator must be reduced to similar fractions before they can be added or subtracted.

95. The sum of two or more similar fractions equals the sum of their numerators divided by their common denominator.

Thus, $\frac{2}{5} + \frac{4}{5} = (2+4)$ fractional units each of which is $\frac{1}{5}$

$$= \frac{2+4}{5}, \text{ or } \frac{6}{5}, \text{ or } 1\frac{1}{5}.$$

96. The difference of two similar fractions equals the difference of their numerators divided by their common denominator.

Thus, $\frac{5}{8} - \frac{3}{8} = (5-3)$ fractional units each of which is $\frac{1}{8}$

$$= \frac{5-3}{8}, \text{ or } \frac{2}{8}, \text{ or } \frac{1}{4}.$$

Illustrative Examples

1. Find the sum of $\frac{3}{8}$, $\frac{3}{4}$, and $\frac{4}{5}$.

l. c. d. = 40.

$$\frac{3}{8}=\frac{5\times 3}{5\times 8}=\frac{15}{40}$$

$$\frac{3}{4}=\frac{10\times 3}{10\times 4}=\frac{30}{40}$$

$$\frac{4}{5}=\frac{8\times 4}{8\times 5}=\frac{32}{40}$$

$$\text{Sum} = \frac{77}{40}=1\tfrac{37}{40}$$

2. Find the difference of $\frac{8}{15}$ and $\frac{3}{20}$.

l. c. d. = 60.

$$\frac{8}{15}=\frac{4\times 8}{4\times 15}=\frac{32}{60}$$

$$\frac{3}{20}=\frac{3\times 3}{3\times 20}=\frac{9}{60}$$

$$\text{Difference} = \frac{23}{60}$$

Exercise 36

(Solve as many as possible at sight.)

Find the sum or difference as indicated:

1. $\frac{1}{4}+\frac{3}{4}$	2. $\frac{1}{5}+\frac{2}{5}$	3. $\frac{2}{5}+\frac{3}{5}$
4. $\frac{5}{6}-\frac{1}{6}$	5. $\frac{7}{8}-\frac{3}{8}$	6. $\frac{3}{8}+\frac{1}{8}$
7. $\frac{4}{4}-\frac{1}{4}$	8. $\frac{5}{6}+\frac{1}{6}$	9. $\frac{8}{4}-\frac{2}{4}$
10. $\frac{3}{8}+\frac{5}{8}$	11. $\frac{5}{2}-\frac{3}{2}$	12. $\frac{3}{8}+\frac{7}{8}$
13. $\frac{3}{8}+\frac{6}{8}$	14. $\frac{5}{6}-\frac{3}{6}$	15. $\frac{1}{12}+\frac{5}{12}$
16. $\frac{7}{6}-\frac{1}{6}$	17. $\frac{7}{12}+\frac{5}{12}$	18. $\frac{9}{12}-\frac{5}{12}$
19. $\frac{1}{10}+\frac{11}{10}$	20. $\frac{12}{12}-\frac{6}{12}$	21. $\frac{7}{12}+\frac{3}{12}$
22. $\frac{9}{12}-\frac{3}{12}$	23. $\frac{12}{12}-\frac{3}{12}$	24. $\frac{1}{10}+\frac{9}{10}$

25. $\frac{11}{12} - \frac{7}{12}$
26. $\frac{7}{16} - \frac{5}{16}$
27. $\frac{3}{12} + \frac{15}{12}$
28. $\frac{7}{10} + \frac{8}{10}$
29. $\frac{5}{24} + \frac{7}{24}$
30. $\frac{9}{24} - \frac{3}{24}$
31. $\frac{15}{24} - \frac{11}{24}$
32. $\frac{19}{24} - \frac{7}{24}$
33. $\frac{15}{18} - \frac{6}{18}$
34. $\frac{1}{2} + \frac{1}{8}$
35. $\frac{1}{2} + \frac{1}{4}$
36. $\frac{1}{4} + \frac{1}{8}$
37. $\frac{1}{3} + \frac{1}{9}$
38. $\frac{1}{3} + \frac{1}{6}$
39. $\frac{1}{4} + \frac{1}{12}$
40. $\frac{1}{5} + \frac{1}{10}$
41. $\frac{1}{6} + \frac{1}{12}$
42. $\frac{1}{15} + \frac{1}{5}$
43. $\frac{1}{2} - \frac{1}{4}$
44. $\frac{1}{3} - \frac{1}{6}$
45. $\frac{1}{3} - \frac{1}{9}$
46. $\frac{1}{6} - \frac{1}{12}$
47. $\frac{1}{5} - \frac{1}{10}$
48. $\frac{1}{8} - \frac{1}{16}$
49. $\frac{1}{6} + \frac{1}{4}$
50. $\frac{1}{2} + \frac{1}{3}$
51. $\frac{1}{2} + \frac{1}{5}$
52. $\frac{1}{3} - \frac{1}{4}$
53. $\frac{1}{3} - \frac{1}{5}$
54. $\frac{1}{4} + \frac{1}{5}$
55. $\frac{1}{3} + \frac{1}{5}$
56. $\frac{1}{6} + \frac{1}{8}$
57. $\frac{1}{8} + \frac{1}{12}$
58. $\frac{1}{2} - \frac{1}{3}$
59. $\frac{1}{2} - \frac{1}{10}$
60. $\frac{1}{4} - \frac{1}{5}$
61. $\frac{1}{2} + \frac{3}{4}$
62. $\frac{1}{2} + \frac{2}{3}$
63. $\frac{1}{2} + \frac{3}{8}$
64. $\frac{5}{6} - \frac{1}{2}$
65. $\frac{3}{4} - \frac{1}{2}$
66. $\frac{5}{6} - \frac{1}{3}$
67. $\frac{2}{3} + \frac{5}{6}$
68. $\frac{3}{4} + \frac{5}{8}$
69. $\frac{2}{3} + \frac{3}{8}$
70. $\frac{3}{4} - \frac{2}{3}$
71. $\frac{3}{4} - \frac{3}{8}$
72. $\frac{5}{6} - \frac{3}{4}$
73. $\frac{3}{4} + \frac{7}{8}$
74. $\frac{3}{4} + \frac{5}{6}$
75. $\frac{7}{12} - \frac{3}{8}$
76. $\frac{4}{5} - \frac{3}{4}$
77. $\frac{7}{8} - \frac{5}{6}$
78. $\frac{5}{12} + \frac{7}{8}$
79. $\frac{5}{8} + \frac{11}{12}$
80. $\frac{9}{10} + \frac{2}{15}$
81. $\frac{5}{8} + \frac{5}{6}$
82. $\frac{5}{4} - \frac{5}{6}$
83. $\frac{3}{2} - \frac{2}{5}$
84. $\frac{15}{16} - \frac{3}{4}$
85. $\frac{5}{12} + \frac{3}{16}$
86. $\frac{7}{16} + \frac{5}{48}$
87. $\frac{3}{16} + \frac{7}{24}$
88. $\frac{17}{30} - \frac{13}{45}$
89. $\frac{7}{24} - \frac{5}{36}$
90. $\frac{8}{25} - \frac{3}{50}$
91. $\frac{9}{16} + \frac{27}{40}$
92. $\frac{17}{25} + \frac{19}{50}$
93. $\frac{5}{16} + \frac{3}{64}$
94. $\frac{9}{15} + \frac{24}{25}$
95. $\frac{25}{32} + \frac{5}{24}$
96. $\frac{13}{24} - \frac{13}{40}$
97. $\frac{1}{4} + \frac{1}{4} + \frac{3}{4}$
98. $\frac{2}{5} + \frac{2}{5} + \frac{4}{5}$
99. $\frac{1}{10} + \frac{1}{10} + \frac{3}{10}$
100. $\frac{7}{8} + \frac{3}{4} + \frac{1}{2}$
101. $\frac{5}{8} + \frac{7}{12} + \frac{9}{16}$
102. $\frac{13}{20} + \frac{11}{30} + \frac{11}{12}$
103. $\frac{2}{3} + \frac{4}{5} + \frac{1}{6}$
104. $\frac{3}{4} + \frac{5}{6} + \frac{7}{16}$
105. $\frac{17}{24} + \frac{9}{16} + \frac{7}{12}$
106. $\frac{9}{10} + \frac{4}{5} + \frac{7}{15}$
107. $\frac{7}{24} + \frac{5}{16} + \frac{3}{8}$
108. $\frac{7}{8} + \frac{9}{10} + \frac{7}{15}$
109. $\frac{9}{10} + \frac{1}{2} - \frac{1}{8}$
110. $\frac{7}{15} - \frac{1}{5} + \frac{1}{3}$
111. $\frac{9}{10} - \frac{3}{100} + \frac{1}{20}$

Illustrative Examples

1. If A can do $\frac{1}{2}$ of a piece of work in one day, B $\frac{1}{4}$ of it in one day, and C $\frac{1}{8}$ of it in one day, what part of it can they together do in one day?

$\frac{1}{2}$ = the part A can do in a day.
$\frac{1}{4}$ = the part B can do in a day.
$\frac{1}{8}$ = the part C can do in a day.
$\therefore \frac{1}{2}+\frac{1}{4}+\frac{1}{8}$, or $\frac{7}{8}$ = the part they together can do in a day.

NOTE. The symbol $\therefore$ is read **therefore**.

2. A can do $\frac{2}{3}$ of a piece of work in one day, and A and B together can do $\frac{7}{8}$ of it; what part can B do in one day?

$\frac{2}{3}$ = the part A can do in one day.
$\frac{7}{8}$ = the part A and B together can do in one day
$\therefore \frac{7}{8}-\frac{2}{3}$, or $\frac{5}{24}$ = the part B can do in one day.

Exercise 37

1. If a man does $\frac{1}{5}$ of a piece of work one day and $\frac{1}{4}$ of it another day, what part of it has he then done?

2. A storekeeper sold at auction $\frac{1}{5}$ of his stock and at private sale $\frac{3}{10}$ of it; what part did he sell at both?

3. How much oats is required to feed one horse $\frac{3}{8}$ of a bushel and another $\frac{3}{4}$ of a bushel?

4. A farmer planted $\frac{1}{2}$ of an acre with potatoes, $\frac{1}{3}$ of an acre with tobacco, and $\frac{1}{4}$ of an acre with sweet corn. How many acres did he plant in all?

5. How much land do three lots contain, if their sizes are $\frac{5}{6}$ of an acre, $\frac{3}{8}$ of an acre, and $\frac{5}{12}$ of an acre?

6. A piece of cloth $\frac{7}{8}$ of a yard long shrunk $\frac{1}{16}$ of a yard in sponging; how long was it then?

7. A can do $\frac{1}{2}$ of a piece of work in one day, and A and B together $\frac{5}{6}$ of it; what part can B do in a day?

8. In a township there were built in one year four pieces of good roads, the first $\frac{5}{8}$ of a mile, the second $\frac{3}{4}$ of a mile, the third $\frac{5}{6}$ of a mile, and the fourth $\frac{1}{2}$ of a mile; how many miles were built in all?

9. A man bought an article for $\$\frac{3}{4}$ and sold it at a profit of $\$\frac{1}{2}$; how much did he receive for it?

10. A man bought an article for $\$\frac{3}{4}$ and sold it at a loss of $\$\frac{1}{20}$; what part of a dollar did he receive for it?

11. If one oil lamp burns $\frac{1}{8}$ of a gallon of oil in a night, another burns $\frac{3}{4}$ of a gallon, and a third burns $\frac{1}{2}$ of a gallon, how much do they all burn?

12. If A can cut $\frac{3}{8}$ of an acre of corn in a day, and his son can cut $\frac{3}{4}$ of an acre, how much can they both cut in a day?

13. A man bought an article for $\$\frac{4}{5}$ and sold it at a profit of $\$\frac{1}{20}$; how much did he receive for it?

14. A man sold an article for $\$\frac{3}{4}$, which was at a loss of $\$\frac{1}{20}$; what part of a dollar did it cost?

15. A man sold an article for $\$\frac{4}{5}$, which was at a gain of $\$\frac{1}{20}$; what part of a dollar did it cost?

Addition and Subtraction of Mixed Numbers

Illustrative Examples

1. Find the sum of $124\frac{7}{8}$, $346\frac{5}{16}$, and $452\frac{7}{32}$.

Form

$124\frac{7}{8} = 124\frac{28}{32}$
$346\frac{5}{16} = 346\frac{10}{32}$
$452\frac{7}{32} = 452\frac{7}{32}$
$\therefore$ Sum $= 923\frac{13}{32}$

SUGGESTION. The sum of the fractions equals $1\frac{13}{32}$. Write the $\frac{13}{32}$, add the 1 to the column of units, and proceed as in addition of whole numbers.

2. Find the difference of $124\frac{3}{8}$ and $98\frac{3}{4}$.

Form

$124\frac{3}{8} = 123\frac{11}{8}$

$98\frac{3}{4} = 98\frac{6}{8}$

$\therefore$ Diff. $= 25\frac{5}{8}$

SUGGESTION. Since $\frac{3}{4}$ is more than $\frac{3}{8}$, take *one* from 124 and add it to $\frac{3}{8}$ before subtracting.

Exercise 38

Find the sum or the difference as indicated:

1. $2\frac{3}{8} + 3\frac{5}{16}$
2. $7\frac{1}{12} + 6\frac{5}{6}$
3. $8\frac{3}{8} + 10\frac{2}{3}$
4. $12\frac{3}{4} + 5\frac{1}{32}$
5. $100 - 33\frac{1}{3}$
6. $66\frac{2}{3} - 12\frac{1}{2}$
7. $24\frac{7}{8} + 19\frac{3}{16}$
8. $86\frac{1}{2} - 18\frac{3}{4}$
9. $70\frac{1}{2} - 25\frac{5}{8}$
10. $33\frac{1}{3} + 16\frac{2}{3}$
11. $32\frac{5}{9} + 12\frac{2}{3}$
12. $56\frac{5}{24} + 32\frac{15}{16}$
13. $25\frac{7}{12} - 13\frac{5}{6}$
14. $47\frac{3}{14} + 20\frac{2}{21}$
15. $45\frac{5}{6} - 32\frac{8}{9}$
16. $85\frac{1}{8} - 32\frac{7}{16}$
17. $128\frac{3}{10} - 39\frac{4}{5}$
18. $100\frac{1}{9} + 37\frac{3}{8}$
19. $12\frac{1}{2} + 13\frac{2}{3} + 15\frac{1}{6}$
20. $16\frac{2}{3} + 12\frac{1}{2} + 18\frac{3}{4}$
21. $42\frac{2}{7} + 35\frac{5}{14} + 40\frac{2}{21}$
22. $15\frac{2}{3} + 18\frac{1}{6} + 20\frac{3}{4}$
23. $28\frac{2}{3} + 19\frac{3}{8} + 56\frac{5}{12}$
24. $16\frac{1}{6} + 33\frac{1}{3} + 66\frac{2}{3}$

Exercise 39

1. A man owned $37\frac{1}{2}$ acres of land and bought $32\frac{3}{4}$ acres more; how much had he then?

2. The four sides of a field measure $42\frac{1}{2}$ rods, 31 rods, $26\frac{1}{2}$ rods, and $38\frac{1}{2}$ rods; what is the distance around the field?

3. Find the weight of four hams, whose weights are $12\frac{3}{4}$ lb., $14\frac{1}{2}$ lb., $16\frac{3}{4}$ lb., and $15\frac{1}{2}$ lb.

4. What is the distance around a room whose length is $18\frac{3}{4}$ ft. and width $15\frac{3}{4}$ ft.?

5. A woman bought $37\frac{1}{2}$ yd. muslin and has used $18\frac{3}{4}$ yd. of it; how much remains?

6. From 40 yd. of carpet, 3 pieces were sold, whose lengths were $10\frac{1}{2}$ yd., $7\frac{3}{4}$ yd., and $8\frac{3}{4}$ yd.; how much remains?

7. How much change should a woman receive from 2 fifty-dollar bills given in payment of a bill of $\$ 72\frac{1}{2}$?

8. $20\frac{3}{4}$ lb. of lard was put into a can weighing $3\frac{1}{2}$ lb.; what was the weight of both?

9. From a barrel containing 42 gallons of oil, $14\frac{1}{2}$ gallons were sold; how much then remained?

10. The width of a rectangular lot is $42\frac{3}{4}$ yd., and its length is $8\frac{3}{4}$ yd. more than its width; what is the distance around it?

11. From a 20-acre tract of land, 3 lots have been sold, containing $2\frac{1}{2}$ acres, $3\frac{5}{8}$ acres, and $6\frac{3}{4}$ acres; how much remains?

12. If a man owes me $\$ 37\frac{1}{2}$ and gives me three five-dollar bills and a $\$ 2\frac{1}{2}$ gold piece, how much does he still owe me?

13. What number must be added to $37\frac{1}{2}$ to make 100?

14. By how much does 200 exceed the sum of $66\frac{2}{3} + 33\frac{1}{3} + 12\frac{1}{2}$?

15. When $\frac{7}{30}$ of June has passed, what part of it remains?

REVIEW

Exercise 40

1. A farmer hauled a load of wheat weighing $2\frac{3}{10}$ tons in a wagon weighing $1\frac{4}{5}$ tons; what weight did he haul?

2. There are three slates in a certain blackboard; the first is $4\frac{2}{3}$ ft. long, the second $3\frac{3}{4}$ ft. long, and the third $5\frac{1}{8}$ ft. long; how long is the blackboard?

3. If a train runs $16\frac{2}{3}$ miles in one hour, $18\frac{3}{4}$ miles the next hour, and $17\frac{5}{6}$ miles the next hour, how far does it run in the three hours?

4. Mr. Smith bought $\frac{1}{24}$ of a certain tract of land, and Mr. Jones $\frac{1}{36}$ of it; what part of it did they together buy?

5. Mr. Hill gave $\frac{1}{16}$ of his money to a church, $\frac{1}{24}$ of it to a hospital, and $\frac{1}{12}$ of it to a college; what part of it did he give away?

6. 3 turkeys together weighed $24\frac{3}{4}$ lb.; if one of them weighed $8\frac{3}{4}$ lb. and another 8 lb., what did the third weigh?

7. A farmer sold two cans of lard, one weighing $35\frac{3}{4}$ lb., the other $32\frac{1}{2}$ lb. One can weighed $2\frac{3}{4}$ lb., the other 2 lb.; what was the weight of the lard?

8. What change should you receive from two 20-dollar bills given in payment for a suit of clothes costing $\$ 27\frac{1}{2}$?

9. Find the sum of $12\frac{1}{2}$, $37\frac{1}{2}$, $62\frac{1}{2}$, and $87\frac{1}{2}$.

10. Arrange the following fractions in the order of their value, writing the largest first to the left: $\frac{1}{2}$, $\frac{5}{6}$, $\frac{8}{9}$, $\frac{9}{10}$, $\frac{9}{15}$.

11. A man bought a box of fruit for $\$ 2\frac{3}{4}$ and sold it at a profit of $\$ 1\frac{1}{2}$; what did he receive for it?

12. How many rods of fence are required to inclose a rectangular field whose length is $16\frac{2}{3}$ rods and width $12\frac{1}{2}$ rods?

13. From a log 40 ft. long, two pieces were cut, the first $10\frac{3}{4}$ ft. long and the second $8\frac{1}{2}$ ft. long; what was the length of the piece remaining?

14. A telegraph pole 60 ft. long was set $5\frac{3}{4}$ ft. in the ground; how many feet of it were above ground?

15. A man has a $\$2\frac{1}{2}$ gold piece and $\$3\frac{3}{4}$ in silver; how much more money does he need to have \$10?

16. An 8-inch spike was driven through a $2\frac{1}{2}$-inch plank into a log; how far did it extend into the log?

17. A rectangular garden is $32\frac{1}{2}$ ft. long and 24 ft. wide; what is the distance around it?

18. By how much does the sum of $12\frac{1}{2}$ and 75 exceed the sum of $37\frac{1}{2}$ and 25?

19. What average mark should a girl receive in Arithmetic, Grammar, and Drawing, if her mark in Arithmetic was $87\frac{1}{4}$, in Grammar $88\frac{1}{4}$, and in Drawing $90\frac{1}{2}$?

20. Find the cost of a horse that sold for \$150 at a loss of $\$37\frac{1}{2}$.

21. Find the selling price of a rope and pulley that were sold at a profit of $\$\frac{2}{5}$, if the pulley cost $\$1\frac{1}{2}$ and the rope $\$1\frac{1}{4}$.

22. Simplify: $(37\frac{1}{2}+18\frac{3}{4}+62\frac{1}{2})-(66\frac{2}{3}+6\frac{1}{6})$.

23. If A can cut $\frac{3}{8}$ of an acre of corn in a day and B $\frac{3}{4}$ of an acre, and C $1\frac{1}{4}$ acres; how much can they all cut in a day?

24. A man gave two \$2 bills and two \$5 bills in pay-

ment for a coat that cost him $\$ 12\frac{1}{2}$; how much change should he receive?

25. A grocer had in the drawer of his cash register, on opening business on a certain day, $\$ 32\frac{1}{2}$; he took in during the day $\$ 112\frac{1}{4}$ and paid out $\$ 18\frac{3}{4}$; how much should the cash drawer contain at the close of business that day?

26. A laborer does $\frac{1}{6}$ of a piece of work in one day, $\frac{1}{4}$ of it the next day, and $\frac{1}{3}$ of it the next day; he finishes it the fourth day. What part of the work does he do the fourth day?

27. $\frac{1}{10}$ part of each of our gold coins is copper and the remaining part pure gold; what part is pure gold?

28. A clerk's expenses for a month were as follows: board $\$ 24$, laundry $\$ 3\frac{3}{4}$, car fare $\$ 2\frac{1}{2}$, and other expenses $\$ 7\frac{1}{2}$; what does he save out of his salary of $\$ 60$?

29. I bought a horse for $\$ 150$, paid in cash $\$ 62\frac{1}{2}$ and the balance in six months. What amount did I pay in six months?

30. Supply the missing number:

$$10 + 62\tfrac{1}{2} + 30 + ? + 8\tfrac{1}{2} = 250.$$

31. A owns $12\frac{1}{2}$ acres of land and B owns $6\frac{1}{4}$ acres more than A; how much do they both own?

32. In a mixture of 50 bushels of corn, oats, and rye there are $16\frac{1}{2}$ bushels of corn and 12 bushels of rye; how many bushels of oats are there?

33. A merchant sold an article at $\$ 12\frac{1}{2}$, which was $\$ 2\frac{1}{4}$ less than he paid for it; how much did he pay for it?

34. The sum of three numbers is 100. If one of them is $22\frac{2}{9}$ and another is $33\frac{1}{3}$, what is the third number?

MULTIPLICATION

Finding the Product of a Whole Number and a Fraction

Multiplying a Fraction by a Whole Number

97. (1) $3 \times \frac{2}{5} = \frac{2}{5} + \frac{2}{5} + \frac{2}{5}$, or $\frac{6}{5}$, or $\frac{3 \times 2}{5}$

(2) $2 \times \frac{3}{8} = \frac{3}{8} + \frac{3}{8}$, or $\frac{6}{8}$, or $\frac{3}{4}$, or $\frac{3}{8 \div 2}$

To multiply a fraction by a whole number, multiply the numerator or divide the denominator by the whole number.

NOTE. Divide the denominator when possible.

Illustrative Example

Multiply $\frac{5}{48}$ by 9.

This shows the different steps: *In practice write only this:*

$$9 \times \frac{5}{48} = \frac{\overset{3}{\cancel{9}} \times 5}{\underset{16}{\cancel{48}}} \text{ or } \frac{15}{16} \qquad \overset{3}{\cancel{9}} \times \frac{5}{\underset{16}{\cancel{48}}} = \frac{15}{16}$$

To reduce the product to its lowest terms, divide 9 and 48 by 3 and cancel the numbers divided.

Exercise 41

(Solve as many as possible at sight.)

Find the product:

1. $3 \times \frac{2}{7}$ Thus, $3 \times \frac{2}{7} = \frac{6}{7}$
2. $2 \times \frac{3}{4}$ Thus, $2 \times \frac{3}{4} = \frac{3}{2}$, or $1\frac{1}{2}$
3. $2 \times \frac{1}{3}$
4. $2 \times \frac{1}{5}$
5. $2 \times \frac{1}{4}$
6. $2 \times \frac{1}{2}$
7. $2 \times \frac{2}{5}$
8. $2 \times \frac{3}{5}$
9. $3 \times \frac{2}{9}$
10. $3 \times \frac{3}{4}$
11. $3 \times \frac{4}{9}$
12. $3 \times \frac{2}{3}$
13. $4 \times \frac{1}{8}$
14. $4 \times \frac{5}{12}$

15. $4 \times \frac{2}{7}$ 16. $4 \times \frac{7}{12}$ 17. $4 \times \frac{3}{9}$

18. $4 \times \frac{5}{6}$ 19. $5 \times \frac{1}{5}$ 20. $5 \times \frac{3}{5}$

21. $5 \times \frac{6}{3}$ 22. $5 \times \frac{2}{3}$ 23. $5 \times \frac{3}{25}$

24. $5 \times \frac{7}{30}$ 25. $5 \times \frac{11}{10}$ 26. $6 \times \frac{1}{6}$

27. $6 \times \frac{1}{12}$ 28. $6 \times \frac{5}{6}$ 29. $6 \times \frac{7}{12}$

30. $6 \times \frac{5}{18}$ 31. $8 \times \frac{5}{8}$ 32. $8 \times \frac{3}{16}$

33. $8 \times \frac{3}{10}$ 34. $8 \times \frac{7}{8}$ 35. $9 \times \frac{5}{9}$

36. $9 \times \frac{7}{24}$ 37. $15 \times \frac{5}{24}$ 38. $18 \times \frac{7}{24}$

39. $14 \times \frac{5}{21}$ 40. $25 \times \frac{7}{15}$ 41. $18 \times \frac{9}{48}$

42. $24 \times \frac{7}{16}$ 43. $25 \times \frac{9}{40}$ 44. $32 \times \frac{11}{24}$

45. $27 \times \frac{5}{18}$ 46. $48 \times \frac{7}{32}$ 47. $45 \times \frac{7}{30}$

48. $48 \times \frac{11}{36}$ 49. $54 \times \frac{17}{48}$ 50. $50 \times \frac{8}{75}$

51. If each side of a square is $\frac{3}{8}$ in. long, what is the distance around it?

52. Find the cost of 4 yd. of muslin at $\$\frac{1}{8}$ a yard.

53. What does a boy pay for 20 newspapers at $\frac{1}{2}¢$ each?

54. A boy who was paid $\$\frac{1}{10}$ for working an hour should be paid how much for working 10 hours?

55. How much milk is there in 4 bottles, each containing $\frac{1}{4}$ of a quart?

56. If a pound of green coffee loses $\frac{1}{10}$ of a pound in roasting, how much should 5 lb. lose?

57. If $\frac{3}{8}$ of an inch is taken to represent 1 foot, how many inches will represent 4 feet?

58. What part of a piece of work should a man do in 4 days if he does $\frac{3}{16}$ of it in 1 day?

59. If a laborer cuts $\frac{7}{8}$ of an acre of corn in a day, how much should he cut in 8 days?

60. A lamp that burns $\frac{3}{8}$ of a quart of oil per night burns how much in a week?

61. On a certain map, $\frac{7}{12}$ of an inch is made to represent a mile; what length on the map represents 25 miles?

62. Find the weight of a bolt of dress flannel of 40 yd., if each yard weighs $\frac{5}{16}$ of a pound.

Multiplying a Whole Number by a Fraction

98. $\frac{2}{3} \times 4 = \frac{2}{3}$ of $(1+1+1+1)$
$= \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3}$
$= 4 \times \frac{2}{3}$

If a whole number is to be multiplied by a fraction, the result may be found by multiplying the fraction by the whole number.

NOTE 1. $\frac{1}{3} \times 4$, $\frac{1}{3}$ of 4, and $4 \div 3$ all have the same meaning.
NOTE 2. $\frac{2}{3} \times 4$ may be read $\frac{2}{3}$ of 4.

Illustrative Example

Multiply 9 by $\frac{7}{24}$.

This shows the different steps:

$$\frac{7}{24} \times 9 = \frac{7 \times \overset{3}{9}}{\underset{8}{24}} = \frac{21}{8}, \text{ or } 2\tfrac{5}{8}$$

In practice write only this:

$$\frac{7}{\underset{8}{24}} \times \overset{3}{9} = \frac{21}{8}, \text{ or } 2\tfrac{5}{8}$$

To reduce the product to its lowest terms, we divide 9 and 24 by 3 and cancel the numbers divided.

Exercise 42

(Solve as many as possible at sight.)

Find the product:

1. $\frac{2}{7} \times 3$ Thus, $\frac{2}{7} \times 3 = \frac{6}{7}$ [§ 97 (1)].
2. $\frac{3}{8} \times 2$ Thus, $\frac{3}{8} \times 2 = \frac{3}{4}$ [§ 97 (2)].
3. $\frac{1}{5} \times 4$
4. $\frac{2}{5} \times 2$
5. $\frac{1}{4} \times 2$
6. $\frac{1}{2} \times 3$
7. $\frac{1}{2} \times 5$
8. $\frac{1}{2} \times 7$
9. $\frac{1}{3} \times 2$
10. $\frac{1}{4} \times 3$
11. $\frac{1}{4} \times 9$
12. $\frac{1}{4} \times 5$
13. $\frac{1}{4} \times 7$
14. $\frac{1}{5} \times 2$
15. $\frac{1}{5} \times 4$
16. $\frac{1}{5} \times 8$
17. $\frac{1}{5} \times 7$
18. $\frac{1}{6} \times 3$
19. $\frac{1}{6} \times 8$
20. $\frac{2}{3} \times 4$
21. $\frac{2}{5} \times 3$
22. $\frac{2}{3} \times 7$
23. $\frac{3}{4} \times 3$
24. $\frac{3}{4} \times 5$
25. $\frac{3}{5} \times 5$
26. $\frac{3}{5} \times 8$
27. $\frac{4}{5} \times 2$
28. $\frac{4}{5} \times 9$
29. $\frac{5}{6} \times 2$
30. $\frac{5}{6} \times 4$
31. $\frac{5}{6} \times 6$
32. $\frac{2}{7} \times 2$
33. $\frac{7}{12} \times 8$
34. $\frac{9}{20} \times 15$
35. $\frac{11}{40} \times 18$
36. $\frac{9}{28} \times 21$
37. $\frac{7}{30} \times 45$
38. $\frac{8}{25} \times 35$
39. $\frac{15}{16} \times 24$
40. $\frac{16}{25} \times 45$
41. $\frac{17}{50} \times 75$
42. $\frac{15}{16} \times 48$
43. $\frac{33}{50} \times 100$
44. $\frac{33}{75} \times 100$
45. $\frac{18}{25} \times 40$
46. $\frac{9}{100} \times 50$
47. $\frac{11}{100} \times 75$
48. $\frac{41}{50} \times 40$
49. $\frac{11}{200} \times 150$
50. $\frac{7}{200} \times 600$

51. Find the cost of $\frac{1}{2}$ a yard of cloth at \$3 a yard.
52. Find the cost of $\frac{3}{4}$ of an acre of land at \$40 an acre.
53. $\frac{1}{2}$ of a foot is how many inches?
54. A man earns \$20 a week and his son earns $\frac{3}{4}$ as much; how much does the son earn a week?
55. How many months are there in $\frac{5}{6}$ of a year?
56. How many hours are there in $\frac{3}{4}$ of a day?

57. Find the cost of $\frac{3}{4}$ of a pound of tea at 60¢ a pound.

58. What should a man be paid for $\frac{3}{4}$ of a day's work, if his wages are \$3 a day?

59. A man had \$40 and spent $\frac{3}{8}$ of it; how much did he spend? How much had he remaining?

Finding the Product of Two Fractions

99. $\frac{2}{3} \times \frac{4}{5}$ means $2 \times \frac{1}{3}$ of $\frac{4}{5}$.

NOTE 1. $\frac{2}{3} \times \frac{4}{5}$ and $\frac{2}{3}$ of $\frac{4}{5}$ have the same meaning.

NOTE 2. $\frac{2}{3} \times \frac{4}{5}$ is usually read $\frac{2}{3}$ of $\frac{4}{5}$.

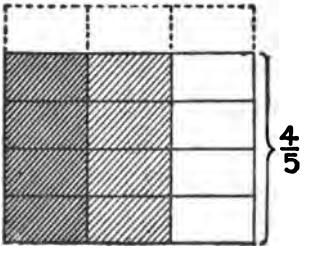
100. $\frac{2}{3} \times \frac{4}{5} = 2 \times \frac{1}{3}$ of $\frac{4}{5}$ (See § 99.)

$= 2 \times \frac{4}{15}$ (See diagram.)

$= \frac{8}{15}$ (See diagram.)

but $\frac{8}{15} = \frac{2 \times 4}{3 \times 5}$

therefore $\frac{2}{3} \times \frac{4}{5} = \frac{2 \times 4}{3 \times 5}$



The product of two fractions equals the product of their numerators divided by the product of their denominators.

Illustrative Example

Multiply $\frac{15}{16}$ by $\frac{8}{9}$.

This shows the different steps:

$$\frac{8}{9} \times \frac{15}{16} = \frac{\overset{1}{\cancel{8}} \times \overset{5}{\cancel{15}}}{\underset{3}{\cancel{9}} \times \underset{2}{\cancel{16}}} = \frac{1 \times 5}{3 \times 2} = \frac{5}{6}$$

In practice write only this:

$$\frac{\overset{1}{\cancel{8}}}{\underset{3}{\cancel{9}}} \times \frac{\overset{5}{\cancel{15}}}{\underset{2}{\cancel{16}}} = \frac{5}{6}$$

To reduce the product to its lowest terms, we divide 8 and 16 by 8, and 15 and 9 by 3, and cancel the numbers divided.

Exercise 43

(Solve as many as possible at sight.)

1. $\frac{1}{3}$ of $\frac{1}{4}$
2. $\frac{1}{2}$ of $\frac{1}{3}$
3. $\frac{1}{2}$ of $\frac{1}{4}$
4. $\frac{1}{2}$ of $\frac{2}{3}$
5. $\frac{1}{2}$ of $\frac{1}{5}$
6. $\frac{1}{3}$ of $\frac{1}{5}$
7. $\frac{1}{5}$ of $\frac{2}{3}$
8. $\frac{2}{3}$ of $\frac{1}{4}$
9. $\frac{3}{4}$ of $\frac{1}{5}$
10. $\frac{2}{3}$ of $\frac{3}{4}$
11. $\frac{2}{3}$ of $\frac{1}{6}$
12. $\frac{2}{3}$ of $\frac{4}{5}$
13. $\frac{4}{5} \times \frac{10}{3}$
14. $\frac{4}{3} \times \frac{3}{5}$
15. $\frac{5}{6} \times \frac{3}{8}$
16. $\frac{2}{3} \times \frac{3}{4}$
17. $\frac{2}{9} \times \frac{6}{5}$
18. $\frac{14}{15} \times \frac{35}{48}$
19. $\frac{25}{24} \times \frac{16}{15}$
20. $\frac{9}{16} \times \frac{20}{21}$
21. $\frac{2}{3} \times \frac{3}{5}$
22. $\frac{3}{5} \times \frac{3}{3}$
23. $\frac{6}{7} \times \frac{9}{2}$
24. $\frac{3}{8} \times \frac{16}{9}$
25. $\frac{39}{40} \times \frac{28}{26}$
26. $\frac{32}{9} \times \frac{16}{27}$
27. $\frac{12}{25} \times \frac{10}{21}$
28. $\frac{7}{12} \times \frac{40}{21}$
29. $\frac{9}{16} \times \frac{8}{27}$
30. $\frac{7}{15} \times \frac{25}{36}$
31. $\frac{44}{45} \times \frac{30}{22}$
32. $\frac{11}{15} \times \frac{45}{33}$
33. $\frac{15}{16} \times \frac{48}{25}$
34. $\frac{18}{25} \times \frac{20}{29}$
35. $\frac{14}{15} \times \frac{30}{49}$
36. $\frac{15}{16} \times \frac{64}{75}$
37. $\frac{32}{33} \times \frac{11}{64}$
38. $\frac{11}{14} \times \frac{21}{44}$
39. $\frac{45}{32} \times \frac{16}{15}$
40. $\frac{32}{45} \times \frac{15}{16}$
41. $\frac{100}{3} \times \frac{3}{200}$
42. $\frac{125}{2} \times \frac{3}{75}$
43. $\frac{9}{25} \times \frac{10}{33}$
44. $\frac{7}{75} \times \frac{150}{14}$
45. $\frac{1}{50} \times \frac{200}{3}$
46. $\frac{32}{75} \times \frac{25}{16}$
47. $\frac{9}{100} \times \frac{150}{6}$
48. $\frac{2}{25} \times \frac{500}{6}$
49. $\frac{32}{35} \times \frac{49}{48}$
50. $\frac{12}{55} \times \frac{77}{18}$
51. $\frac{63}{100} \times \frac{75}{54}$

52. A quart is $\frac{1}{4}$ of a gallon and a pint is $\frac{1}{2}$ of a quart; what part of a gallon is a pint?

53. What part of a dollar should $\frac{1}{2}$ of a yard of cloth cost, if 1 yard cost $\frac{1}{2}$ of a dollar?

54. A peck is $\frac{1}{4}$ of a bushel and a quart is $\frac{1}{8}$ of a peck; what part of a bushel is a quart?

55. A man bought $\frac{1}{4}$ of an acre of land and sold $\frac{1}{4}$ of what he bought; what part of an acre did he sell?

56. A man had a $\frac{2}{3}$ interest in a mine and sold $\frac{1}{6}$ of his share; what part did he then own?

57. A yard is $\frac{2}{11}$ of a rod, and a foot is $\frac{1}{3}$ of a yard; what part of a rod is a foot?

58. A contracted to build $\frac{3}{4}$ of a mile of road: he has built $\frac{2}{3}$ of it; how much has he yet to build?

59. A dealer bought $\frac{7}{8}$ of an acre of land and sold $\frac{1}{2}$ of it to one man and $\frac{1}{4}$ of it to another; what part of an acre has he remaining?

Finding the Product when the Multiplicand or the Multiplier is a Mixed Number

Finding the Product when the Multiplier is a Whole Number

101. $3 \times 2\frac{1}{5} = 2\frac{1}{5} + 2\frac{1}{5} + 2\frac{1}{5}$, or $6\frac{3}{5}$, or $3 \times 2 + 3 \times \frac{1}{5}$.

Also, $3 \times 2\frac{1}{5} = 3 \times \frac{11}{5}$, or $\frac{33}{5}$, or $6\frac{3}{5}$.

To multiply a mixed number by a whole number, multiply the integral part and the fractional part separately and add the products; or reduce the mixed number to an improper fraction and multiply it by the whole number.

Illustrative Example

Multiply $456\frac{5}{6} \times 25$.

This shows the different steps:

$$\begin{array}{r} 456\frac{5}{6} \\ 25 \\ \hline 2280 \quad = 5 \times 456 \\ 912 \quad = 20 \times 456 \\ 20\frac{5}{6} = 25 \times \frac{5}{6} \\ \hline 11420\frac{5}{6} = 25 \times 456\frac{5}{6} \end{array}$$

In practice write only this:

$$\begin{array}{r} 456\frac{5}{6} \\ 25 \\ \hline 2280 \\ 912 \\ 20\frac{5}{6} \\ \hline 11420\frac{5}{6} \end{array}$$

Exercise 44

(Solve as many as possible at sight.)

1. $2 \times 1\frac{1}{3}$	2. $2 \times 1\frac{1}{2}$	3. $2 \times 1\frac{1}{4}$
4. $2 \times 1\frac{2}{5}$	5. $2 \times 1\frac{3}{8}$	6. $3 \times 1\frac{1}{3}$
7. $3 \times 2\frac{2}{3}$	8. $3 \times 2\frac{2}{7}$	9. $3 \times 2\frac{3}{7}$
10. $3 \times 2\frac{1}{2}$	11. $4 \times 1\frac{1}{4}$	12. $4 \times 3\frac{3}{4}$
13. $4 \times 2\frac{7}{8}$	14. $5 \times 3\frac{1}{5}$	15. $5 \times 4\frac{3}{10}$
16. $5 \times 2\frac{1}{2}$	17. $5 \times 7\frac{1}{2}$	18. $6 \times 2\frac{1}{2}$

19. Find the length of 2 strips of carpet each $2\frac{1}{4}$ yd. long.

20. How far did a car run in three hours, if it ran $12\frac{1}{4}$ miles each hour?

21. One and one-eighth bushels of seed were sowed on each of 2 acres; how much was sowed on both?

22. If a boy reads on an average one page of a certain book in $1\frac{1}{6}$ minutes, what time will he require to read 5 pages?

23. What length of board is required to make 3 pieces of board each $2\frac{1}{2}$ ft. long?

24. I bought 3 pieces of carpet each $12\frac{1}{2}$ yd. long; how much carpet did I buy?

Find the cost of:

25. 2 bushels of seed at $\$ 2\frac{1}{2}$ a bushel.
26. 6 lb. of sugar at $6\frac{1}{2}¢$ a pound.
27. 6 lb. of starch at $6\frac{1}{4}¢$ a pound.
28. 8 yd. of carpet at $\$ 2\frac{1}{4}$ a yard.
29. 4 yd. of calico at $8\frac{1}{4}¢$ a yard.
30. 6 oranges at $3\frac{1}{3}¢$ apiece.
31. 12 apples at $3\frac{1}{3}¢$ apiece.

32. 20 ft. of wire fence at $1\frac{3}{10}$¢ a foot.

33. 3 yd. of serge at \$ $1\frac{1}{2}$ a yard.

34. 4 yd. of carpet at \$ $3\frac{1}{8}$ a yard.

35. Find the selling price of a flock of 20 sheep that sold at an average price of \$ $5\frac{3}{4}$.

36. Allowing each of 50 sailors $1\frac{3}{4}$ lb. of meat daily, how much meat must be supplied for a cruise of 10 days?

37. A cubic foot of water weighs $62\frac{1}{2}$ pounds, and platinum is about 22 times as heavy as water; find the weight of a cubic foot of platinum.

Finding the Product when the Multiplier is a Mixed Number

102. 1. When the multiplier or the multiplier and the multiplicand both are small mixed numbers, the product may be readily found; thus,

Multiply $2\frac{2}{5}$ by $3\frac{1}{4}$.

$$2\tfrac{2}{5} \times 3\tfrac{1}{4} = \frac{\overset{3}{\cancel{12}}}{5} \times \frac{13}{\cancel{4}} = \frac{39}{5} = 7\tfrac{4}{5}$$

2. When the multiplier is a large mixed number and the multiplicand an integer, the product may be readily found; thus,

Multiply 235 by $32\frac{2}{3}$.

This shows the different steps:

$$\begin{array}{r} 235 \\ 32\frac{2}{3} \\ \hline 470 = 2 \times 235 \\ 705 = 30 \times 235 \\ 156\frac{2}{3} = \frac{2}{3} \times 235 \\ \hline 7676\frac{2}{3} = 32\frac{2}{3} \times 235 \end{array}$$

In practice write only this:

$$\begin{array}{r} 235 \\ 32\frac{2}{3} \\ \hline 470 \\ 705 \\ 156\frac{2}{3} \\ \hline 7676\frac{2}{3} \end{array}$$

3. When the multiplier and the multiplicand are both large mixed numbers, the product may be readily found; thus,

Multiply $127\frac{1}{4} \times 46\frac{2}{3}$.

This shows the different steps:

$$
\begin{array}{rl}
127\frac{1}{4} & \\
46\frac{2}{3} & \\
\hline
762 & = \quad 6 \times 127 \\
508 & = \quad 40 \times 127 \\
11\frac{1}{2} & = \quad 46 \times \frac{1}{4} \\
84\frac{2}{3} & = \quad \frac{2}{3} \times 127 \\
\frac{1}{6} & = \quad \frac{2}{3} \times \frac{1}{4} \\
\hline
5938\frac{1}{3} & = 46\frac{2}{3} \times 127\frac{1}{4}
\end{array}
$$

In practice write only this:

$$
\begin{array}{r}
127\frac{1}{4} \\
46\frac{2}{3} \\
\hline
762 \\
508 \\
11\frac{1}{2} \\
84\frac{2}{3} \\
\frac{1}{6} \\
\hline
5938\frac{1}{3}
\end{array}
$$

Exercise 45

1. $\frac{2}{3} \times 1\frac{1}{2}$
2. $\frac{3}{5} \times 2\frac{2}{3}$
3. $\frac{6}{7} \times 4\frac{1}{2}$
4. $\frac{3}{8} \times 1\frac{7}{9}$
5. $2\frac{3}{4} \times \frac{2}{9}$
6. $3\frac{3}{8} \times \frac{3}{4}$
7. $7\frac{1}{3} \times \frac{2}{9}$
8. $4\frac{1}{2} \times \frac{5}{6}$
9. $1\frac{1}{2} \times 1\frac{1}{3}$
10. $4\frac{1}{2} \times 3\frac{2}{3}$
11. $7\frac{1}{3} \times 2\frac{1}{2}$
12. $3\frac{1}{3} \times 3\frac{1}{2}$
13. $4\frac{2}{3} \times 5\frac{1}{7}$
14. $8\frac{2}{3} \times 7\frac{1}{2}$
15. $4\frac{3}{4} \times 2\frac{1}{4}$

16. $\begin{array}{r} 226 \\ 16\frac{1}{2} \\ \hline \end{array}$
17. $\begin{array}{r} 128 \\ 23\frac{2}{3} \\ \hline \end{array}$
18. $\begin{array}{r} 150 \\ 32\frac{3}{4} \\ \hline \end{array}$
19. $\begin{array}{r} 153 \\ 34\frac{1}{4} \\ \hline \end{array}$
20. $\begin{array}{r} 335 \\ 24\frac{5}{6} \\ \hline \end{array}$
21. $\begin{array}{r} 216 \\ 52\frac{2}{5} \\ \hline \end{array}$
22. $\begin{array}{r} 324 \\ 45\frac{1}{2} \\ \hline \end{array}$
23. $\begin{array}{r} 306 \\ 9\frac{2}{3} \\ \hline \end{array}$
24. $\begin{array}{r} 235 \\ 32\frac{1}{3} \\ \hline \end{array}$
25. $\begin{array}{r} 430 \\ 4\frac{2}{3} \\ \hline \end{array}$
26. $\begin{array}{r} 430 \\ 47\frac{1}{4} \\ \hline \end{array}$
27. $\begin{array}{r} 217 \\ 18\frac{5}{6} \\ \hline \end{array}$
28. $\begin{array}{r} 216\frac{1}{2} \\ 19\frac{2}{3} \\ \hline \end{array}$
29. $\begin{array}{r} 117\frac{2}{3} \\ 26\frac{1}{2} \\ \hline \end{array}$
30. $\begin{array}{r} 421\frac{3}{4} \\ 14\frac{1}{6} \\ \hline \end{array}$
31. $\begin{array}{r} 518\frac{1}{2} \\ 32\frac{5}{6} \\ \hline \end{array}$

32. $143\frac{1}{2} \times 25\frac{2}{3}$

33. $175\frac{3}{4} \times 16\frac{2}{3}$

34. $625\frac{1}{4} \times 37\frac{1}{5}$

35. $409\frac{1}{2} \times 48\frac{1}{2}$

36. $810\frac{3}{4} \times 36\frac{1}{2}$

37. $300\frac{1}{2} \times 52\frac{2}{3}$

38. $177\frac{1}{2} \times 44\frac{5}{6}$

39. $333\frac{4}{5} \times 35\frac{1}{2}$

40. What was the cost of $2\frac{1}{2}$ doz. eggs at 30¢ a dozen?

41. Find the cost of $2\frac{1}{2}$ lb. of sugar at 6¢ a pound.

42. Find the cost of $1\frac{1}{2}$ lb. of butter at 40¢ a pound.

43. Find the cost of $1\frac{3}{4}$ lb. of dried beef at 32¢ a pound.

44. If a man receives \$2.50 for one day's work, how much should he receive for $3\frac{1}{2}$ days' work?

45. If 1 in. on a certain drawing represents $2\frac{1}{2}$ mi., what should $5\frac{1}{2}$ in. on the same drawing represent?

46. Find the cost of $18\frac{3}{4}$ acres of land at \$75 an acre.

47. A gallon of water weighs about $8\frac{1}{3}$ lb.; find the weight of a barrel of water, if it contains $31\frac{1}{2}$ gallons.

48. In one rod there are $5\frac{1}{2}$ yards; how many yards are there in $12\frac{1}{2}$ rods?

49. When eggs are selling at 30¢ a dozen, how much should be paid for $10\frac{1}{2}$ doz.?

50. What should be paid for $12\frac{1}{2}$ cords of wood at \$$1\frac{1}{2}$ a cord?

51. If a farmer sows $2\frac{1}{2}$ bu. of wheat to the acre, how much does he sow on a field containing $2\frac{1}{2}$ A.?

52. There are $16\frac{1}{2}$ feet in a rod; how many feet long is a field whose length is $35\frac{1}{2}$ rods?

REVIEW

Exercise 46

1. What is the distance around a square, if each side of it is $\frac{3}{4}$ ft. long?

2. What is the cost of $2\frac{1}{4}$ acres of land at \$75 an acre?

3. A merchant asked \$15 for an article, but sold it at $\frac{1}{4}$ of this price; for what did he sell it?

4. What was the length of the iron rod used to make 10 bolts, each $5\frac{1}{4}$ in. long?

5. What was the cost of $12\frac{1}{2}$ doz. eggs at 32¢ a dozen?

6. A cubic foot of water weighs $62\frac{1}{2}$ lb. and copper is about 9 times as heavy as water; what is the weight of a cubic foot of copper?

7. Find the cost of cleaning 20 yd. of carpet at $2\frac{1}{2}$¢ per yard.

8. Find the cost of thrashing 60 bu. of oats at $2\frac{1}{2}$¢ a bushel.

9. What should a mechanic be paid for $\frac{3}{4}$ of a day's work at $\$3\frac{1}{2}$ a day?

10. A laborer who is paid $\$1\frac{3}{5}$ for working ten hours a day should be paid how much for six hours of work?

11. When a machinist is earning $\$\frac{3}{8}$ an hour, how much is he earning each working day of 10 hours?

12. Find the cost of a ham weighing $12\frac{3}{4}$ lb. at 16¢ a pound.

13. Two pieces each $10\frac{1}{4}$ ft. long were cut from a log 30 ft. long; what was the length of the piece that remained?

14. If a man takes, on an average, 110 steps per minute, each step $2\frac{1}{2}$ ft. long, how many feet does he walk per hour?

15. What do $21\frac{3}{4}$ yd. of cloth cost at $\$3\frac{3}{5}$ a yard?

16. A woman bought $3\frac{1}{2}$ dozen eggs at 30¢ a dozen and $1\frac{1}{4}$ pounds of dried beef at 40¢ a pound; what was her bill?

17. During a certain month a mail carrier drove $22\frac{3}{4}$ miles each day for 23 days; how many miles did he drive in all?

18. A and B together purchased 24 acres of land at $\$50$ an acre; how much should each pay, if A took $15\frac{1}{2}$ acres and B the remainder?

19. Find the product of $33\frac{1}{3} \times 12\frac{1}{2} \times \frac{1}{3} \times \frac{1}{8}$.

20. How much money must a contractor pay a force of 18 men for the six days of a week, if each works 10 hr. a day at $\$\frac{4}{5}$ an hour?

DIVISION

Dividing a Fraction by a Whole Number

103. 1. $\frac{4}{5} \div 2 = \frac{4 \div 2}{5}$, because in $\frac{4 \div 2}{5}$ there are one half as many fractional units as there are in $\frac{4}{5}$, and the fractional units are of the same size as those in $\frac{4}{5}$.

2. $\frac{3}{4} \div 2 = \frac{3}{2 \times 4}$, because in $\frac{3}{2 \times 4}$ there are as many fractional units as there are in $\frac{3}{4}$, but the fractional units are only one half as large as those in $\frac{3}{4}$.

To divide a fraction by a whole number, divide the numerator or multiply the denominator by the whole number.

NOTE 1. Divide the numerator when possible.

NOTE 2. $\frac{4}{5} \div 2$ and $\frac{1}{2}$ of $\frac{4}{5}$ have the same meaning.

NOTE 3. When a mixed number is to be divided by a whole number the quotient may be found by dividing the integral part and the fractional part separately and adding the results; or by reducing the mixed number to an improper fraction and dividing as stated in Sec. 103.

Illustrative Example

Divide $\frac{27}{50}$ by 45.

$$\frac{27}{50} \div 45 = \frac{\overset{3}{\cancel{27}}}{\underset{5}{\cancel{45}} \times 50} = \frac{3}{250}$$

Exercise 47

(Solve as many as possible at sight.)

Find the quotient:

1. $\frac{2}{5} \div 2$ Thus, $\frac{2}{5} \div 2 = \frac{1}{5}$ [§ 103, 1]
2. $\frac{3}{5} \div 2$ Thus, $\frac{3}{5} \div 2 = \frac{3}{10}$ [§ 103, 2]
3. $\frac{2}{3} \div 2$
4. $\frac{3}{8} \div 2$
5. $\frac{2}{7} \div 2$
6. $\frac{4}{5} \div 2$
7. $\frac{1}{5} \div 2$
8. $\frac{1}{2} \div 3$
9. $\frac{3}{8} \div 3$
10. $\frac{4}{5} \div 3$
11. $\frac{4}{3} \div 2$
12. $\frac{4}{3} \div 3$
13. $\frac{1}{5} \div 3$
14. $\frac{6}{7} \div 3$
15. $\frac{5}{8} \div 2$
16. $\frac{4}{5} \div 5$
17. $\frac{3}{4} \div 5$
18. $\frac{1}{2} \div 5$
19. $\frac{1}{6} \div 5$
20. $\frac{5}{6} \div 5$
21. $\frac{4}{7} \div 3$
22. $\frac{4}{3} \div 6$
23. $1\frac{1}{2} \div 3$
24. $\frac{4}{5} \div 4$
25. $\frac{3}{10} \div 5$
26. $\frac{5}{6} \div 4$
27. $\frac{3}{10} \div 2$
28. $\frac{3}{10} \div 4$
29. $\frac{7}{10} \div 5$
30. $\frac{3}{10} \div 8$
31. $3\frac{9}{10} \div 3$
32. $\frac{9}{10} \div 10$
33. $1\frac{1}{2} \div 2$
34. $2\frac{1}{3} \div 2$
35. $3\frac{3}{5} \div 12$
36. $\frac{5}{12} \div 12$
37. $1\frac{1}{2} \div 10$
38. $\frac{11}{12} \div 22$
39. $\frac{12}{5} \div 24$
40. $\frac{11}{12} \div 33$
41. $\frac{4}{25} \div 16$

42. $\frac{14}{25} \div 28$ 43. $21\frac{3}{5} \div 36$ 44. $\frac{42}{55} \div 35$

45. $\frac{35}{66} \div 70$ 46. $18\frac{3}{4} \div 25$ 47. $\frac{58}{75} \div 29$

48. $\frac{77}{100} \div 55$ 49. $27\frac{1}{9} \div 50$ 50. $\frac{34}{100} \div 51$

51. If 2 yd. of cloth cost $\$\frac{4}{5}$, what does 1 yd. cost?

52. $\frac{5}{8}$ of an acre of land was divided into 5 lots of equal size; what part of an acre did each lot contain?

53. A boy who worked 2 hours for $\frac{1}{5}$ of a dollar earned what part of a dollar per hour?

54. If $\frac{1}{2}$ bu. of grass seed was sowed to 4 acres, what part of a bushel was sowed to the acre?

55. A druggist filled two bottles of equal size with $\frac{1}{4}$ of a pint of oil; what part of a pint did he put into each?

56. $\frac{3}{8}$ of a yard of lace was divided into 3 equal pieces; what part of a yard was there in each piece?

57. $\frac{3}{4}$ of an acre of land was divided into 6 lots of equal size; how much land did each lot contain?

58. A line $\frac{7}{8}$ of an inch long was divided into two equal parts; how long was each part?

59. If $\frac{3}{8}$ of a bushel of lime is used to whitewash both sides of a fence, how much was used to whitewash one side of it?

60. If $\frac{1}{2}$ lb. of shot is divided into 8 equal loads, what part of a pound is there in each load?

61. A laborer cut $\frac{7}{4}$ acres of corn in 2 da.; how much was that per day?

62. If $\frac{1}{4}$ lb. of tea is used for 8 meals, the same quantity for each, how much is that for each meal?

63. If 5 times a number is $37\frac{1}{2}$, what is the number?

64. A man paid $\$12\frac{1}{2}$ for 5 yards of cloth; how much was that a yard?

65. A cord $33\frac{1}{3}$ feet long was cut into 10 equal parts; find the length of each part.

66. A man was paid $\$112\frac{1}{2}$ wages for working 3 months; how much was that per month?

67. $52\frac{1}{2}$ bushels of wheat were put into 21 sacks, in each sack the same amount; how much was put into each sack?

Dividing by a Fraction

104. If $2 \times$ a number $= 8$, the number $= 8 \div 2$.

If $2 \times$ a number $= \frac{2}{3}$, the number $= \frac{2}{3} \div 2$.

If $\frac{3}{4} \times$ a number $= \frac{2}{3}$, **the number** $= \frac{2}{3} \div \frac{3}{4}$.

If $\frac{3}{4}$ of a number $= \frac{2}{3}$,

$\frac{1}{4}$ of the number $= \frac{1}{3}$ of $\frac{2}{3}$;

and $\frac{4}{4}$ of the number $= 4 \times \frac{1}{3}$ of $\frac{2}{3}$, or $\frac{4}{3}$ of $\frac{2}{3}$.

That is, **the number** $= \frac{4}{3} \times \frac{2}{3}$.

But **the number** $= \frac{2}{3} \div \frac{3}{4}$.

Therefore, $\frac{2}{3} \div \frac{3}{4} = \frac{4}{3} \times \frac{2}{3}$.

The quotient of two fractions equals the product of the dividend and the inverse of the divisor.

NOTE. Since a whole number may be regarded as a fraction whose denominator is 1 and a mixed number may be reduced to a fraction, the principle stated in Sec. 104 applies in finding the quotient when the dividend is an integer or a mixed number and the divisor a fraction or a mixed number.

Illustrative Examples

1. Divide $\frac{4}{5}$ by $\frac{2}{3}$.

$$\frac{4}{5} \div \frac{2}{3} = \frac{3}{\cancel{2}} \times \frac{\overset{2}{\cancel{4}}}{5} = \frac{6}{5} = 1\frac{1}{5}$$

2. Divide 4 by $\frac{3}{5}$.

$$4 \div \tfrac{3}{5} = \tfrac{5}{3} \times 4 = \tfrac{20}{3} = 6\tfrac{2}{3}$$

3. Divide $12\frac{1}{2}$ by $\frac{5}{8}$.

$$12\tfrac{1}{2} \div \tfrac{5}{8} = \frac{25}{2} \div \frac{5}{8} = \frac{\overset{4}{\cancel{8}}}{\underset{1}{\cancel{5}}} \times \frac{\overset{5}{\cancel{25}}}{\underset{1}{\cancel{2}}} = 20$$

4. Divide $3\frac{1}{3}$ by $2\frac{1}{2}$.

$$3\tfrac{1}{3} \div 2\tfrac{1}{2} = \frac{10}{3} \div \frac{5}{2} = \frac{\overset{2}{\cancel{2}}}{\underset{1}{\cancel{5}}} \times \frac{\overset{2}{\cancel{10}}}{3} = \frac{4}{3} = 1\tfrac{1}{3}$$

Exercise 48

(Solve as many as possible at sight.)

Divide:

1. $1 \div \frac{1}{3}$ Thus, $1 \div \frac{1}{3} = 3$
2. $5 \div \frac{1}{2}$ Thus, $5 \div \frac{1}{2} = 10$
3. $1 \div \frac{2}{5}$ Thus, $1 \div \frac{2}{5} = \frac{5}{2}$, or $2\frac{1}{2}$

4. $1 \div \frac{1}{5}$	5. $1 \div \frac{1}{6}$	6. $1 \div \frac{1}{8}$	7. $1 \div \frac{1}{10}$
8. $1 \div \frac{1}{12}$	9. $1 \div \frac{1}{16}$	10. $1 \div \frac{1}{15}$	11. $1 \div \frac{1}{14}$
12. $1 \div \frac{1}{32}$	13. $1 \div \frac{1}{7}$	14. $1 \div \frac{1}{9}$	15. $1 \div \frac{1}{50}$
16. $3 \div \frac{1}{2}$	17. $4 \div \frac{1}{3}$	18. $5 \div \frac{1}{8}$	19. $6 \div \frac{1}{2}$
20. $3 \div \frac{1}{5}$	21. $2 \div \frac{1}{8}$	22. $8 \div \frac{1}{2}$	23. $4 \div \frac{1}{4}$
24. $2 \div \frac{1}{10}$	25. $5 \div \frac{1}{2}$	26. $6 \div \frac{1}{8}$	27. $4 \div \frac{1}{10}$
28. $8 \div \frac{1}{3}$	29. $4 \div \frac{1}{8}$	30. $8 \div \frac{1}{5}$	31. $5 \div \frac{1}{10}$
32. $1 \div \frac{3}{4}$	33. $1 \div \frac{4}{5}$	34. $1 \div \frac{2}{3}$	35. $1 \div \frac{5}{6}$
36. $1 \div \frac{4}{3}$	37. $1 \div \frac{3}{2}$	38. $1 \div \frac{7}{8}$	39. $1 \div \frac{6}{5}$
40. $1 \div \frac{3}{8}$	41. $1 \div \frac{5}{12}$	42. $1 \div \frac{7}{12}$	43. $1 \div \frac{3}{16}$

44. $1 \div 1\frac{1}{2}$ SUGGESTION. $1 \div 1\frac{1}{2} = 1 \div \frac{3}{2}$

45. $1 \div 2\frac{1}{2}$ 46. $1 \div 3\frac{1}{3}$ 47. $1 \div 3\frac{1}{2}$ 48. $1 \div 4\frac{1}{2}$

49. $1 \div 1\frac{2}{3}$ 50. $1 \div 1\frac{3}{4}$ 51. $1 \div 8\frac{1}{3}$ 52. $1 \div 6\frac{2}{3}$

53. $2 \div \frac{3}{4}$ 54. $3 \div \frac{2}{3}$ 55. $2 \div \frac{4}{5}$ 56. $3 \div \frac{5}{4}$

57. $2 \div \frac{3}{8}$ 58. $2 \div \frac{3}{5}$ 59. $2 \div \frac{3}{2}$ 60. $5 \div \frac{2}{3}$

61. $4 \div \frac{3}{4}$ 62. $5 \div \frac{3}{2}$ 63. $4 \div \frac{2}{3}$ 64. $2 \div \frac{3}{10}$

65. $2 \div 1\frac{1}{3}$ SUGGESTION. $2 \div 1\frac{1}{3} = 2 \div \frac{4}{3}$

66. $3 \div 1\frac{1}{2}$ 67. $3 \div 1\frac{2}{3}$ 68. $2 \div 1\frac{3}{4}$ 69. $2 \div 1\frac{3}{5}$

70. $\frac{2}{3} \div \frac{3}{4}$ 71. $\frac{4}{5} \div \frac{3}{4}$ 72. $\frac{6}{7} \div \frac{8}{9}$ 73. $\frac{9}{10} \div \frac{3}{2}$

74. $\frac{15}{16} \div \frac{3}{4}$ 75. $\frac{7}{8} \div \frac{9}{10}$ 76. $\frac{9}{16} \div \frac{11}{12}$ 77. $\frac{5}{9} \div \frac{25}{24}$

78. $2 \div \frac{2}{3}$ 79. $3 \div \frac{4}{5}$ 80. $7 \div \frac{3}{4}$ 81. $5 \div \frac{5}{4}$

82. $4 \div 1\frac{1}{3}$ 83. $9 \div 3\frac{1}{2}$ 84. $8 \div 3\frac{2}{3}$ 85. $4 \div 7\frac{1}{2}$

86. $1\frac{1}{2} \div 1\frac{1}{3}$ 87. $3\frac{1}{3} \div 1\frac{3}{4}$ 88. $2\frac{1}{2} \div 7\frac{1}{2}$ 89. $3\frac{1}{3} \div 2\frac{1}{2}$

90. $1\frac{3}{4} \div 5\frac{1}{4}$ 91. $1\frac{2}{3} \div 6\frac{2}{3}$ 92. $12\frac{1}{2} \div 6\frac{1}{4}$ 93. $2\frac{6}{7} \div 14\frac{2}{7}$

94. $3 \div 2\frac{2}{3}$ 95. $4 \div 1\frac{1}{2}$ 96. $10 \div 3\frac{1}{3}$

97. At $\$\frac{1}{8}$ a yard, how many yards of cloth can be bought for $\$12$?

98. How many $\frac{3}{4}$-acre lots can be made from 15 acres of land?

99. If $\frac{3}{4}$ of an inch in a drawing represents a mile, what distance is represented by a line 3 inches long?

100. How many cords $\frac{2}{3}$ of a foot long can be made from a piece $12\frac{2}{3}$ ft. long?

101. At $\$\frac{3}{4}$ each, how many yards of cloth can be bought for $\$12\frac{1}{2}$?

102. How many bushels of seed can be bought for $\$12\frac{1}{2}$ at $\$2\frac{1}{2}$ a bushel?

Dividing by Large Mixed Numbers

105. When the divisor is a large mixed number, the following is a desirable way of finding the quotient; thus,

Divide $233\frac{1}{3}$ by $12\frac{1}{2}$.

$12\frac{1}{2}$ $233\frac{1}{3}$
6 6
$75)1400(18\frac{2}{3}$
75
650
600
50
$\frac{50}{75} = \frac{2}{3}$

Multiply both dividend and divisor by the l. c. d. of the fractions before dividing.

Exercise 49

Divide:

1. 100 by $14\frac{2}{7}$
2. $87\frac{1}{2}$ by $62\frac{1}{2}$
3. $133\frac{1}{3}$ by $66\frac{2}{3}$
4. $112\frac{1}{2}$ by $6\frac{1}{4}$
5. $127\frac{1}{2}$ by $25\frac{1}{2}$
6. $144\frac{4}{9}$ by $6\frac{2}{3}$
7. $262\frac{1}{2}$ by $18\frac{3}{4}$
8. $183\frac{1}{3}$ by $26\frac{2}{3}$
9. $237\frac{1}{2}$ by $33\frac{1}{3}$
10. $666\frac{2}{3}$ by $55\frac{5}{9}$
11. $150\frac{1}{2}$ by $16\frac{2}{3}$
12. $133\frac{1}{3}$ by $18\frac{3}{4}$
13. $141\frac{1}{2}$ by $56\frac{3}{5}$
14. $366\frac{1}{4}$ by $48\frac{5}{6}$
15. $444\frac{4}{9}$ by $33\frac{1}{3}$
16. $666\frac{2}{3}$ by $18\frac{3}{4}$
17. $333\frac{1}{3}$ by $16\frac{2}{3}$
18. $652\frac{1}{2}$ by $32\frac{5}{8}$
19. $600\frac{5}{12}$ by $55\frac{5}{9}$.
20. $889\frac{8}{9}$ by $55\frac{5}{9}$

21. 20 bu. of grain are sowed on a field, $1\frac{1}{2}$ bu. to each acre; how many acres are in the field?

22. At $6\frac{1}{4}$ ¢ a pound, how many pounds of sugar can be bought for 25 ¢?

23. How many bags are required to hold 100 bu. of seed, if each bag holds $1\frac{1}{4}$ bu.?

24. If the distance around a carriage wheel is $12\frac{1}{2}$ ft., how many times will it turn in going 150 ft.?

25. There are $5\frac{1}{2}$ yd. in a rod; how many rods are there in 110 yd.?

26. A cubic foot of water weighs $62\frac{1}{2}$ lb.; how many cubic feet of water are there in a ton of 2000 lb.?

27. A barrel contains $31\frac{1}{2}$ gal. and a hogshead 63 gal. How many barrels are there in a hogshead?

28. How many $\$ 2\frac{1}{2}$ gold pieces should be given in exchange for $\$ 62\frac{1}{2}$?

29. How many times can a bucket holding $4\frac{3}{4}$ gal. be filled from a barrel holding $47\frac{1}{2}$ gal.?

30. At $\$ 1\frac{1}{3}$ a yard, how many yards of silk can be bought for $\$ 18\frac{1}{2}$?

REVIEW

Exercise 50

1. $\$ 22\frac{1}{2}$ was paid for three sheep, for each the same sum; what was paid for each?

2. At $\$ \frac{4}{5}$ a yard, how many yards of cloth can be bought for $\$ 5$?

3. How many days will 10 bu. of oats last a horse that eats $\frac{3}{8}$ of a bushel per day?

4. If $\frac{3}{4}$ of a bushel of seed cost $\$ 6$, how much was that per bushel?

5. When $\frac{1}{4}$ bu. of potatoes sells for $\$ 0.30$, how much is that a bushel?

6. How many $\frac{1}{8}$-acre plats of land can be made from $2\frac{1}{4}$ acres?

7. If $\frac{1}{2}$ an acre of wheat yields $18\frac{3}{4}$ bu., what is the yield per acre?

8. If a $4\frac{1}{2}$-lb. dressed chicken cost 63 ¢, how much is that a pound?

9. $5\frac{1}{2}$ gal. of sirup will fill how many bottles, each holding $\frac{1}{8}$ of a gallon?

10. If a woman was charged \$ $10\frac{1}{2}$ for a week's board, how much was that a day?

11. How many strips of carpet, each $\frac{3}{4}$ of a yard wide, are needed to cover a floor 18 ft. wide?

12. How many strips of wall paper $2\frac{1}{4}$ ft. wide are required to cover a ceiling 27 ft. wide?

13. How many pounds of copper can be bought for \$ 6 at \$ $\frac{3}{4}$ a pound?

14. A rod is $5\frac{1}{2}$ yards; how many rods are there in 1100 yd.?

15. How many trees can be bought for \$ $10\frac{1}{2}$ at \$ $1\frac{3}{4}$ each?

16. How many $3\frac{1}{2}$-ft. boards can be sawed from a board 24 ft. long? What length will there be left over?

17. A piece of iron 8 inches long was cut into bolts $\frac{3}{4}$ of an inch long; how many bolts did it make, and what was the length of the piece left over?

18. What average does a girl receive in four studies, if she receives a mark of 80 in one, $82\frac{1}{2}$ in another, $83\frac{3}{4}$ in another, and $87\frac{1}{2}$ in another?

19. A quart of milk weighs about $2\frac{1}{2}$ lb.; how many quarts of milk will weigh 2000 lb.?

20. If a laborer was paid \$ 2 for $1\frac{1}{4}$ days' work, what should he be paid for $2\frac{1}{2}$ days' work?

COMPLEX FRACTIONS

106. A complex fraction is an expression in the form of a fraction, one or both of whose terms are either fractions or mixed numbers.

Thus, $\frac{\frac{3}{4}}{2}$, $\frac{2}{\frac{3}{4}}$, $\frac{\frac{2}{3}}{\frac{3}{4}}$, and $\frac{2\frac{1}{2}}{3\frac{1}{3}}$ are complex fractions.

NOTE. A complex fraction may be regarded as denoting that the numerator is to be divided by the denominator.

Illustrative Examples

1. Reduce $\frac{\frac{3}{4}}{2}$ to a simple fraction.

SOLUTION. $\frac{\frac{3}{4}}{2} = \frac{3}{4 \times 2}$ [§ 103] $= \frac{3}{8}$.

2. Reduce $\frac{2}{\frac{3}{4}}$ to a simple fraction.

SOLUTION. $\frac{2}{\frac{3}{4}} = \frac{4}{3}$ of 2 [§ 104, note] $= \frac{8}{3}$, or $2\frac{2}{3}$.

3. Reduce $\frac{\frac{2}{3}}{\frac{3}{4}}$ to a simple fraction.

SOLUTION. $\frac{\frac{2}{3}}{\frac{3}{4}} = \frac{2}{3} \div \frac{3}{4} = \frac{4}{3} \times \frac{2}{3}$ [§ 104] $= \frac{8}{9}$.

4. Reduce $\frac{2\frac{1}{2}}{3\frac{1}{3}}$ to a simple fraction.

SOLUTION. $\frac{2\frac{1}{2}}{3\frac{1}{3}} = \frac{\frac{5}{2}}{\frac{10}{3}} = \frac{3}{10} \times \frac{5}{2} = \frac{3}{4}$.

5. Reduce $\frac{3\frac{1}{5} - 2\frac{1}{2}}{3\frac{1}{4} + 1\frac{1}{2}}$ to a simple fraction.

SOLUTION. $\frac{3\frac{1}{5} - 2\frac{1}{2}}{3\frac{1}{4} + 1\frac{1}{2}} = \frac{20(3\frac{1}{5} - 2\frac{1}{2})}{20(3\frac{1}{4} + 1\frac{1}{2})} = \frac{64 - 50}{65 + 30} = \frac{14}{95}$ [§ 89]

Exercise 51

Reduce to simple fractions:

1. $\frac{\frac{4}{5}}{2}$
2. $\frac{10}{\frac{3}{8}}$
3. $\frac{4}{\frac{8}{3}}$
4. $\frac{5}{\frac{9}{20}}$
5. $\frac{10\frac{5}{12}}{5}$
6. $\frac{3\frac{1}{3}}{10}$
7. $\frac{100}{16\frac{2}{3}}$
8. $\frac{50}{33\frac{1}{3}}$
9. $\frac{\frac{4}{5}}{\frac{7}{8}}$
10. $\frac{12\frac{1}{2}}{\frac{3}{8}}$
11. $\frac{\frac{7}{8}}{62\frac{1}{2}}$
12. $\frac{12\frac{1}{2}}{6\frac{1}{4}}$
13. $\frac{16\frac{2}{3}}{83\frac{1}{3}}$
14. $\frac{37\frac{1}{2}}{87\frac{1}{2}}$
15. $\frac{14\frac{2}{7}}{71\frac{3}{7}}$
16. $\frac{66\frac{2}{3}}{4\frac{1}{6}}$
17. $\frac{2+\frac{2}{3}}{2-\frac{2}{3}}$
18. $\frac{4+2\frac{1}{12}}{5-1\frac{5}{6}}$
19. $\frac{\frac{8}{9}-\frac{2}{3}}{\frac{5}{9}-\frac{1}{3}}$
20. $\frac{5\frac{3}{4}-\frac{2}{3}}{3\frac{1}{4}+\frac{1}{12}}$
21. $\frac{12\frac{1}{2}+6\frac{1}{4}}{18\frac{3}{4}-3\frac{1}{8}}$
22. $\frac{\frac{3}{100}+\frac{9}{50}+\frac{4}{25}+\frac{3}{5}}{\frac{92}{100}-\frac{23}{50}+\frac{12}{25}-\frac{16}{25}}$
23. $\frac{\frac{3-\frac{1}{5}}{4}-\frac{3+\frac{1}{5}}{8}}{16}$
24. $\frac{(\frac{2}{3}+\frac{3}{4})\times(\frac{3}{4}-\frac{2}{3})}{(\frac{9}{16}-\frac{4}{9})\div(\frac{2}{3}+\frac{3}{4})}$
25. $\frac{\frac{2}{3}\times\frac{3}{4}}{\frac{1}{4}\div\frac{1}{2}}$
26. $\frac{12\frac{1}{2}+37\frac{1}{2}}{50\div100}$
27. $\frac{87\frac{1}{2}+12\frac{1}{2}}{\frac{1}{2}\times\frac{1}{4}\div\frac{1}{8}}$
28. $\frac{16\frac{2}{3}\times\frac{1}{10}\times\frac{1}{2}}{1\div\frac{1}{2}}$
29. $\frac{\frac{5}{6}\text{ of }\frac{2}{5}\div5}{\frac{1}{2}+\frac{2}{5}}$
30. $\frac{(\frac{1}{2}+\frac{1}{4})-(\frac{1}{2}\text{ of }\frac{1}{4})}{2\frac{1}{2}+1\frac{1}{4}}$
31. $\frac{\frac{1}{2}\text{ of }\frac{1}{4}\div\frac{1}{8}+2\frac{1}{2}\div5}{2\frac{1}{2}\div5}$

REVIEW OF FRACTIONS

Exercise 52

1. Read and tell what each means:

$\frac{1}{2}$; $\frac{1}{4}$; $\frac{1}{6}$; $\frac{1}{10}$

2. What is a fractional unit?

3. Read, and tell what each means:

$\frac{4}{5}$; $\frac{3}{4}$; $\frac{5}{6}$; $\frac{9}{10}$

4. What is a fraction?

5. What does the denominator of a fraction show? the numerator?

6. What are the numerator and denominator of a fraction called?

7. In the following, name the proper fractions; the improper fractions; the mixed numbers:

$\frac{2}{3}$; $\frac{1}{8}$; $\frac{2}{2}$; $1\frac{1}{2}$; $\frac{8}{9}$; $\frac{9}{8}$; $3\frac{1}{3}$; $\frac{9}{4}$; $\frac{5}{4}$; $\frac{3}{2}$; $2\frac{1}{3}$; $\frac{11}{10}$

8. What is a proper fraction? an improper fraction? a mixed number?

Reduce to improper fractions:

9. $18\frac{3}{4}$	10. $66\frac{2}{3}$	11. $50\frac{1}{2}$	12. $88\frac{1}{8}$
13. $56\frac{3}{4}$	14. $88\frac{8}{9}$	15. $870\frac{5}{8}$	16. $96\frac{7}{8}$
17. $112\frac{1}{2}$	18. $137\frac{1}{2}$	19. $133\frac{1}{3}$	20. $266\frac{2}{3}$

Reduce to whole or mixed numbers:

21. $\frac{100}{3}$	22. $\frac{100}{6}$	23. $\frac{200}{8}$	24. $\frac{100}{12}$
25. $\frac{100}{16}$	26. $\frac{200}{4}$	27. $\frac{200}{6}$	28. $\frac{200}{12}$
29. $\frac{300}{16}$	30. $\frac{300}{32}$	31. $\frac{500}{16}$	32. $\frac{600}{16}$

33. When is a fraction in its lowest terms?

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34. What is the shortest way of reducing a fraction to its lowest terms?

Reduce to lowest terms:

35. $\frac{180}{195}$ 36. $\frac{144}{124}$ 37. $\frac{270}{288}$ 38. $\frac{182}{168}$

39. $\frac{125}{175}$ 40. $\frac{279}{288}$ 41. $\frac{232}{548}$ 42. $\frac{369}{423}$

43. $\frac{405}{450}$ 44. $\frac{108}{324}$ 45. $\frac{180}{48}$ 46. $\frac{510}{330}$

47. What is the least common denominator of two or more fractions in their lowest terms?

Reduce to fractions having the least common denominator:

48. $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{7}{12}$

49. $\frac{4}{9}$, $\frac{7}{21}$, $\frac{5}{28}$, and $\frac{7}{36}$

50. $\frac{2}{3}$, $\frac{4}{5}$, $\frac{3}{10}$, and $\frac{2}{15}$

51. $\frac{3}{7}$, $\frac{5}{14}$, and $\frac{2}{28}$

52. $\frac{12}{13}$, $\frac{5}{26}$, and $\frac{4}{39}$

53. $\frac{9}{15}$, $\frac{8}{20}$, and $\frac{15}{25}$

54. $\frac{8}{63}$ and $\frac{2}{105}$

55. $\frac{24}{125}$, and $\frac{48}{175}$

56. Tell what is meant by similar fractions. Illustrate.

57. State how to find the sum of two or more similar fractions.

58. What is the sum of $\frac{3}{8}$, $\frac{1}{8}$, and $\frac{5}{8}$?

59. State how the sum of two or more fractions is found, if they do not have a common denominator.

Find the sum of:

60. $\frac{2}{15}+\frac{3}{10}+\frac{5}{6}$

61. $\frac{7}{10}+\frac{2}{15}+\frac{2}{3}$

62. $\frac{5}{8}+\frac{7}{12}+\frac{1}{24}$

63. $\frac{2}{5}+\frac{7}{15}+\frac{7}{10}$

64. $\frac{5}{12}+\frac{3}{16}+\frac{5}{24}$

65. $\frac{7}{15}+\frac{8}{9}+\frac{5}{12}$

Add upward; add across:

66. $\frac{1}{4}+\frac{1}{3}+\frac{1}{2}=?$
$\frac{5}{6}+\frac{2}{3}+\frac{5}{12}=?$
$?+?+?=?$

67. $\frac{5}{6}+\frac{7}{8}+\frac{2}{3}=?$
$\frac{17}{24}+\frac{19}{48}+\frac{2}{3}=?$
$?+?+?=?$

68. State how to subtract two similar fractions.

69. What is the difference between $\frac{7}{8}$ and $\frac{3}{8}$?

70. State how to subtract two fractions with unlike denominators.

Subtract:

71. $\frac{1}{12} - \frac{1}{15}$ 72. $\frac{1}{20} - \frac{1}{30}$ 73. $\frac{13}{15} - \frac{6}{25}$

74. $\frac{17}{18} - \frac{5}{24}$ 75. $\frac{7}{12} - \frac{7}{20}$ 76. $\frac{9}{20} - \frac{9}{25}$

77. $\frac{1}{30} - \frac{1}{35}$ 78. $\frac{1}{8} - \frac{1}{12}$ 79. $\frac{8}{9} - \frac{7}{12}$

80. $\frac{6}{7} - \frac{9}{35}$ 81. $\frac{3}{10} - \frac{3}{11}$ 82. $\frac{23}{24} - \frac{5}{16}$

Find the value of:

83. $\frac{3}{4} + \frac{5}{6} - \frac{7}{8}$ 84. $\frac{5}{3} - \frac{5}{9} + \frac{5}{6}$ 85. $1 - (\frac{3}{26} + \frac{5}{13})$

86. $\frac{5}{8} + \frac{9}{16} - \frac{7}{24}$ 87. $\frac{4}{5} - \frac{7}{10} + \frac{3}{40}$ 88. $\frac{7}{8} - (\frac{4}{5} - \frac{2}{3})$

Add:

89. $18\frac{3}{4}$, $56\frac{7}{8}$, $19\frac{3}{16}$, $7\frac{1}{2}$

90. $17\frac{4}{5}$, $26\frac{3}{10}$, $5\frac{7}{15}$, $19\frac{2}{3}$

91. $33\frac{1}{3}$, $16\frac{2}{3}$, $22\frac{2}{9}$, $4\frac{1}{6}$

92. $42\frac{9}{16}$, $7\frac{7}{12}$, $13\frac{5}{24}$, $9\frac{2}{3}$

Add upward; subtract across:

93.
$100 - 66\frac{2}{3} = ?$
$88\frac{8}{9} - 16\frac{2}{3} = ?$
$33\frac{1}{3} - 6\frac{1}{6} = ?$
$20 - 8\frac{1}{3} = ?$
$? - ? = ?$

94.
$50 - 6\frac{1}{4} = ?$
$100 - 16\frac{1}{4} = ?$
$87\frac{1}{2} - 6\frac{1}{4} = ?$
$133\frac{1}{3} - 8\frac{1}{3} = ?$
$? - ? = ?$

95. Tell how to multiply a fraction by a whole number.

Name at sight the product:

96. $2 \times \frac{2}{9}$ 97. $3 \times \frac{7}{9}$ 98. $4 \times \frac{5}{16}$

99. $5 \times \frac{3}{25}$ 100. $5 \times \frac{7}{10}$ 101. $5 \times \frac{10}{3}$

102. Tell how to multiply a whole number by a fraction.

Name at sight the product:

103. $\frac{3}{4} \times 3$ 104. $\frac{3}{4} \times 4$ 105. $\frac{3}{8} \times 5$

106. $\frac{3}{10} \times 5$ 107. $\frac{3}{8} \times 8$ 108. $\frac{7}{6} \times 6$

109. Tell how to find the product of two fractions.

Name at sight the product:

110. $\frac{1}{2} \times \frac{1}{3}$ 111. $\frac{1}{5} \times \frac{2}{3}$ 112. $\frac{2}{3} \times \frac{2}{5}$ 113. $\frac{2}{3} \times \frac{3}{5}$

114. $\frac{3}{5} \times \frac{5}{3}$ 115. $\frac{5}{4} \times \frac{4}{9}$ 116. $\frac{3}{8} \times \frac{4}{5}$ 117. $\frac{5}{6} \times \frac{18}{25}$

Multiply, expressing the answer in the simplest form:

118. $\frac{14}{15} \times \frac{25}{21}$ 119. $\frac{2}{3} \times \frac{3}{4} \times \frac{4}{5}$ 120. $\frac{12}{25} \times \frac{35}{36} \times \frac{8}{9}$

121. $\frac{20}{21} \times \frac{14}{15}$ 122. $\frac{3}{8} \times \frac{16}{15} \times \frac{5}{4}$ 123. $\frac{7}{16} \times \frac{32}{49} \times \frac{7}{8}$

124. Tell how to multiply a mixed number by a whole number.

Multiply at sight:

125. $2 \times 3\frac{2}{5}$ 126. $2 \times 3\frac{2}{7}$ 127. $2 \times 2\frac{3}{8}$

128. $3 \times 2\frac{1}{9}$ 129. $3 \times 2\frac{1}{2}$ 130. $3 \times 2\frac{5}{6}$

Multiply:

131. $58 \times 26\frac{1}{4}$ 132. $98 \times 32\frac{1}{4}$ 133. $98 \times 38\frac{3}{4}$ 134. $76 \times 52\frac{2}{3}$

135. $216\frac{2}{3} \times 33\frac{1}{3}$ 136. $128\frac{5}{6} \times 54\frac{2}{3}$ 137. $132\frac{1}{4} \times 29\frac{1}{3}$ 138. $296\frac{3}{8} \times 35\frac{2}{3}$

139. Tell how to divide a fraction by a whole number.

Divide at sight:

140. $\frac{4}{5} \div 2$ 141. $\frac{8}{3} \div 4$ 142. $\frac{2}{3} \div 5$ 143. $\frac{2}{3} \div 6$

144. $\frac{7}{8} \div 5$ 145. $\frac{3}{5} \div 5$ 146. $\frac{10}{3} \div 5$ 147. $\frac{10}{3} \div 3$

148. Tell how to find the quotient of two fractions; also how to find the quotient if a whole number or a mixed number is to be divided by a fraction.

149. Name at sight the quotients:

$1 \div \frac{1}{3}$; $1 \div \frac{1}{6}$; $1 \div \frac{1}{10}$; $1 \div \frac{1}{12}$.

150. Name at sight the quotients:

$3 \div \frac{1}{6}$; $3 \div \frac{1}{8}$; $5 \div \frac{1}{5}$; $7 \div \frac{1}{10}$.

151. Name at sight the quotients:

$1 \div \frac{5}{6}$; $1 \div \frac{9}{10}$; $1 \div \frac{5}{2}$; $1 \div \frac{7}{3}$.

152. Name at sight the quotients:

$2 \div \frac{3}{5}$; $3 \div \frac{3}{4}$; $5 \div \frac{10}{3}$; $4 \div \frac{4}{5}$;

$2 \div 1\frac{1}{2}$; $3 \div 1\frac{1}{3}$; $4 \div 1\frac{1}{4}$; $2 \div 1\frac{1}{5}$.

Divide:

153. $\frac{9}{10} \div \frac{24}{35}$ 154. $75 \div \frac{3}{4}$ 155. $9 \div \frac{7}{15}$

156. $\frac{15}{16} \div \frac{25}{27}$ 157. $100 \div \frac{25}{2}$ 158. $100 \div \frac{100}{3}$

Divide:

159. $66\frac{2}{3} \div 16\frac{2}{3}$ 160. $37\frac{1}{2} \div 16\frac{1}{4}$

161. $555\frac{5}{9} \div 66\frac{2}{3}$ 162. $666\frac{2}{3} \div 77\frac{7}{9}$

163. A woman who charges $\$1\frac{3}{4}$ a day for board should charge how much for 15 days' board?

164. When coal costs \$ 5 a ton, what part of a ton can be bought for \$ 1? $\$2\frac{1}{2}$? $\$1\frac{1}{4}$?

165. How many yards of cloth can be bought for $\$2\frac{1}{2}$ at $\$\frac{1}{2}$ a yard?

166. If 4 lb. of spices were put up in 16 equal packages, what was the weight of each package?

167. When eggs are selling at 30¢ a dozen, what is the price per score?

168. What is the distance around a tree if a string $1\frac{7}{8}$ yards long will reach three times around it?

169. A lady paid 6 cents for $\frac{3}{4}$ of a yard of muslin; what was the price per yard?

170. If 24 bu. of potatoes were raised on $\frac{3}{4}$ of an acre, what was the yield per acre?

171. If $\frac{3}{4}$ of a bushel of oats weighs 24 pounds, what does a bushel weigh?

172. After spending 12 cents, a boy finds he has spent $\frac{6}{7}$ of his money; how much money had he?

173. A man, after walking 8 miles, finds he has yet $\frac{5}{9}$ of his journey to go; how long is his journey?

174. If the distance around a carriage wheel is $7\frac{1}{2}$ feet, how many times has it turned in going 1 mile, or 5280 feet?

175. A cubic foot of water weighs $62\frac{1}{2}$ lb.; how many cubic feet of water are there in 2 tons of 2000 lb. each?

176. If the cloth for a suit cost $\$15\frac{3}{4}$ at $\$2\frac{1}{4}$ a yard, how many yards were bought for the suit?

177. How many pounds of lard at $11\frac{1}{4}¢$ a pound can be bought for 6 dozen eggs at $12\frac{1}{2}¢$ a dozen?

178. How many times can a bucket holding $2\frac{1}{4}$ gallons be filled from a barrel holding $31\frac{1}{2}$ gallons?

179. I bought $2\frac{1}{2}$ dozen eggs at $20¢$ a dozen and $\frac{3}{4}$ of a pound of dried beef at 24 cents a pound; what was my bill?

180. If $3\frac{1}{2}$ yards of ribbon are cut into pieces $\frac{1}{4}$ of a yard long, how many pieces are made?

181. If 2 lb. of cheese can be bought for $\frac{1}{4}$ of a dollar, how many pounds can be bought for $\frac{7}{8}$ of a dollar?

182. A quart of milk weighs about $2\frac{1}{2}$ pounds; how many quarts are there in 100 pounds?

183. How long will $4\frac{3}{4}$ bushels of oats last a horse which is fed $\frac{3}{4}$ of a bushel a day?

184. A owned $\frac{2}{3}$ of an acre of land and sold $\frac{3}{4}$ of it; what part of an acre had he then?

185. Black tea forms, by weight, $\frac{2}{5}$ of a mixture of black and green tea. There are $2\frac{1}{4}$ pounds of green tea in the mixture; what is the weight of the mixture?

186. From an acre on which $2\frac{1}{2}$ bushels of seed were sown, the yield is 20 bushels; what is the yield from each bushel of seed?

187. A merchant paid 4 times as much for a pound of coffee as for a pound of sugar, and for both he paid $31\frac{1}{4}$ cents; find the cost of each per pound.

188. $\frac{2}{7}$ of the weight of a bushel of corn equals $\frac{1}{2}$ of the weight of a bushel of oats, and the difference of their weights is 24 pounds; find the weight of a bushel of each.

189. A dealer bought an article for \$ 20 and sold it at a gain of $\frac{2}{5}$ of the cost; find the selling price.

190. The gain made in selling a pair of shoes was $\$ 1\frac{2}{5}$; if this was $\frac{7}{20}$ of the cost, what was the selling price?

191. One pipe can carry off the water in a tank in 6 hours, and another pipe can carry it off in 4 hours; what part of the water can both carry off in an hour?

192. There are 2000 pounds in a short ton, which is $\frac{25}{28}$ as heavy as a long ton; how many pounds are there in a long ton?

193. At $\$15\frac{3}{5}$ a ton, find the cost of $12\frac{1}{2}$ tons of hay.

194. A bankrupt's property is $\$3000$, and with it he can pay $\frac{2}{3}$ of his debts; what are his debts?

195. A farmer raised $286\frac{2}{3}$ bushels of wheat on $14\frac{1}{3}$ acres; what was the yield per acre?

196. Find the value of $\frac{8}{9} \times \frac{3}{4} + \frac{1}{3} \div \frac{1}{4} \times 2\frac{1}{2} - 1\frac{1}{4}$.

197. $\frac{3}{4}$ of a number was diminished by 10, and then there remained $\frac{7}{12}$ of the number; find the number.

198. Express in words $\frac{3}{6}\frac{0}{2}\frac{4}{2}$.

199. 60 pounds of good wheat produce about $43\frac{5}{9}$ pounds of flour; how many pounds of wheat will produce a barrel of 196 pounds of flour?

200. From a piece of cloth containing 24 yards there were sold $2\frac{1}{2}$ yards, $3\frac{3}{4}$ yards, and $12\frac{1}{8}$ yards. Find the value of the remainder at $\$1\frac{1}{3}$ a yard.

201. A laborer who earns $\$1\frac{2}{3}$ a day spends $\$\frac{3}{4}$ a day for support and saves the remainder. If he has already saved $\$24\frac{1}{4}$, how many days more must he work to save his rent, which is $\$60$?

202. A merchant sold $\$15$ worth of potatoes at $\$\frac{3}{4}$ a bushel, thereby making a profit of $\$5$. What did he pay for them per bushel?

203. What part of a gallon is $2\frac{1}{2}$ qt.?

204. The sum of three numbers is 100. If the first is $37\frac{1}{2}$, and the second is $12\frac{1}{2}$ more than the first, what is the third?

205. At $\$33\frac{1}{3}$ per month, in how many years would the rent of a house amount to $\$4000$?

GENERAL REVIEW

Exercise 53

1. Divide at sight each of the following numbers by 100, stating the quotient and the remainder:

711; 910; 1565; 7706; 8109; 9001.

2. What was the average gain per head on 25 horses bought for $4375 and sold for $5000?

3. A farmer mixed an equal number of bushels of corn and of oats for horse feed and found that the mixture weighed 1760 lb. How many bushels of feed had he, a bushel of corn weighing 56 lb. and a bushel of oats 32 lb.?

4. Write $7\frac{3}{4}$ million in figures.

5. How much has a man left out of $1000 after paying two bills, one of $275.50 and the other of $185?

6. A man bought 20 cows at $40 a head and sold 10 of them at $45 a head; what average price must he receive for the remainder of them to gain $250 on the transaction?

7. What is the dividend, if the divisor is 25, the quotient 25, and the remainder 20?

8. What is the least number that any of the numbers 2, 3, 4, 5, or 6 will exactly divide?

9. In an election in which A, B, and C were the candidates, A received 160 votes, B 120 votes, and C sufficient to elect him by a majority of 12; how many votes were cast?

10. What is the least sum that can be paid either in \$ 10 bills, \$ 20 bills, or \$ 50 bills?

11. Name all the composite numbers from 30 to 50 inclusive.

12. Name the prime factors of 12; 18; 21; 24; 25; 27; 28; 30; 33.

13. How many quarter-acre lots can be made from $6\frac{1}{2}$ acres of land?

14. A field was formed from 13 half-acre lots. How many acres did the field contain?

15. A bar of metal $1\frac{1}{2}$ in. long was cut into pieces, each $\frac{1}{12}$ in. long. How many did it make?

16. If $\frac{2}{3}$ of a number is 4, what is the number?

17. If $\frac{8}{9}$ of a number is 16, what is the number? What is $\frac{2}{3}$ of the number?

18. What should be paid for 2 yards of cloth, if 4 yards cost \$ 8?

SUGGESTION. 2 yards are what part of 4 yards?

19. Simplify:

$$116 \times 4 \div 8 - 84 \div 12 \times 3 + 63 \div (9 \times 7).$$

20. What should be paid for 5 yd. of cloth, if $2\frac{1}{2}$ yd. cost \$10?

SUGGESTION. 5 yards are how many times $2\frac{1}{2}$ yards?

21. From a board 12 ft. long 2 pieces were cut, the first $2\frac{1}{2}$ ft. long, and the second $2\frac{3}{4}$ ft. long. What was the length of the piece that remained?

22. When plums are selling at the rate of 2 qt. for \$ $\frac{1}{4}$, how many quarts can be bought for \$ $\frac{3}{4}$?

23. The product of 3 fractions is $1\frac{1}{15}$. Two of them are $\frac{3}{2}$ and $\frac{4}{5}$. What is the third?

24. During a certain year (365 da.) which began on Saturday, a rural mail carrier traveled $24\frac{1}{2}$ mi. each week day, excepting February 22 and July 4. How many miles did he travel that year in delivering his mail?

25. Supply the missing number:

$$7\tfrac{1}{2} + 18\tfrac{3}{4} + 33\tfrac{1}{3} + \ ? = 150.$$

26. If A can do a piece of work in 4 days, what part of it can he do in 1 day? What part of it can he do in 3 days?

27. If A can do a piece of work in 4 days and B can do it in 3 days, what part of the work can each do in 1 day? What part can each do in 2 days? What part can both do in 1 day? What part can both do in $1\frac{1}{2}$ days?

28. If A can do a piece of work in 4 days and B can do it in 3 days, how long will both, working together, require to do the work?

29. What part of $\frac{1}{2}$ is $\frac{1}{4}$?

SUGGESTION. Reduce $\frac{1}{2}$ and $\frac{1}{4}$ to a common denominator and compare the numerators.

30. A tenant, working a farm, receives for his labor $\frac{1}{3}$ of all the grain he raises, and the owner $\frac{2}{3}$. If the tenant's share of the crop is 104 bu. of oats, 121 bu. of wheat, 320 bu. of corn, and $30\frac{1}{2}$ bu. of rye, what is the owner's share?

31. A dealer paid \$45 for potatoes at $\$\frac{3}{4}$ a bushel; what must he sell them for per bushel to gain \$30?

32. 56 lb. of sweet corn were put up in $\frac{1}{2}$-pound, $\frac{1}{4}$-pound, and $\frac{1}{8}$-pound packages, an equal number of each; how many packages of each kind were made?

33. A wage rate of men on farms in Montana is \$ 49 per month without board and \$ 35 per month with board. What fractional part of the \$49 is deducted for board?

34. The following are the ingredients and quantities necessary to make enough apple pudding for 3 persons. How much of each of these ingredients should be used to make enough apple pudding for 12 persons?

1 cup of flour
$\frac{1}{4}$ teaspoonful of salt
$1\frac{1}{2}$ teaspoonfuls of baking powder
$1\frac{1}{2}$ tablespoonfuls of sugar
1 egg
$\frac{3}{4}$ cup of milk
2 tablespoonfuls of butter
2 apples
$\frac{1}{4}$ teaspoonful of cinnamon

35. What should $2\frac{1}{2}$ lb. of tea cost if $\frac{1}{2}$ lb. cost \$ $\frac{1}{4}$?

36. A dealer weighed 3 drafts of lard in a can which weighed $2\frac{3}{4}$ lb. The weight of the first draft and can was $21\frac{1}{2}$ lb., the weight of the second draft and can was $18\frac{1}{2}$ lb., the weight of the third draft and can was $20\frac{3}{4}$ lb.; what was the weight of the lard in the three drafts?

37. Find the cost of $7\frac{1}{2}$ bushels of clover seed at \$ $12\frac{1}{2}$ a bushel.

38. Smith owns $14\frac{1}{2}$ acres of land, Brown owns $5\frac{3}{4}$ acres more than Smith, and White owns $9\frac{1}{2}$ acres more than Brown; how much do they all own?

39. From a barrel containing $31\frac{1}{2}$ gallons of vinegar $9\frac{3}{4}$ gallons were drawn; what is the value of the remainder at $\$\frac{1}{4}$ a gallon?

40. Simplify: $\dfrac{1-\frac{1}{4}\times 1+\frac{1}{4}\div 1+\frac{1}{4}\times 1-\frac{1}{4}}{1\times\frac{1}{4}+1+\frac{1}{4}}$.

41. A dealer bought 3 lots for $\$1000$. He sold one of them for $\$500$, and another for $\$250$; what must he receive for the third so that he may have a profit of $\$200$ on the 3 lots?

42. Divide 160 into two such parts that $\frac{2}{3}$ of the smaller shall equal $\frac{2}{5}$ of the larger.

SUGGESTION. $\frac{2}{3}$ of the smaller $= \frac{2}{5}$ of the larger.
$\frac{1}{3}$ of the smaller $= \frac{1}{5}$ of the larger.
$\frac{3}{3}$ of the smaller $= \frac{3}{5}$ of the larger.
$\frac{5}{5}$ of the larger = the smaller and the larger, or 160.

43. In 1910 twice as many hogs were packed in Chicago as in Kansas City; 5,133,578 hogs were packed in Chicago that year. How many were packed in both cities?

44. Dried apples weigh about 26 lb. per bushel; how many bushels are there in 273 lb.?

45. How many days must a man labor to save $\$20$ if his wages are $\$1\frac{3}{4}$ a day, and his expenses are $\$1\frac{1}{8}$ a day?

46. At $20\,\text{¢}$ an hour how much will a man earn in a week who works 8 hours on Monday, $7\frac{1}{2}$ hours on Tuesday, 8 hours on Wednesday, $9\frac{1}{2}$ hours on Thursday, 7 hours on Friday, and $4\frac{1}{2}$ hours on Saturday?

47. From a cheese containing 18 lb., $3\frac{1}{2}$ lb. were sold to one person and $2\frac{1}{4}$ lb. to another; how much then remained?

48. Find a grocer's gain on 100 lb. of sugar bought at $5\frac{3}{8}¢$ a pound and retailed at $6¢$ a pound.

49. When plums are selling at the rate of 2 qt. for $\$\frac{1}{4}$, how many quarts can be bought for $\$1\frac{1}{2}$?

50. Simplify: $\frac{2\frac{1}{2} \times 25 \times \frac{1}{2} \times 37\frac{1}{2}}{1\frac{1}{4} \times 12\frac{1}{2} \times 12\frac{1}{2} \times \frac{1}{8}}$. (Cancel.)

51. Divide 50 into two such parts that $\frac{1}{3}$ of the larger shall equal $\frac{1}{2}$ of the smaller.

52. What part of $\frac{4}{5}$ is $\frac{3}{4}$?

53. If one lamp burns $\frac{3}{4}$ of a quart of oil in a night and another burns $1\frac{1}{4}$ quarts, how many nights will a gallon of oil last both lamps?

54. Name the quotient obtained by dividing 100 by each of the following numbers in turn:

$$12\tfrac{1}{2};\ 33\tfrac{1}{3};\ 6\tfrac{1}{4};\ 16\tfrac{2}{3};\ 8\tfrac{1}{3}.$$

55. A dealer offered $\$5\frac{1}{2}$ apiece for 5 lambs, but finally agreed to pay $\$2\frac{1}{4}$ more for them; how much did he pay for each?

56. Show that cancelling a factor common to the denominators of two fractions does not change their quotient.

57. Show that cancelling a factor common to the numerators of two fractions does not change their quotient.

58. If a train runs 30 miles in 45 minutes, at the same rate how far does it run in 1 hour?

59. Simplify: $1 + \frac{1}{4} \times \frac{1}{2} + \frac{1}{2} \div 2 \times \frac{1}{4} - \frac{1}{4} \div \frac{1}{2}$.

60. Simplify: $\frac{1}{2} \div \frac{1}{4} \times 2 + \frac{1}{4} \times 2 + 2 - \frac{1}{2} \div \frac{1}{4}$.

DECIMALS

DEFINITIONS

107. One tenth of a unit is *a tenth;* one tenth of a tenth is *a hundredth;* one tenth of a hundredth is *a thousandth;* and so on.

108. A tenth, a hundredth, a thousandth, and so on, are called **decimal units.** A tenth is a decimal unit of the *first order;* a hundredth is a decimal unit of the *second order;* a thousandth is a decimal unit of the *third order;* and so on.

109. Any number of decimal units of the same order is called a **decimal fraction,** or simply a **decimal.**

Thus, *7 tenths, 21 hundredths, and 325 thousandths* are decimals.

NOTATION AND NUMERATION

110. If the line of notation in the decimal system is extended rightward from units' place, and a point (.), called the **decimal point,** is placed after the units' order, a figure written in the first place to the right of this point expresses *tenths;* in the second place, *hundredths;* in the third place, *thousandths;* in the fourth place, *ten-thousandths;* in the fifth place, *hundred-thousandths;* in the sixth place, *millionths.*

111. Table

Millions	Hundred-thousands	Ten-thousands	Thousands	Hundreds	Tens	Units	.	Tenths	Hundredths	Thousandths	Ten-thousandths	Hundred-thousandths	Millionths
5	3	8	7	4	2	3	.	2	3	5	6	4	8

REMARK. In practice a decimal unit less than one ten-thousandth is seldom used.

Reading and Writing Decimals of One Order

Exercise 54

1. Name the first place to the right of the decimal point; the second; the fourth; the fifth; the third; the sixth.

2. Beginning at the decimal point, name the decimal places in their order to millionths.

3. Beginning at the right, name the places to the left of the decimal point.

4. Counting from the decimal point, what is the number of the tenths' place? the hundred-thousandths'? the thousandths'? the millionths'? the hundredths'? the ten-thousandths'?

5. In the number written in the table, Sec. 111, how many tenths are expressed? how many thousandths? how many hundred-thousandths? how many tens? how many hundreds? how many thousands? how many millionths? how many millions?

Read:

6. .1; .03; .09
7. .7; .008; .007
8. .08; .001; .9
9. .006; .4; .05
10. .7; .05; .003
11. .09; .002; .3
12. .004; .9; .06
13. .2; .02; .005
14. .01; .0001; .6
15. .00001; .001
16. .5; .0006; .09
17. .00008; .009

Write in words:

18. .5; .04; .009
19. .8; .07; .006
20. .004; .6; .03
21. .08; .002; .7
22. .01; .005; .4
23. .9; .06; .008
24. .001; .0004; .2
25. .007; .05; .00002

Write as decimals:

26. $\frac{2}{10}$
27. $\frac{5}{100}$
28. $\frac{7}{1000}$
29. $\frac{9}{1000}$
30. $\frac{9}{10}$
31. $\frac{1}{10}$
32. $\frac{1}{100}$
33. $\frac{1}{1000}$
34. $\frac{7}{10}$
35. $\frac{4}{10}$
36. $\frac{4}{100}$
37. $\frac{4}{1000}$
38. $\frac{6}{10}$
39. $\frac{3}{100}$
40. $\frac{8}{1000}$
41. $\frac{7}{10000}$
42. $\frac{5}{10000}$
43. $\frac{9}{10000}$
44. $\frac{1}{100000}$
45. $\frac{4}{100000}$

Reading and Writing Decimals of more than One Order

112. $.275 = \frac{2}{10} + \frac{7}{100} + \frac{5}{1000} = \frac{200}{1000} + \frac{70}{1000} + \frac{5}{1000} = \frac{275}{1000}$ = 275 thousandths.

113. To read a decimal, read the number expressed by the figures of the decimal and pronounce the name of the place occupied by the right-hand figure.

Exercise 55

Read:

1. .025 Thus, *twenty-five thousandths.*
2. 7.32 Thus, *seven and thirty-two hundredths.*

3. .25	4. .201	5. .42	6. .572
7. .304	8. .27	9. .750	10. .820
11. .09	12. .74	13. .047	14. .274
15. 4.07	16. 8.47	17. 5.704	18. 6.549
19. 7.4	20. 3.742	21. 2.523	22. 9.248
23. 13.07	24. 7.245	25. .2004	26. 4.375
27. 4.25	28. 4.75	29. 4.625	30. 3.875

31. $.02\frac{1}{4}$ Thus, *two and one fourth hundredths.*
32. $3.12\frac{1}{2}$ Thus, *three and twelve and one half hundredths.*

33. $.06\frac{1}{4}$	34. $.12\frac{1}{2}$	35. $.37\frac{1}{2}$	36. $.62\frac{1}{2}$
37. $.87\frac{1}{2}$	38. $.16\frac{2}{3}$	39. $.08\frac{1}{3}$	40. $.037\frac{1}{2}$
41. $.33\frac{1}{3}$	42. $6.66\frac{2}{3}$	43. $4.18\frac{3}{4}$	44. $5.07\frac{1}{2}$

Exercise 56

Write in decimal form:

1. $\frac{25}{100}$	2. $\frac{75}{100}$	3. $\frac{80}{100}$	4. $\frac{50}{100}$
5. $\frac{45}{100}$	6. $\frac{25}{1000}$	7. $\frac{50}{1000}$	8. $\frac{375}{1000}$
9. $\frac{205}{1000}$	10. $\frac{280}{1000}$	11. $\frac{500}{1000}$	12. $\frac{675}{1000}$

13. 24 hundredths SUGGESTION. Write 24 and so place the decimal point that 4 stands in *hundredths'* place; thus, .24.

14. 15 tenths SUGGESTION. Write 15 and so place the decimal point that 5 stands in *tenths'* place; thus, 1.5.

15. 12 hundredths
16. 25 thousandths
17. 16 thousandths
18. 30 thousandths
19. 15 hundredths
20. 50 hundredths
21. 22 hundredths
22. 12 tenths
23. 67 hundredths
24. 132 thousandths
25. 206 thousandths
26. 212 hundredths
27. 404 hundredths
28. 607 thousandths
29. 25 tenths
30. 40 thousandths
31. 86 hundredths
32. 164 thousandths
33. 107 thousandths
34. 145 hundredths
35. 18 hundredths
36. 19 millionths

37. 15 ten-thousandths
38. 185 hundred-thousandths
39. 25 thousandths
40. 325 ten-thousandths
41. 607 thousandths
42. Twenty-eight hundredths
43. Six and twenty-one thousandths
44. Two hundred four millionths
45. Twenty and seven tenths
46. Two hundred six thousandths
47. Two hundred and six thousandths

SUGGESTION. Two hundred is an integer.

48. Four hundred eight thousandths
49. Four hundred and eight thousandths
50. Seven hundred and seven tenths
51. Four hundred eight hundredths
52. Four hundred and eight hundredths

Reading Decimals Approximately

Exercise 57

1. Tell for what amount of money each figure stands in $8.73.
2. Which is $8.73 nearer, $8.70 or $8.80?
3. Read $8.73 correct to the nearest dime.
4. Which is $8.73 nearer, $7 or $8?
5. Read $8.73 correct to the nearest dollar.
6. Read 6.7 correct to the nearest unit.
7. Read 8.2 correct to the nearest unit.
8. Read 6.28 correct to the nearest tenth.
9. Read 3.71 correct to the nearest tenth.
10. Read 3.78 correct to the nearest unit.
11. Read 7.829 correct to the nearest hundredth.
12. Read 1.7381 correct to the nearest hundredth.
13. Read $25.56 correct to the nearest dime.
14. Read $7.92 correct to the nearest dollar.
15. Read $3.12 correct to the nearest dime.
16. Read $3.07 correct to the nearest dollar.

REDUCTION

Reducing Decimals to Different Denominations

114. $\frac{6}{10}=\frac{60}{100}=\frac{600}{1000}=\frac{6000}{10000}$ [§ 89].

That is, $.6=.60=.600=.6000.$

115. 1. **Annexing ciphers to a decimal does not change its value.**

2. **Removing ciphers from the right of a decimal does not change its value.**

Exercise 58

Reduce:

1. .5 to hundredths. Thus, .5 = .50 [§ 115, 1]
2. .4000 to tenths. Thus, .4000 = .4 [§ 115, 2]
3. .08 to thousandths.
4. .420 to hundredths.
5. .85 to thousandths.
6. .3000 to hundredths.
7. .90 to tenths.
8. .04 to thousandths.
9. .240 to hundredths.
10. .30 to tenths.
11. .37 to thousandths.
12. .7 to hundredths.
13. .6 to thousandths.
14. .05 to thousandths.
15. .400 to tenths.
16. .8000 to hundredths.
17. .55 to thousandths.
18. .6000 to tenths.
19. .64 to thousandths.
20. .255 to ten-thousandths.
21. .5000 to tenths.
22. .240 to hundredths.
23. .07 to thousandths.
24. .2 to thousandths.
25. .200 to tenths.
26. .005 to millionths.

Reducing Decimals to Common Fractions or Mixed Numbers

Illustrative Examples

116. 1. Reduce .375 to a common fraction.

Thus, $.375 = \frac{375}{1000}$, or $\frac{3}{8}$.

2. Reduce $.16\frac{2}{3}$ to a common fraction.

Thus, $.16\frac{2}{3} = \frac{16\frac{2}{3}}{100} = \frac{3 \times 16\frac{2}{3}}{3 \times 100} = \frac{50}{300}$, or $\frac{1}{6}$.

Exercise 59

Reduce to common fractions or mixed numbers:

1. .8
2. .75
3. 7.6
4. .4
5. .04
6. 3.75

7. .25	8. .625	9. .85
10. .6	11. .50	12. 1.125
13. .225	14. 7.75	15. 8.50
16. 7.45	17. 9.60	18. 10.45
19. .004	20. .075	21. .275
22. .450	23. .0025	24. 7.5
25. 1.08	26. 12.012	27. 6.775
28. $.8\frac{1}{3}$	29. $.6\frac{1}{4}$	30. $.12\frac{1}{2}$
31. $.16\frac{2}{3}$	32. $.14\frac{2}{7}$	33. $.33\frac{1}{3}$
34. $.66\frac{2}{3}$	35. $.18\frac{3}{4}$	36. $.87\frac{1}{2}$
37. $.83\frac{1}{3}$	38. $.28\frac{4}{7}$	39. $.42\frac{6}{7}$

Reducing Common Fractions to Decimals

117. Any common fraction may be reduced to a decimal; thus:

Reduce $\frac{3}{4}$ to a decimal.

$$4\overline{)3.00}\quad .75$$

$\frac{3}{4}$ equals $\frac{1}{4}$ of 3, which equals $\frac{1}{4}$ of 30 tenths.
$\frac{1}{4}$ of 30 tenths is 7 tenths, and 2 tenths remaining.
2 tenths equals 20 hundredths.
$\frac{1}{4}$ of 20 hundredths is 5 hundredths.
Therefore, $\frac{3}{4}$ equals .75.

To reduce a common fraction to a decimal:

118. Annex ciphers to the numerator, divide by the denominator, and point off as many decimal places in the quotient as there are ciphers annexed.

NOTE. A common fraction that can easily be reduced to tenths, hundredths, thousandths, and so on, may readily be reduced to a decimal.

Thus, 1. $\frac{6}{10} = .6$. 2. $\frac{75}{100} = .75$ 3. $\frac{17}{500} = \frac{34}{1000} = .034$

Exercise 60

(Reduce the first 20 examples as suggested in note, Sec. 118.)

Reduce to decimals:

1. $\frac{1}{2}$	2. $\frac{4}{5}$	3. $\frac{3}{50}$	4. $\frac{2}{25}$
5. $\frac{1}{4}$	6. $\frac{7}{50}$	7. $\frac{9}{25}$	8. $\frac{15}{50}$
9. $\frac{1}{500}$	10. $\frac{9}{200}$	11. $\frac{17}{500}$	12. $\frac{7}{500}$
13. $\frac{17}{20}$	14. $\frac{11}{25}$	15. $\frac{9}{10}$	16. $\frac{3}{5}$
17. $\frac{4}{5}$	18. $\frac{3}{25}$	19. $\frac{3}{50}$	20. $\frac{1}{5}$
21. $\frac{3}{8}$	Thus, $8\overline{)3.000}$ $.375$		
22. $\frac{1}{8}$	23. $\frac{5}{8}$	24. $\frac{7}{8}$	25. $\frac{13}{40}$
26. $\frac{21}{80}$	27. $\frac{3}{16}$	28. $\frac{13}{16}$	29. $\frac{9}{16}$
30. $\frac{9}{125}$	31. $\frac{27}{125}$	32. $\frac{15}{32}$	33. $\frac{21}{32}$
34. $\frac{5}{16}$	35. $\frac{11}{64}$	36. $\frac{33}{64}$	37. $\frac{51}{64}$

Reduce:

38. $\frac{5}{6}$ to hundredths. Thus, $6\overline{)5.00}$ $.83\frac{1}{3}$

39. $\frac{8}{9}$ to thousandths. Thus, $9\overline{)8.000}$ $.888\frac{8}{9}$

40. $\frac{1}{3}$ to tenths.

41. $\frac{2}{9}$ to hundredths.

42. $\frac{2}{3}$ to hundredths.

43. $\frac{5}{12}$ to thousandths.

ADDITION AND SUBTRACTION

Illustrative Examples

1. Find the sum of 2.5, 13.75, .018, and 12.56.

$$\begin{array}{r} 2.5 \\ 13.75 \\ .018 \\ 12.56 \\ \hline 28.828 \end{array}$$

Write the numbers so that units of the same order stand in the same column, and add as in addition of whole numbers. Place the decimal point between units and tenths in the sum.

2. From 12.75 take 7.9.

$$\begin{array}{r} 12.75 \\ 7.9 \\ \hline 4.85 \end{array}$$

3. From 28.7 take 1.245.

$$\begin{array}{r} 28.700 \\ 1.245 \\ \hline 27.455 \end{array}$$

Write the numbers so that units of the same order stand in the same column, and subtract as in subtraction of whole numbers. Place the decimal point between units and tenths in the difference.

Exercise 61

Add:

1. 6.14, .186, 31.4, .561

2. 1.004, .31, 25.06, .9

3. .403, 4.25, .117, 12.075

4. .4127, .8562, .3791, .5314, .1746, .1942

5. .9118, .7456, .3984, .6798, .7436, .5679

6. 1.3407, 4.7321, 3.9456, 6.1684, 8.1215, 5.2019

7. 27.4348, 14.8653, 86.9720, 29.5438, 32.2936, 64.4875

8. 98.4416, 10.1, 6.76, 75.243, 61.13, 40.0007, 66.6699

9. 38.1729, 59.6743, 20.2, .9236, 9.6438, .7491, 43.24

10. 77.5888, .7, 46., 96.4711, .108, 2.3, .678

11. 6.0001, 89., .8, 29.44, 1.263, 7.7, 76.4328

12. .6, .074, and .73.
13. 2.19, 7.005, and 15.3.
14. .45, 3.77, .017, and .711.
15. 30.03, 114.250, .09, .02.
16. 7 tenths, 52 hundredths, and 19 thousandths.
17. 3 and 1 tenth, 21 thousandths, and 3 and 8 hundredths.
18. 4 tenths, 78 hundredths, and 220 thousandths.
19. 13 and 21 thousandths, 6 and 6 hundredths, and 8 tenths.

Subtract:

20. 5.632	21. 4.000	22. 7.4	23. 8.
1.16	2.341	3.100	4.237
24. 2.40	25. 6.10	26. 52.1	27. 32.07
1.10	.57	2.04	13.005
28. .2956	29. .7890	30. .9481	31. .8541
.1432	.5421	.9409	.6897
32. 1.4687	33. 9.7648	34. 21.4262	35. 80.
1.2345	8.8059	19.8371	79.4387

Find the difference:

36. $3 - .431$
37. $3 - .011$
38. $6.101 - .122$
39. $8.52 - .006$
40. $8.13 - .021$
41. $25.2 - .112$

Find the difference between:

42. 72 hundredths and 57 thousandths.
43. 3 and 4 tenths and 55 hundredths.

44. 73 hundredths and 21 thousandths.

45. 9 and 8 tenths and 36 hundredths.

46. A merchant had $172.40 cash on hand at the beginning of a day. During the day he received $264.18 and paid out $86.12. What was his cash balance at the close of the day?

47. A dealer bought a horse for $175.75, paid $2.50 for his feed, and then sold him for $190. Find the gain.

48. A man's store bill for January was $30.42, for February $28.95, and for March $29.16; what was his store bill for the three months?

49. Read, supplying the missing numbers:

(*a*) $1+.3=$ (*c*) $1+.06=$

(*b*) $1-.4=$ (*d*) $1-.05=$

50. Find the cost of the following bill of hardware:

1 stove	$30.50
1 saw	2.25
1 pail	.25
1 pound of nails	.08
1 file	.09
Total	

51. At a certain point a river rose to within 3.7 feet of the high-water mark, which was 12.1 feet above low-water mark; how high above low-water mark was the river at that time?

52. Take the sum of 5 tenths and 5 hundredths from 6 and 6 tenths.

53. What was the average cost of 3 sheep, if one of them cost $5.25, another $4.25, and another $5.50?

54. Henry has $6.50; he owes $3.85. How much will he have left after paying the bill?

55. A girl buys a hat for $5.50 and a pair of gloves for $1.25; how much change should she receive, if she gives in payment a $10 bill?

56. Which is nearer to $10 and by how much, $9.37 or $10.79?

57. The distance from A to B is 15.4 mi., from B to C 14.6 mi., and from C to D 65.3 mi. What is the distance from A to D by way of B and C?

58. A man paid for his house in 3 payments; the first was $350.50, the second $750.25, and the third $560.75. What did the house cost?

59. How much greater is 1 than .3875?

60. What decimal added to .3756 will make the sum .4?

61. $(.04+.005)-(.04-.005)=?$

62. What is the area of Greater New York, which is composed of the following boroughs, with the area of each as here given:

Manhattan	21.93 sq. mi.
Brooklyn	77.62 sq. mi.
The Bronx	40.65 sq. mi.
Queens	129.50 sq. mi.
Richmond	57.19 sq. mi.

How much larger is Brooklyn than The Bronx?

How much larger is Richmond than Manhattan?

63. Find the sum of three hundred twenty-five ten-thousandths, six hundred four thousandths, six tenths, and six and fourteen hundredths.

64. Here is a kitchen outfit arranged by a large department store. Find the total cost.

Two-burner Gas Stove	$1.35	Flour Dredger	$ .15
Wash Tub	1.25	Salt Dredger	.05
Wash Board	.35	Pepper Dredger	.05
Clothes Wringer, 1-year Guarantee	3.50	Grater	.08
Clothes Line, 75 feet	.40	Bread Toaster	.10
Ironing Board, 4 feet	.65	Egg Beater	.10
Step Ladder, 3 feet	.54	Cake Turner	.10
Salt Box	.25	Tin Colander	.27
Rolling Pin	.15	Tin Dipper	.10
Meat Board	.20	Tin Gem Pan	.15
Potato Masher	.18	Strainer	.10
Flour Bucket	.35	Funnel	.05
Soap Cup	.12	Pie Pans, three	.21
Slaw Cutter	.30	Jelly Cake Pans, two	.20
Kitchen Table, 36 inches	1.50	Agate Cooking Pot, 4 quarts	.58
Clothes Pins, 5 dozen	.15	Agate Lipped Sauce Pan, 2 quarts	.30
Clothes Basket	1.30	Agate Teakettle, 5 quarts	1.00
Galvanized Coal Bucket	.35	Agate Coffeepot, 2 quarts	.60
Galvanized Water Pail, 12 quarts	.25	Agate Teapot, 1 quart	.50
Griddle, Iron	.40	Broom	.45
Frying Pan, Iron	.35	Dust Brush	.45
Roasting Pan	.50	Floor Sweeping Brush	1.00
Bread Raiser	.65	Scrubbing Brush	.12
Bread Box	.65	Food Chopper	.75
Sugar Can	.60	Kitchen Knife	.50
Coffee Canister, 1 pound	.25	Paring Knife	.15
Tea Canister, ½ pound	.20	Total	
Sugar Dredger	.15		

Exercise 63

Multiply:

1. $.22 \times 4$
2. 4.12×32
3. 1.28×40
4. $.117 \times 25$
5. 1.627×213
6. $.047 \times 320$

7. .22 by 3
8. .56 by 4
9. .421 by 4
10. 2.13 by 9
11. .63 by 7
12. .117 by 3
13. 4.606 by 13
14. 1.145 by 22
15. 21.16 by 25
16. .173 by 54
17. 8.60 by 32
18. 3.124 by 33
19. 9.20 by 123
20. .16 by 321
21. .313 by 620
22. 2.175 by 108
23. 1.220 by 116
24. .104 by 320
25. 14.56 by 200
26. .785 by 140
27. .25 by 7
28. 3.25 by 5
29. .035 by 9
30. 2.45 by 12
31. .095 by 15
32. .075 by 14
33. 8.36 by 40
34. .375 by 30
35. .508 by 25
36. .0086 by 54
37. 54 by .06
38. 4.8 by 7.8
39. .66 by .06
40. .32 by 20
41. 1.75 by 240
42. .654 by 2500
43. .375 by .0004
44. 100.5 by .25
45. 3.96 by .00256
46. .879 by 7.56

47. Find the weight of 9 hams if their average weight is 15.75 lb.

48. What was the yield of 20 acres of wheat averaging 18.75 bu. per acre?

49. A cubic foot of water weighs 62.5 lb.; what is the weight of 30 cubic feet of water?

50. A merchant failed in business and could pay only $ 0.625 on each dollar he owed. How much did he pay, if his debts were $ 5000?

51. A gallon of water weighs 8.36 lb.; find the weight of 68 gal. of water.

52. Find the cost of 6.75 yd. of cloth at $ 1.25 per yard.

53. A franc is worth 19.3¢; how much are 65 francs worth?

54. A cubic foot of water weighs 62.5 lb. Iron is 7.788 times as heavy as water. What is the weight of a cubic foot of iron?

55. Find the cost of 25.75 A. of land at $65.50 per acre.

56. Find the weight of a cubic foot of oak, which is 1.17 times as heavy as water (62.5 lb. per cubic foot).

DIVISION

Dividing by 10, 100, 1000, etc.

123. 1. $.04 \div 10 = \frac{4}{100} \div 10 = \frac{4}{1000}$, or .004

2. $.4 \div 100 = \frac{4}{10} \div 100 = \frac{4}{1000}$ or .004.

3. $4 \div 1000 = \frac{4}{1000}$, or .004

In 1, 2, and 3 observe the position of the decimal points in the dividend and in the quotient.

124. Moving the decimal point one place to the left divides by 10, moving it two places to the left divides by 100, moving it three places to the left divides by 1000.

Exercise 64

Read, supplying at sight the missing quotients:

1. $5 \div 10$ Thus, $5 \div 10 = .5$
2. $25 \div 100$ Thus, $25 \div 100 = .25$
3. $36 \div 1000$ Thus, $36 \div 1000 = .036$
4. $6 \div 10$
5. $2 \div 10$
6. $15 \div 10$
7. $.3 \div 10$
8. $.5 \div 10$
9. $2.5 \div 10$
10. $1.2 \div 10$
11. $.02 \div 10$
12. $.75 \div 10$
13. $.82 \div 10$
14. $.07 \div 10$
15. $.32 \div 10$
16. $7 \div 100$
17. $1 \div 100$
18. $.2 \div 100$
19. $1.5 \div 100$
20. $2.7 \div 100$
21. $4.2 \div 100$
22. $25 \div 100$
23. $36 \div 100$
24. $3.5 \div 100$
25. $5.5 \div 100$
26. $20 \div 100$
27. $7 \div 1000$
28. $80 \div 1000$
29. $900 \div 1000$
30. $8.4 \div 1000$
31. $.7 \div 1000$
32. $45 \div 1000$
33. $86.5 \div 10000$

34. Find the cost of 500 cabbage plants at \$ 0.75 per C.

SUGGESTION. $500 = 5C$; therefore, $5 \times \$0.75 =$ the required cost.

35. Find the cost of 300 rails at \$ 6.50 per C.
36. Find the cost of 250 shad at \$ 30 per C.
37. Find the cost of 2000 gun wads at \$ 0.48 per M.
38. Find the cost of 360 paper bags at \$ 0.08 per C.
39. Find the cost of 2500 envelopes at \$ 2.25 per C.

40. Find the cost of 600 posts at $18 per C.

41. Find the cost of 200 bolts at $0.49 per C.

42. Find the cost of 2000 paper fasteners at $0.35 per C.

43. Find the cost of 2600 balls of string at $35 per M.

Dividing by Integers and by Decimals

125. 1. $.015 \div 3 = \frac{15}{1000} \div 3$

$= \frac{5}{1000}$, or .005.

That is, $.015 \div 3 = .005$. (A)

2. $.008 \div .04 = \frac{8}{1000} \div \frac{4}{100}$

$= \frac{100}{4} \times \frac{8}{1000}$

$= \frac{1}{4} \times \frac{8}{10}$, or $.8 \div 4$.

That is, $.008 \div .04 = .8 \div 4$. (B)

3. $.8 \div 20 = \frac{8}{10} \div \frac{20}{1}$

$= \frac{1}{20} \times \frac{8}{10}$

$= \frac{1}{2} \times \frac{8}{100}$, or $.08 \div 2$.

That is, $.8 \div 20 = .08 \div 2$. (C)

Observe in (A) that the divisor is a whole number and that the quotient contains as many decimal places as there are in the dividend.

Observe in (B) and (C) that the decimal point may be moved to the right or to the left in both dividend and divisor the same number of places without changing the quotient.

From (A), (B), and (C) it follows that, in division of decimals, we may:

126. Move the decimal point to the right or to the left in both dividend and divisor such a number of places as will make the divisor a whole number not ending in ciphers; then divide and point off in the quotient as many decimal places as there are in the dividend.

Illustrative Examples

Divide:

1. .625 by .25; 2. 6.25 by .025; 3. 6.25 by 250.

$$.625 \div .25 = 25\overline{)62.5}\quad 2.5,\ 50,\ 125,\ 125$$

$$6.25 \div .025 = 25\overline{)6250}\quad 250,\ 50,\ 125,\ 125,\ 0$$

$$6.25 \div 250 = 25\overline{).625}\quad .025,\ 50,\ 125,\ 125$$

Check: $.25 \times 2.5 = .625$ $.025 \times 250 = 6.25$ $250 \times .025 = 6.25$

Exercise 65

(Solve as many as possible at sight.)

Divide:

1. .005 by 5
2. 3.6 by 9
3. .63 by 7
4. .72 by 8
5. .024 by 6
6. .012 by 6
7. 9.18 by 9
8. .2 by 5
9. .04 by 8
10. .003 by 6
11. .012 by 12
12. 7.014 by 7
13. 31.5 by 35
14. 6.25 by 25
15. .225 by 15
16. 2.6 by 20

17. 8 by 40
18. 1.2 by 60
19. .15 by 30
20. .06 by 30
21. 42 by 60
22. .025 by .5
23. .075 by .15
24. .16 by .04
25. .036 by .06
26. 4.8 by 80
27. 3 by 60
28. .45 by 90
29. 4.5 by 50
30. 27.6 by 30
31. 2.79 by 90
32. 1.44 by .02
33. .048 by .12
34. .144 by .012
35. .0375 by .15
36. 28 by 200
37. 360 by 300
38. 4.8 by 200
39. 72 by 600
40. .1728 by .036
41. 30.75 by .075
42. 22.5 by 150
43. 6.25 by 2500
44. 14.4 by 120
45. 720 by 3000
46. .02 by .002
47. .5 by .0005
48. .125 by .025
49. 1 by .08
50. 1250 by 5000
51. 75 by 150
52. 1250 by 2500
53. 3750 by 5000
54. 1.44 by .036
55. 86.4 by .24
56. 256 by 200
57. 2.25 by 150
58. .9 by 360
59. 48.32 by 16
60. 461.5 by 142
61. 98.1 by 3000
62. 191.52 by 4200
63. 180 by 50
64. .0045 by 1500
65. 3.6 by 1500
66. 5 by .005
67. .005 by .5
68. 8.64 by 2.4
69. .6048 by .48
70. .1728 by .0144
71. 4816 by .16
72. .9 by .0036
73. 1 by .125
74. 15.39 by 4.75

75. If I divide $2.25 equally among 9 boys, how much will each boy receive?

76. If a man walked 14 miles in 4 hr., how many miles was that an hour?

77. A farmer sold 10 bushels of wheat which weighed 585 lb.; what was the weight per bushel?

78. If 5 yd. of silk cost $6.25, how much is that a yard?

79. If a hack driver receives $1.75 a day, in how many days will he earn $40.25?

80. A merchant failed in business and could pay only $0.625 on each dollar he owed; how much did he owe a creditor to whom he paid $500?

81. A cubic foot of water weighs 62.5 lb.; how many cubic feet are there in 2000 lb. of water?

Finding the Quotient Approximately

127. When the division does not terminate, it is seldom carried in practice beyond the fifth decimal place in the quotient, which is usually expressed to the nearest *ten-thousandth* (.0001).

Thus, the quotient of $7.4 \div 35$ expressed correct to the nearest ten-thousandth is found as follows:

$.21142^{+} = .2114$, correct to the nearest .0001.

$$35\overline{)7.40000}$$

$$\begin{array}{r} 70 \\ \hline 40 \\ 35 \\ \hline \text{etc.} \end{array}$$

NOTE. If the quotient were $.21146^{+}$ expressed to the nearest .0001, it would be .2115.

Exercise 66

Express to the nearest ten-thousandth the value of:

1. $2.573 \div 6$	2. $25\ 38 \div 8$	3. $.075 \div 6$
4. $37.52 \div 9$	5. $.046 \div .06$	6. $1 \div 15$
7. $.8 \div 11$	8. $325.5 \div 16.5$	9. $7 \div 15$
10. $9 \div 28$	11. $7.5 \div 22$	12. $6.75 \div 34$
13. $.05 \div 32$	14. $.075 \div 48$	15. $2.04 \div 65$
16. $.384 \div 81$	17. $.147 \div 300$	18. $15 \div 38$
19. $8 \div 75$	20. $1 \div 300$	21. $4 \div 270$
22. $2.1 \div 275$	23. $7.5 \div 45$	24. $9.35 \div 280$

UNITED STATES MONEY

128. The standard unit of value in the United States is the **dollar.**

Table

10 mills (m.)	**= 1 cent (ct., ¢, or c)**
10 cents	**= 1 dime (d.)**
10 dimes	**= 1 dollar ($)**
10 dollars	**= 1 eagle (E.)**

Since a dime equals $\frac{1}{10}$ of a dollar, a cent $\frac{1}{100}$ of a dollar, and a mill $\frac{1}{1000}$ of a dollar, any sum may be expressed in dollars without reduction by placing the number of dimes in the tenths' place, the number of cents in the hundredths' place, and the number of mills in the thousandths' place.

Thus, 25 dollars 4 dimes 6 cents 5 mills may be expressed $25.465, and read *25.465 dollars*, or *25 dollars 46 cents 5 mills.*

Exercise 67

1. How many mills are there in 2¢? 5¢? 7¢? 1 d.?
2. How many cents are there in 3 d.? 5 d.? $\frac{1}{2}$ d.?
3. How many mills are there in $\frac{1}{2}$¢? in $2\frac{1}{2}$¢? in $1\frac{1}{2}$¢?
4. How many cents are there in $2? $4? $10?
5. How many cents are there in $\$\frac{1}{2}$? $\$\frac{1}{4}$? $\$\frac{3}{4}$? $\$\frac{2}{5}$?
6. How many cents are there in $\$\frac{3}{8}$? $\$\frac{5}{8}$? $\$\frac{7}{8}$? $\$\frac{1}{12}$?
7. How many cents are there in $\$\frac{1}{3}$? $\$\frac{2}{3}$? $\$\frac{1}{6}$?
8. How many dollars are there in 300¢? 150¢? 1000¢?
9. What part of a dollar is 1 d.? 3 d.? 5 d.? 9 d.?
10. What part of a dollar is 25¢? 50¢? 75¢?
11. What part of a dollar is 20¢? 30¢ 60¢?
12. How many dimes are there in $2.50?
13. Name our gold coins, and tell how many dollars each is worth.
14. Name all the coins of the United States that are now being made. Name those that are silver.
15. Find the gain on 200 shad bought at $30 per C, and sold at $0.50 each. What was the gain on 150 of them?
16. Find the gain on 2000 envelopes bought at $0.60 per M, and sold at the rate of 25 for 5¢.
17. Find the gain on 10,000 shingles bought at $12 per M, and sold at $22.50 per M.
18. If a merchant buys 150 baskets of peaches at $\$1.12\frac{1}{2}$, and sells them at $1.30, how much does he make?
19. At $42.50 a front foot, what will be the cost of a piece of land having a frontage of 250 ft.?

MISCELLANEOUS TABLES

129. Paper

24 sheets = 1 quire
20 quires = 1 ream

130. Counting

12 units = 1 dozen (doz.)
12 dozen = 1 gross (gro.)
12 gross = 1 great gross (G. gro.)
20 units = 1 score

Exercise 68

1. How many sheets of paper are there in 4 quires?
2. How many sheets of paper are there in $\frac{1}{2}$ a quire?
3. How many sheets of paper are there in $3\frac{1}{2}$ quires?
4. Find the cost of $2\frac{1}{2}$ doz. eggs at 30¢ a dozen.
5. What is the cost of 9 oranges at 40¢ a dozen?
6. How many crayons are there in 5 boxes each containing a gross?
7. What should a woman receive for $15\frac{1}{2}$ doz. eggs at \$ 0.32 a dozen?
8. When oranges are selling at the rate of 6 for 15¢, how much should be paid for 2 doz.?
9. How old is a man whose age is 4 score and 10 years?
10. How many score of eggs are there in 5 doz.?
11. When matches are selling at 3 boxes for 10¢, how much is that per gross?

12. When paper is bought at 20¢ per quire, how much is that per ream?

13. What profit is made on 10 doz. notebooks bought at $0.90 a dozen and sold at 10¢ each?

14. 10 eggs weigh about 1 pound; find the weight of 20 doz. eggs.

Find the gain on:

15. 8 doz. notebooks, bought at $1.60 per dozen and sold at 15¢ each.

16. 3 doz. rulers, bought at 50¢ per dozen and sold at 5¢ each.

17. 4 gro. erasers, bought at $3.20 per gross and sold at 5¢ each.

18. 2 gro. pencils, bought at $3.00 per gross and sold at 2 for 5¢.

19. 1½ reams water-color paper, bought at $20 per ream, and sold at 5¢ per sheet.

20. 5 reams drawing paper, bought at 35¢ per ream and sold at 3 sheets for 1¢.

21. 1 quire blotters, bought at 60¢ and sold at 3¢ each.

BILLS AND ACCOUNTS

131. A **bill** is a written statement of the names, quantities, and prices of goods sold or services rendered, together with the names of the parties and the date of the transaction.

132. An **account** is a record of business transactions between two parties at a given date.

133. The **debtor** (Dr.) is the party who buys the goods or receives the services.

134. The **creditor** (Cr.) is the party who sells the goods or renders the services.

135. A **debit** (Dr.) is an entry in a bill or an account against the debtor.

136. A **credit** (Cr.) is an entry in a bill or an account in favor of the debtor.

137. An **item** is any separate debit or credit made in a bill or an account.

138. A bill is **filled** by writing in it the cost of each item, and **footed** by adding the cost of the different items to find the total cost.

139. A Receipted Bill for Goods purchased on the Same Date

SILVER, BURDETT & COMPANY
PUBLISHERS OF SCHOOL AND COLLEGE TEXTBOOKS
TWO TWENTY-ONE COLUMBUS AVENUE
BOSTON, MASSACHUSETTS

Date July 31, 1912. No. Via American Express

Mr. Alexander Hamilton,
30 Greene Street,
Washington, D.C.

10 Progressive Road to Reading, Book 3	@ .48	4.80	
5 First Year Latin	@ 1.00	5.00	
10 Literature in the Schools	@ 1.25	12.50	
12 In Fableland	@ .45	5.40	
20 New Normal Music, Book 2	@ .40	8.00	35.70

RECEIVED PAYMENT
Aug. 14, 1913.
SILVER, BURDETT & CO.
Per *R. S. M.*

140. A Receipted Bill for the Entire Purchases of One Month

MARSHALL FIELD & COMPANY

IMPORTERS, RETAILERS, MANUFACTURERS

STATE, WASHINGTON, RANDOLPH & WABASH

CHICAGO, DEC. 2, 1912

Sold to Mr. FRANK P. PRICE,
124 MAIN ST.,
GALESBURG, ILL.

31/2365

NOTICE. — Merchandise to be credited on current bill must be returned before the last day of the month.

Nov.				DAILY TOTALS	CREDITS
2	2 yd. lace	75	1 50		
	1 pr. blankets		4 75		
	2 doz. napkins	4 00	8 00	14 25	
16	1 pr. blankets				4 75
19	1 dining table			24 00	
23	2 yd. cloth	3 85	7 70		
	1 pr. shoes		4 00		
	4 yd. ribbon	18	72		
	1 hand bag		2 00	14 42	
25	1 pr. shoes				4 00
				52 67	8 75
				8 75	
				43 92	

RECEIVED PAYMENT
Jan. 29, 1913,
MARSHALL. FIELD & CO
Per *L. B. M.*

What is the total amount of the purchases made Nov. 2? Nov. 19? Nov. 23? What is the total amount of the purchases for the month of November?

What is the total amount of the credits. What amount was paid Jan. 29, 1913?

141. **An Account**

DENVER, COLO., September 3, 1912.

John Clark,

In account with B. B. WITMER

Dr.					
15 yd. White Lawn	*$0.25*	*3*	*75*		
18 lb. Sugar	*.06*	*1*	*08*		
20 yd. Carpet	*.80*	*16*	*00*		
10 yd. Lace	*.15*	*1*	*50*	*22*	*33*
Cr.					
15 doz. Eggs	*.30*	*4*	*50*		
16 lb. Butter	*.36*	*5*	*76*		
25 lb. Chicken	*.18*	*4*	*50*	*14*	*76*
Balance,				*7*	*57*

Exercise 69

1. Extend the items in the following bill and find the total:

WOODBURY, N.J., June 15, 1912.

Mr. James Wilson,

To G. G. GREENE, *Dr.*

Terms Cash.

36 yd. Muslin	*$0.09*				
1 doz. Men's Linen Hdkfs.	*1.80*				
2 pr. Gloves	*.75*				
4 pr. Socks	*.25*				
10 yd. Fancy Ribbon	*.12*				
2 Silk Waists	*6.00*				

2. Extend the items in the following account and find the balance:

BALTIMORE, MD., April 1, 1912.

Jacob M. Frantz,

In account with STEINMAN & CO.

Dr.			
1 Gasoline Stove		*4*	*45*
1 Teakettle and Steamer		*2*	*00*
2 Preserving Kettles	*$0.75*		
2 doz. Bread Pans	*1.20*		
Cr.			
2 bu. Timothy Seed	*$2.75*		
3 bu. Seed Potatoes	*.80*		
½ bu. Seed Corn	*.90*		
Balance,			

Rule bill forms and on them make out bills for the following articles; use your own name for that of the buyer, and the name of some firm that deals in the articles purchased for that of the seller. Fill, foot, and receipt the bills, dating them appropriately.

3.

½ lb. tea	@	80¢
3 loaves bread	@	5¢
3 lb. butter	@	35¢
1 bottle olives	@	60¢
2 cans corn	@	20¢
5½ gal. vinegar	@	30¢

4.

1½ gal. vinegar	@	30¢
3½ lb. ham	@	12¢
3 glasses jelly	@	18¢
¼ doz. bottles sirup	@	$2.00
8¼ lb. rice	@	12¢
3 lb. white grapes	@	20¢

5.

1 lb. cheese	@	25¢
5 lb. coffee	@	30¢
10 lb. sugar	@	7¢
2 lb. lard	@	16¢
15½ doz. eggs	@	30¢
1 lb. English walnuts	@	75¢

6.

2 qt. cranberries	@	15¢
4 lb. French prunes	@	25¢
5 doz. Fla. oranges	@	50¢
6 bottles sirup per doz.		$ 2.75
4 glasses Guava jelly	@	35¢
5½ lb. raisins	@	16¢

7.

80 bu. wheat	@	96¢
62 bu. corn	@	72¢
100 bu. oats	@	55¢
10 bu. rye	@	85¢
8½ bu. potatoes	@	90¢
2 bu. clover seed	@	$ 10.00

8.

¼ lb. nutmegs	@	60¢
6 pkgs. cornstarch	@	5¢
100 lb. sugar	@	5¢
7 lb. coffee	@	35¢
5½ lb. ham	@	15¢
3 lb. figs	@	20¢

Render each of these bills for the month named in the current year; receipt the bills.

9.	Mar. 14.	1 ladder	@	$ 1.80
		2 egg beaters	@	.20
		1 market basket	@	1.00
		2 potato mashers	@	.18
	Mar. 25.	3 cans corn	@	$ .10
		4 lb. cheese	@	.18
		2 bottles of olives	@	.30
	Mar. 30.	4 yd. silk	@	$ .50
		3 pr. silk hose	@	.75
		4½ yd. silk ribbon	@	.30
		½ doz. napkins	@	$ 3.60

10.	Apr. 2.	1 hot water bottle	@	$1.00
		2 tooth brushes	@	.25
	Apr. 4.	1 pr. skates	@	$1.10
		3 skate straps	@	.10
	Apr. 6.	1 raincoat	@	$ 2.50
		1 pr. rubbers	@	.55
		2 umbrellas	@	2.50
11.	May 4.	4 yd. gingham	@	$ 0.12
		6 yd. percale	@	.12½
		8 yd. calico	@	.06
		2 yd. lawn	@	.30
	May 8.	4 lb. butter	@	$ 0.30
		3 lb. rice	@	.10
		5 lb. soup beans	@	.08
		1 lb. coffee	@	.30
	May 10.	4 cans of tomatoes	@	$ 0.10
		3 cans of peaches	@	.25
		2 cans of beans	@	.15
		2 cans of peas	@	.12

12. Rule account form and on it make out the following account and find the balance:

Oct. 1, 1912, James Wilson bought of Hiram Leonard, Pittsburg, Pa., 40 bbl. Winter Patent Flour @ $5.60 per barrel; 12 bbl. Winter Straight Flour @ $5.40 per barrel; 14 bbl. Penna Flour @ $4.90 per barrel. At the same time Mr. Leonard bought from James Wilson 60 bu. No. 2 Red Wheat @ $1.13 per bushel; 60 bu. Corn @ $0.85 per bushel; 50 bu. Oats @ $0.60 per bushel.

13. Copy the following account of the expenditures and receipts of a ten-acre field of corn, and supply the missing amounts.

		EXPENDITURES		
		Interest on capital per acre	$2.00	
Apr.	1–9	Plowing, man and team $6\frac{1}{2}$ da. @	3.00	
"	10	Harrowing, man and team, 7 hr. @	.30	
"	25	Harrowing, man and team, 7 hr. @	.30	
May	10	Harrowing, man and team, 7 hr. @	.30	
"	10	Planting, man and team, 14 hr. @	.30	
"	10	Seed 5 pecks @	.30	
"	25	Cultivating, man and team, $1\frac{2}{3}$ da. @	3.00	
June	5	Cultivating, man and team, $1\frac{2}{3}$ da. @	3.00	
"	16	Cultivating, man and team, $1\frac{2}{3}$ da. @	3.00	
"	20	Cultivating, man and team, $1\frac{2}{3}$ da. @	3.00	
Sept.	20	Cutting and shocking, 4 men, $2\frac{1}{2}$ da., per day	1.50	
Oct.	25	Husking, 4 men 4 da., per day	1.50	
"	25	Cribbing, man and team, $1\frac{2}{3}$ da. @	3.00	
		TOTAL		
		RECEIPTS		
Feb.	25	500 bu. corn @	.58	
"	25	25 T. Stover @	2.50	
		TOTAL		
		Expenditures to be deducted		
		Net profit from 10 A.		
		" " per acre		

14. Copy the following account of the expenditures and receipts of a ten-acre field of wheat, and supply the missing amounts.

	Expenditures			
	Interest on capital	per acre	$ 2.00	
Sept. 5	Plowing, man and team,	6½ da.	@ 3.00	
" 12	Harrowing, man and team,	7 hr.	@ .30	
" 25	Harrowing, man and team,	7 hr.	@ .30	
Oct. 12	Harrowing, man and team,	7 hr.	@ .30	
" 15	Drilling, man and team,	1 da.	@ 3.00	
" 15	Seed	1½ bu. per acre	@ 1.15	
July 5	Harvesting	per acre	.60	
" 5	Twine	per acre	.25	
" 10	Stacking	per acre	.50	
" 30	Threshing	250 bu.	@ .04	
" 30	Labor	1 da. 4 men	@ 1.50	
	Total			
	Receipts			
Aug. 1	250 bu. wheat		@ $ .90	
" 1	20 T. straw		@ 2.00	
	Total			
	Expenditures to be deducted			
	Net profit from 10 acres			
	" " per acre			

Note. Pupils should be encouraged to learn whether or not the prices and facts stated in such examples as 13 and 14 are in accord with those prevailing in their community, and to make examples corresponding to those given, using such prices and facts.

BUSINESS PROBLEMS

Domestic Postage

142. DOMESTIC POSTAGE RATES

(Apply to Mail Matter for Canada, Canal Zone, Cuba, Guam, Hawaii, Mexico, Philippines, Porto Rico, Republic of Panama, Tutuila, and the U. S. Postal Agency at Shanghai. The Domestic Rate for Letters, but not for Other Articles, applies also to Germany, Great Britain, and Ireland, and Newfoundland.)

First-class.	Letters and sealed matter: 2 cts. for each ounce or fraction. Postal cards and Post cards: 1 ct. each.
Second-class.	Newspapers and periodicals: 1 ct. for each four ounces or fraction.
Third-class.	Miscellaneous printed matter, except books weighing over eight ounces: 1 ct. for each two ounces or fraction.
Fourth-class.	All matter not included in first three classes and weighing four ounces or less: 1 ct. for each ounce or fraction, regardless of distance.
Special Delivery.	A special-delivery stamp (or 10 cts. in ordinary stamps when the letter or parcel is marked "Special Delivery") in addition to postage, secures immediate delivery within the carrier limits of city delivery offices between the hours of 7 A.M. and 11 P.M. and within one mile of all other United States post offices between the hours of 7 A.M. and 7 P.M.

REMARK. Fourth-class matter weighing more than four ounces is mailable at the rates shown in the table under Parcel Post, page 152.

Exercise 70

1. What is the postage on a letter weighing 2 ounces?
2. What is the postage on a letter weighing $1\frac{1}{2}$ ounces?
3. What is the postage on a letter weighing 5 ounces?
4. What is the postage on a letter weighing $4\frac{1}{2}$ ounces?
5. What is the postage on a package of newspapers weighing 8 ounces?
6. What is the postage on a package of newspapers weighing 7 ounces?
7. What is the postage on a magazine weighing 12 ounces?
8. What is the postage on a magazine weighing 10 ounces?

9. What is the postage on a book weighing 8 ounces?

10. What is the postage on a book weighing 7 ounces?

11. What is the postage on a book weighing 5 ounces?

12. What is the postage on a book weighing $7\frac{1}{2}$ ounces?

13. What is the postage on a package of merchandise weighing 4 ounces?

14. What is the postage on a 2-ounce box of jewelry?

15. What is the postage on a box of merchandise weighing $1\frac{1}{2}$ ounces?

16. What is the postage on a box of medicine weighing $2\frac{1}{2}$ ounces?

17. What is the postage on a 3-ounce letter which also bears a special delivery stamp?

18. What is the postage on a 6-ounce book if the package has also a special delivery stamp upon it?

To insure a piece of mail against loss not exceeding \$ 50, it may be *registered*, the rate being 10¢ on each piece in addition to the regular postage. A piece of registered mail must bear the name and address of the sender.

19. Find the cost of sending a registered letter weighing $1\frac{1}{2}$ ounces.

20. Find the cost of sending a registered letter weighing 3 ounces.

21. Find the cost of sending a registered parcel containing merchandise weighing $2\frac{1}{2}$ ounces.

22. Find the cost of sending a registered parcel containing merchandise weighing $3\frac{1}{2}$ ounces.

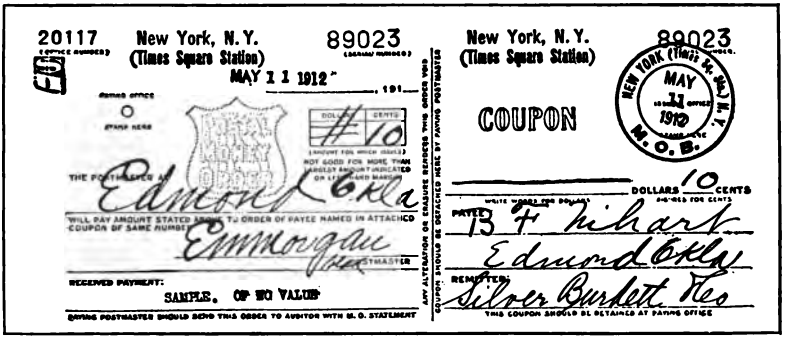
23. Find the cost of sending a registered letter weighing $2\frac{1}{2}$ ounces.

Money Orders

United States Post Office Money Orders

143. Any sum of money not exceeding $100 may be paid at a distance by means of a postal money order which may be purchased at a post office. This order directs the postmaster at another post office to pay a certain sum of money to a certain person named in the order.

Postal Money Order



FEES FOR MONEY ORDERS DRAWN ON DOMESTIC FORM

144. Payable in the United States (which includes Hawaii and Porto Rico) and its possessions, comprising the Canal Zone (Isthmus of Panama), Guam, the Philippines and Tutuila, Samoa; also for Orders payable in Bermuda, British Guiana, British Honduras, Canada, Cuba, Mexico, Newfoundland, the United States Postal Agency at Shanghai (China), the Bahama Islands, and certain other islands in the West Indies mentioned in Register of Money Order Post Offices.

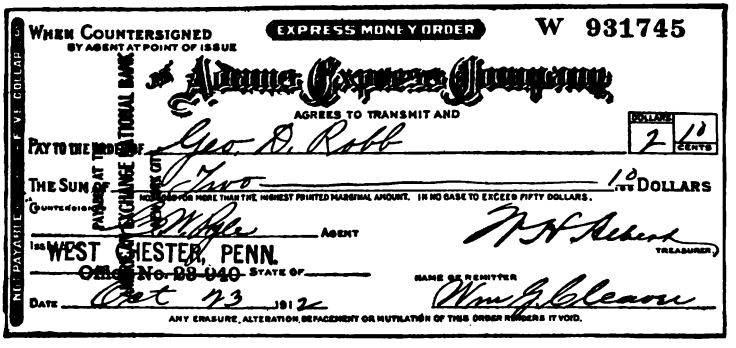
For orders From $0.01 to $2.50 . . . 3 cents.
From $2.51 to $5.00 . . . 5 cents.

For orders	From $ 5.01 to $ 10.00	. . .	8 cents.
	From $10.01 to $ 20.00	. . .	10 cents.
	From $20.01 to $ 30.00	. . .	12 cents.
	From $30.01 to $ 40.00	. . .	15 cents.
	From $40.01 to $ 50.00	. . .	18 cents.
	From $50.01 to $ 60.00	. . .	20 cents.
	From $60.01 to $ 75.00	. . .	25 cents.
	From $75.01 to $100.00	. . .	30 cents.

TO THE TEACHER. Pupils should be encouraged to secure and fill out properly the forms used in making application for postal money orders.

Express Money Orders

145. Money may be paid at a distance by means of express money orders, which may be obtained at the offices of express companies.


WHEN COUNTERSIGNED BY AGENT AT POINT OF ISSUE
EXPRESS MONEY ORDER
W 931745
Adams Express Company
AGREES TO TRANSMIT AND
PAY TO THE ORDER OF Geo. D. Robb
DOLLARS 2 CENTS 10
THE SUM OF Two 10/100 DOLLARS
NOT TO EXCEED FIFTY DOLLARS.
Countersigned
Agent
WEST CHESTER, PENN.
Office No. 23-040 State of
Date Oct 23 1912
NAME OF REMITTER
Wm G Cleaver
ANY ERASURE, ALTERATION, DEFACEMENT OR MUTILATION OF THIS ORDER RENDERS IT VOID.
NOT PAYABLE IF COLLAP
PAYABLE AT EXCHANGE NATIONAL BANK

146. RATES FOR EXPRESS MONEY ORDERS

Not over $ 2.50	. . . 3c.	Not over $ 40.00	. . . 15c.
Not over $ 5.00	. . . 5c.	Not over $ 50.00	. . . 18c.
Not over $10.00	. . . 8c.	Not over $ 60.00	. . . 20c.
Not over $20.00	. . . 10c.	Not over $ 75.00	. . . 25c.
Not over $30.00	. . . 12c.	Not over $100.00	. . . 30c.

Over $100.00 at above rates.

Exercise 71

1. I bought a pair of skates for $3 and a sled for $2.50. What was the fee for the post-office money order sent in payment? What was the total cost?

2. What is the fee for a post-office money order for $1.85? for $2.40? for $2.60? for $5.70? for $25? for $32? for $75? for $0.75?

3. What is the total cost of the post-office money order made in payment of 20 yd. of carpet bought at $1.45 a yard?

4. A man living in Philadelphia ordered the following bill of goods from a New York store: 2 saws each costing $2, a hammer costing $.75, and a hatchet costing $1.00. What fee did he pay for the express money order sent in payment of the goods? What was the total cost?

5. I bought the following bill of goods: one refrigerator costing $23.85, one couch costing $10.35, a chair for $2.75, and a pair of shoes for $3. Find what fee I paid for the express money order sent in payment of the bill, and the total cost.

Parcel Post

147. The parcel post law provides:

"That fourth-class mail matter shall embrace all other matter, including farm and factory products and books, not now embraced by law in either the first, second, or third class, not exceeding fifty pounds in weight (twenty pounds if intended for delivery in zones beyond the second), nor greater in size than seventy-two inches in length and girth combined, nor in form or kind likely to injure the

person of any postal employee or damage the mail equipment or other mail matter and not of a character perishable within a period reasonably required for transportation and delivery."

148. For the purpose of carrying the law into effect the United States is divided into zones with different rates of postage applicable to each, except in the case of parcels weighing four ounces or less and books weighing eight ounces or less.

Parcels weighing four ounces or less are mailable at the rate of *one cent for each ounce or fraction thereof*, regardless of distance, and books weighing eight ounces or less are mailable at the rate of *one cent for each two ounces or fraction thereof*, regardless of distance. Parcels weighing more than four ounces, and books weighing more than eight ounces are mailable at the pound rate, as shown in the following table, and when mailed at this rate, any fraction of a pound is considered a full pound:

Weight	1st zone		2d zone, 50 to 150 miles	3d zone, 150 to 300 miles	4th zone, 300 to 600 miles	5th zone, 600 to 1000 miles	6th zone, 1000 to 1400 miles	7th zone, 1400 to 1800 miles	8th zone, all over 1800 miles
	Local rate	Zone rate, 50 miles	*Rate*	*Rate*	*Rate*	*Rate*	*Rate*	*Rate*	*Rate*
1 pound	$0.05	$0.05	$0.05	$0.06	$0.07	$0.08	$0.09	$0.11	$0.12
2 pounds	.06	.06	.06	.08	.11	.14	.17	.21	.24
3 pounds	.06	.07	.07	.10	.15	.20	.25	.31	.36
4 pounds	.07	.08	.08	.12	.19	.26	.33	.41	.48
5 pounds	.07	.09	.09	.14	.23	.32	.41	.51	.60
	and 1 cent for each additional 2 pounds or fraction thereof	and 1 cent for each additional pound or fraction thereof	and 1 cent for each additional pound or fraction thereof	and 2 cents for each additional pound or fraction thereof	and 4 cents for each additional pound or fraction thereof	and 6 cents for each additional pound or fraction thereof	and 8 cents for each additional pound or fraction thereof	and 10 cents for each additional pound or fraction thereof	and 12 cents for each additional pound or fraction thereof
	Weight limit 50 pounds		Weight limit 20 pounds						

The local rate is applicable to parcels intended for delivery at the office of mailing or on a rural route starting therefrom.

Exercise 72

From the table, state at sight the postage on the following:

	Weight of Parcel	Distance carried from Office of Mailing
1.	2 pounds	60 miles
2.	4 pounds	30 miles
3.	8 pounds	350 miles
4.	10 pounds	750 miles
5.	2 pounds	2000 miles
6.	4 pounds	350 miles
7.	2 pounds	Within sixth zone
8.	11 pounds	20 miles
9.	3 pounds	Within third zone
10.	$2\frac{1}{2}$ pounds	100 miles
11.	1 pound	60 miles
12.	$7\frac{1}{2}$ pounds	400 miles
13.	9 pounds	210 miles
14.	$4\frac{1}{2}$ ounces	1500 miles
15.	9 ounces	1250 miles
16.	20 ounces	40 miles
17.	30 ounces	60 miles
18.	3 ounces	600 miles
19.	4 ounces	1000 miles

20. What is the postage on a parcel weighing 4 pounds, mailed for delivery at the local office?

21. What is the postage on a parcel weighing 11 pounds, mailed for delivery at the local office?

22. What is the postage on a parcel weighing 6 pounds, mailed for delivery on a rural route starting from the office of mailing?

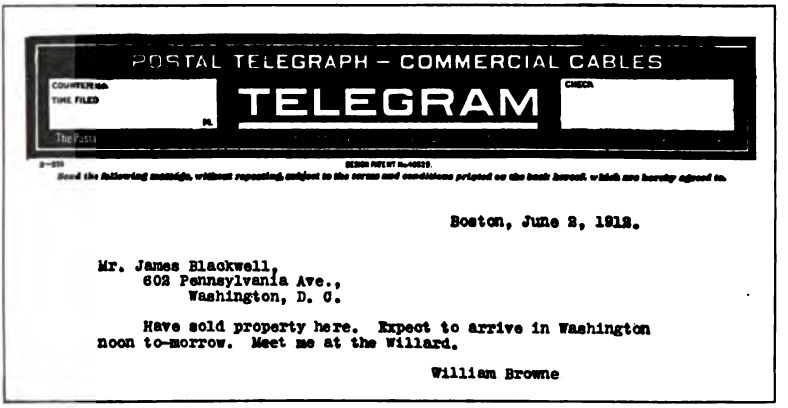
23. What is the postage on a parcel weighing 10 pounds, mailed for delivery on a rural route starting from the office of mailing?

24. What is the postage on a parcel weighing $3\frac{1}{2}$ ounces, mailed for delivery on a rural route starting from the office of mailing?

Messages

Telegrams

149. Find the cost of sending the following telegram, the day rate from Boston to Washington being 40¢ for ten words or less, and 3¢ for each word over ten (address and one signature free):



POSTAL TELEGRAM – COMMERCIAL CABLES

TELEGRAM

Boston, June 2, 1913.

Mr. James Blackwell,
602 Pennsylvania Ave.,
Washington, D. C.

Have sold property here. Expect to arrive in Washington noon to-morrow. Meet me at the Willard.

William Browne

NOTE. The night rate for telegrams is usually lower than the day rate.

Cablegrams

150. Find the cost of sending the following cablegram, the rate from New York to London being 25¢ per word, all words except those of the date to be counted and charged for. In the address the name of "*place to*" and the name of the territorial division of the country in which the "*place to*" is situated are each counted as one word without regard to number of letters they contain. Punctuation marks are counted as one word each.

Form 1

WESTERN UNION *Cable Message* WESTERN UNION

THE WESTERN UNION TELEGRAPH COMPANY

INCORPORATED

THEO. N. VAIL, PRESIDENT BELVIDERE BROOKS, GENERAL MANAGER

THE LARGEST TELEGRAPH AND CABLE SYSTEM IN EXISTENCE. CABLE SERVICE TO ALL THE WORLD

25,000 OFFICES AND 50,000 ADDITIONAL TELEGRAPH AND TELEPHONE CONNECTIONS IN NORTH AMERICA

DIRECT AMERICAN CABLES NEW YORK TO GREAT BRITAIN
CONNECTS ALSO WITH ANGLO-AMERICAN AND DIRECT U. S. ATLANTIC CABLES
DIRECT COMMUNICATION WITH GERMANY AND FRANCE, CUBA, WEST INDIES, MEXICO AND CENTRAL AND SOUTH AMERICA
WITH PACIFIC CABLES TO ALASKA, HONOLULU, AUSTRALIA, GUAM, THE PHILIPPINES, JAPAN, ETC.
MONEY TRANSFERRED BY TELEGRAPH AND CABLE TO ALL THE WORLD

BRANCH OFFICES IN PRINCIPAL CITIES OF GREAT BRITAIN AND THE EUROPEAN CONTINENT. ALL FOREIGN TELEGRAPH STATIONS ACCEPT MESSAGES TO BE SENT "VIA WESTERN UNION"

New York, June 5, 1914

SEND THE FOLLOWING MESSAGE, SUBJECT TO TERMS ON BACK HEREOF, WHICH ARE HEREBY AGREED TO

To A. T. Smith

820 Oxford St. London

We sail Tuesday morning

George Hallett

Night Letters

151. Telegraph companies will receive not later than midnight **night letters** to be transmitted only for delivery

on the morning of the next ensuing business day, at rates still lower than their standard night message rates, and usually stated as follows:

"The standard day rate for ten words shall be charged for the transmission of fifty words or less, and one fifth of the standard day rate for ten words shall be charged for each additional ten words or less."

Find the cost of sending the following night letter, the *day rate* from Albany, N.Y., to Elizabeth, N.J., being 30¢ for ten words or less and 2¢ for each word over ten (address and one signature free):

NIGHT LETTER

THE WESTERN UNION TELEGRAPH COMPANY

26,000 OFFICES IN AMERICA CABLE SERVICE TO ALL THE WORLD

THEO. N. VAIL, PRESIDENT BELVIDERE BROOKS, GENERAL MANAGER

RECEIVED at | TIME FILED | CHECK

SEND the following NIGHT LETTER subject to the terms on back hereof, which are hereby agreed to | Albany, N.Y., June 5, 1913

To Hitchings & Company,

Elizabeth, N.J.

We have just sold for you four greenhouses to be shipped to Quarryville, Pa., at the earliest date possible. We must guarantee that they will be up and ready for use in thirty days. Ship in our name and send workmen on at once to put the houses together. Send us fifty more booklets: we expect more large sales to-morrow.

J. M. Thompson & Co.

Day Letters

152. Telegraph companies will receive **day letters**, to be transmitted at rates lower than their standard day message rates, and usually stated as follows:

"One and one half times the standard night letter rate shall be charged for the transmission of fifty (50) words or less, and one fifth of the initial rate for fifty words shall be charged for each additional ten (10) words or less."

Find the cost of sending the following day letter, the rate from Minneapolis, Minn., to Lexington, Ky., being 50¢ for ten words or less, and 3¢ for each word over ten (address and one signature free):

DAY LETTER Form 2300

THE WESTERN UNION TELEGRAPH COMPANY
INCORPORATED

25,000 OFFICES IN AMERICA CABLE SERVICE TO ALL THE WORLD

THEO. N. VAIL, PRESIDENT — BELVIDERE BROOKS, GENERAL MANAGER

RECEIVER'S No.	TIME FILED	CHECK

SEND the following DAY LETTER subject to the terms on back hereof which are hereby agreed to } Minneapolis, Minn., June 5, 1912.

Mr. G. L. Brown,

Lexington, Kentucky.

Notify me as soon as you have shipped horses. I understand that you are shipping two carloads. There is ready sale here for good horses. Prices are advancing.

F. K. Lewis.

Exercise 73

In the columns headed respectively "Day" and "Night," in the following, the figures before the hyphen indicate rates for ten words or less (address and signature free), and the figures after the hyphen indicate the rates for each word over ten, for messages sent between Philadelphia and the cities named; find the missing costs.

Place	Number of Words	Rate		Cost of Day Telegram	Cost of Night Telegram	Cost of Night Letter	Cost of Day Letter
		Day	Night				
1. Mobile, Ala.	17	60-4	50-3				
2. Phœnix, Ariz.	22	100-7	100-7				
3. Los Angeles, Cal.	25	100-7	100-7				
4. Denver, Colo.	18	75-5	60-4				
5. Hartford, Conn.	60	30-2	25-1				
6. Wilmington, Del.	21	25-2	25-1				
7. Jacksonville, Fla.	40	60-4	50-3				
8. Atlanta, Ga.	15	60-4	50-3				
9. Boise, I.	24	100-7	100-7				
10. Chicago, Ill.	35	50-3	40-3				
11. Columbus, O.	19	50-3	40-3				
12. Des Moines, Ia.	8	60-4	50-3				
13. Kansas City, Mo.	40	60-4	50-3				
14. Louisville, Ky.	26	50-3	40-3				
15. Bar Harbor, Me.	50	50-3	40-3				
16. Baltimore, Md.	16	25-2	25-1				
17. Boston, Mass.	32	35-2	25-1				
18. Albany, N.Y.	45	30-2	25-1				
19. Harrisburg, Pa.	27	25-2	25-1				

Shipping Problems

Exercise 74

1. Find the cost of shipping by freight (1st class) 250 lb. of carpet, from Chicago to Boston, the rate being $ 0.82 per 100 lb. What would it cost to ship the carpet by express, the rate being $ 2.50 per 100 lb.?

2. Find the cost of shipping by freight (1st class) 675 lb. of furniture from Chicago to Cincinnati, the rate being $ 0.40 per 100 lb. What would it cost to ship this furniture by express, the rate being $ 1.50 per 100 lb.?

3. Find the cost of shipping by freight (2d class) 350 lb. of hardware from Chicago to Savannah, the rate being $ 1.10 per 100 lb.

4. Find the cost of shipping by freight (3d class) 560 lb. of farm implements from Chicago to Baltimore, the rate being $ 0.47 per 100 lb.

5. Find the cost of shipping by freight (4th class) 1260 lb. steel roofing from Chicago to Wheeling at $ 0.30 per 100 lb.

6. Find the cost of shipping by freight (1st class) 125 lb. of electrical goods from Chicago to Seattle, the rate being $ 3.00 per 100 lb. What would it cost to ship these goods by express, the rate being $ 11.50 per 100 lb.?

7. Find the cost of shipping by freight (1st class) 210 lb. of sporting goods from Chicago to Kansas City, the rate being $ 0.80 per 100 lb. What would it cost to ship these goods by express, the rate being $ 2.00 per 100 lb.?

REVIEW OF DECIMALS

Exercise 75

1. Each of two boys has $0.20; how much more must they together get to have $1?

2. If 3 lb. of coffee cost $0.90, what should $\frac{1}{2}$ lb. cost at the same rate?

3. A merchant had on hand, at the beginning of a day's sales, $52.75. During the day he took in $215.18 cash and paid out $32.28 cash. How much money had he on hand at the close of the day?

4. A contractor paid $30 to 3 men for 5 days' labor. To one he paid $9 and to another $12; how much did he pay the third?

5. A boy bought 4 lb. of sugar at $6\frac{1}{2}¢$ per pound and 1 lb. of coffee at $30¢$; how much change should he receive, if he gave in payment $1?

6. A boy has 20 nickels, 5 dimes, and 6 quarter dollars; how much money has he?

7. A boy sells 60 lb. of scrap iron at the rate of 4 lb. for $1¢$; how much does he receive for it?

8. Two pairs of shoes cost $6; if one pair cost $3.75, what did the other pair cost?

9. How much must be added to the sum of $25 and $16.25 to make $100?

10. In 1910 Portugal produced .253 as much wine as Spain, which produced 428,000,000 gal.; how much did Portugal produce?

11. A merchant failed in business and could pay only \$ 0.875 on the dollar; how much would a man lose to whom the merchant owed \$ 350?

12. Beef scraps for poultry can be bought at the rate of 5 lb. for 25¢. 100 lb. can be bought for \$ 3.50. How much more must be paid for 100 lb. bought in quantities of 5 lb. rather than as one quantity of 100 lb.?

13. How many bushels of corn, worth 90¢ a bushel, must be given for 5 barrels of flour worth \$ 5.40 a barrel?

14. Find the amount of the following sales of potatoes:

10 bu. @ \$ 1.30 per bushel,
12 bu. @ \$ 0.20 per peck,
8 bu. @ \$ 0.15 per half peck.

15. Find the wages due for 6 days' work of $12\frac{1}{2}$ hours per day, if the wages paid were \$ $0.37\frac{1}{2}$ per hour.

16. What number multiplied by 1.2 will produce 17.28?

17. The wage rate of men on farms in Michigan, in a recent year, was \$ 29.37 per month without board, and \$ 21.57 per month with board; what was the estimated value of board per year?

18. If a man's income is \$ 1500 for a year and his expenses are \$ 875, what are his average savings per month?

19. Water weighs 62.5 lb. per cubic foot, and silver is 10.47 times as heavy as water; what does a cubic foot of silver weigh?

20. A farmer planted 12.5 bu. of seed and the yield was 378.125 bu. of grain; what was the yield for each bushel sowed?

21. Express each of the following as a decimal part of a dollar, and find their sum: 12.5 cents, 18.75 mills, 1.75 cents, 14 cents, 6.5 mills, and $\$\frac{1}{2}$.

22. First-class fitters and cutters in factories of large cities receive $\$25$ per week, while plain sewers receive but $\$5$ per week. What wages do 10 fitters and cutters and 6 plain sewers earn in one week?

23. The weight of dressed hogs is about 80 lb. to each 100 lb. of their live weight; how much should a hog whose live weight is 350 lb. weigh when dressed?

24. How much more is paid per year for *The Saturday Evening Post* bought at 5¢ per copy, 52 numbers per year, than if it were subscribed for at $\$1.50$ per year?

25. Simplify: $\dfrac{.001 \div .1}{.01 \div 10}$.

26. I have ten dollars in 5-cent pieces; how many 5-cent pieces have I?

27. Find the cost of 360 cabbage plants at $\$1.75$ per hundred.

28. Simplify: $\dfrac{.02 \div .002}{.002 \div .02}$.

29. Find the cost of sending a registered letter weighing $3\frac{1}{2}$ oz.

30. Find the cost of sending a registered parcel containing merchandise and weighing $1\frac{1}{2}$ oz.

31. Milch cows should be fed each day 2.25 lb. of hay or its equivalent for each 100 lb. of their live weight. How much should a cow weighing 800 lb. be fed each day?

32. One cubic foot of water at the rate prescribed by the Philadelphia council costs .03¢. The average family should use not more than about 2400 cu. ft. per month. What must it pay for water, at the rate prescribed, for one year?

33. A man who earns $2.50 a day earns how much during the working days of September, the first day of which is Tuesday?

34. If a man is paid $2.75 for working a 10-hour day, what should he be paid for working during a certain day from 7 A.M. till noon, and from 2 P.M. to 4 P.M.

35. There are 1728 cubic inches in a cubic foot and 231 cubic inches in a gallon. Estimate to the nearest hundredth the number of gallons in a cubic foot.

36. A cubic foot of water weighs 62.5 lb.; milk is 1.03 times as heavy as water. What is the weight of 1 cubic foot of milk?

37. At $24.50 per 1000 feet, what is the cost of 1275 feet of boards?

38. Estimating that a cubic foot of milk equals 7.48 gallons, and that milk is 1.03 times as heavy as water, find, correct to the nearest hundredth of a pound, the weight of a gallon of milk, water weighing 62.5 lb. per cubic foot.

39. Cast iron is 7.2 times as heavy as water; how many cubic feet of cast iron are in a ton of 2000 lb.?

40. When 2.5 bushels of seed yield a crop of 28 bushels of grain, what is the yield for each bushel sowed?

41. A mechanic who receives $3.60 for a 10-hour day, is paid $1\frac{1}{2}$ times his regular wages for each hour he works

over 10 hours; how much should be paid him for working during a certain day from 7 A.M. till noon, and from 1 P.M. to 8 P.M.?

42. A gallon contains 231 cubic inches and a bushel 2150.42 cubic inches; find, correct to the nearest hundredth, the number of gallons in a bushel.

43. Simplify: $\frac{.125 + .25}{.625 + .375}$.

44. When 9¢ is paid for the loan of \$2, the sum paid is what decimal part of the loan?

45. A mechanic is receiving \$3 a day (10 hours) and 50¢ an hour for overtime. During a certain week he works each working day from 7 A.M. till noon, from 1 P.M. to 6 P.M., and from 7 P.M. to 10 P.M.; what are his wages for that week?

46. The average age of the 24 girls in a certain school is 13.55 years; what will be the average age to the nearest hundredth, of the girls in this school, if 6 more enter whose ages average 14.62 years?

47. Two bushels of hair are used for each coat in a hundred square yards of plastering. How many bushels of hair are needed to give 5760 sq. yd. of surface two coats of plaster?

48. Of what denomination is the product when tenths are multiplied by tenths? hundredths by hundredths? tenths by hundredths? thousandths by hundredths?

49. Of what denomination is the quotient when tenths are divided by tenths? hundredths by tenths? thousandths by hundredths?

GENERAL REVIEW

Exercise 76

1. Write in figures $1\frac{1}{2}$ million.

2. Find the cost of sawing 27,560 ft. of lumber at \$ 6 per 1000 ft.

3. Under ordinary conditions tomatoes yield 300 bu. to the acre; how many acres will yield 1350 bu.?

4. The dressed weight of a hog is estimated to be .80 of its live weight; estimate what was the live weight of a hog whose dressed weight was 142 lb.

5. If envelopes are bought at the rate of 20¢ a hundred and sold at the rate of 2 for 1¢, what is the profit on 1000 envelopes?

6. Express as fractions each of the following in two other ways: $\frac{6}{8}$, $\frac{8}{12}$, $\frac{5}{15}$.

7. The following are the ingredients necessary to make a filling for a small lemon pie for a family of 4 persons; how much of each of these ingredients must be used to make a dozen such pies?

$1\frac{1}{2}$ tablespoonfuls of cornstarch
$\frac{1}{2}$ cup of sugar
$\frac{1}{2}$ cup of boiling water
1 teaspoonful of butter
$\frac{1}{2}$ lemon
1 egg
$\frac{1}{2}$ teaspoonful of pulverized sugar

8. A man has $4000 in the bank; if he earns $500 a year and spends $750 a year, in how many years will all his money be gone?

9. To a kitchen $18\frac{1}{2}$ ft. long and 16 ft. wide, an addition is built 9 ft. long and 16 ft. wide. What is the distance around the enlarged kitchen?

10. A dealer bought a live turkey weighing 14 lb. at $12\frac{1}{2}¢$ a pound. If it loses $\frac{1}{4}$ of its weight in dressing, at what price per pound must the dressed turkey be sold that the gain may be $35¢$?

11. The value of 8 bu. of wheat and 6 bu. of corn is $12. If the value of each bushel of wheat is $90¢$, what is the value of each bushel of corn?

12. If a man earns $2 per day for 6 days of the week and saves $3 per week, how much does he spend per week?

13. Under ordinary conditions sweet corn will yield 2000 lb. to the acre; how many pounds should $\frac{3}{8}$ acre yield?

14. A boy won 6 marbles, which were $\frac{3}{5}$ of what he already had; what was then his total number of marbles?

A good mixed feed for chickens from 3 days to 3 weeks old can be made from the following formula:

Formula

28 lb. cracked wheat
20 lb. granulated oatmeal
30 lb. fine screened cracked corn
4 lb. fine cracked peas
4 lb. broken rice
10 lb. chick grit
4 lb. fine charcoal (chick size)

Refer to formula in answering questions 15–21.

15. What part of the mixture is cracked wheat?

16. What part of the mixture is chick grit?

17. What part of the mixture is cracked corn?

18. What part of the mixture is granulated oatmeal?

19. How many pounds of each ingredient are there in 500 lb. of the mixture?

20. How many pounds of each ingredient are there in a ton of the mixture?

21. How many pounds of each ingredient are there in 50 lb. of the mixture?

22. Simplify $.006 + .02 \times .02 + .006 + .002 \times 60 + .12$.

23. How many pounds of butter fat should a herd of 25 cows produce in a week, if they average 24 lb. of milk daily, and the milk tests .04 butter fat?

24. A dealer bought 10,000 cigars at \$3.50 per 100 and retailed them at 4 for 25¢. Find his profit.

25. The weight of a common brick is about 4.5 lb. What is the weight of a load of 750 bricks?

26. In what time should 2 men finish a piece of work $\frac{3}{4}$ of which 4 men have done in 1 da.?

27. In a recent World's Championship Series in baseball $\frac{8}{9}$ of \$147,572.28 was divided equally among 22 players of the winning team. Find, correct to the nearest cent, how much each of these players received.

28. A dealer marked a hat to gain 50¢, but he finally sold it for \$1.75, which was at a loss of 25¢; find the marked price.

29. A laborer's wages for a month were \$50. He spent $\frac{5}{8}$ of it and saved the remainder; how much did he save?

30. A man receives $\$\frac{2}{5}$ an hour and during a certain week puts in time as follows: Monday, $7\frac{1}{2}$ hr.; Tuesday, 8 hr.; Wednesday, $8\frac{1}{2}$ hr.; Thursday, 10 hr.; Friday, $12\frac{1}{2}$ hr.; Saturday, 6 hr.; what does he receive that week?

31. The amount of timber standing in Oregon is estimated at 400,000,000,000 ft., or one sixth of the standing timber of the United States; estimate the amount of standing timber in the United States.

32. A piece of wire $\frac{7}{8}$ in. long was divided into 7 equal parts; how many of these parts together measure $\frac{3}{4}$ of an inch in length?

33. What is the value of 102,569.5 lb. of butter fat at 25.92¢ per pound?

34. What is the postage on a letter weighing 2 oz., if the envelope bears also a special delivery stamp?

35. What should be charged for 2240 lb. of coal at $\$0.32$ per 100 lb.?

36. Anna has $\$2.75$ and Harriet has $\$0.15$ more than 3 times the amount that Anna has. How much money have both?

37. If a dealer gains $\$5.50$ on $\$75$ invested, find, correct to the nearest cent, his gain on each dollar invested.

38. The annual output of grapes in a certain state is estimated at 4,500,000 lb., valued at $\$135{,}000$; what is the estimated value per pound?

39. Simplify:

$$\frac{\frac{3}{4} \div \frac{1}{2} - \frac{1}{2} + \frac{3}{4}}{62\frac{1}{2} + 100}$$

40. Estimating the amount of standing timber in the United States at 2,400,000,000,000 ft., and the average price on the stump at $2.44 per 1000 ft., what is the value of the standing timber in the United States?

41. Each bushel of wheat raised is estimated to use up approximately the following amount of fertilizing substances from the soil: nitrogen, 20 oz.; phosphoric acid, 8 oz.; potash, 5 oz.; how many pounds of each of these substances are used up by 10 A. of wheat averaging 25 bu. per acre?

42. Simplify:

$$\frac{100 \div .01}{.01 \times 100} + \frac{.1 \div .01}{.01 \div 2.5}$$

43. In an experiment made on a large potato farm it was found that the price of producing an acre of potatoes, including seed, labor, land rental, etc., was $26.37. The average yield per acre was 127 bu., which were sold at 40¢ per bushel; what was the net income per acre?

44. The factors of the dividend are 100, .1, .01, and .125; the divisor is $.125 \div .005 \times .0025$. Find the quotient.

45. A laborer was offered $20 per month and board for a certain year, or $1.25 per day without board. Which was the better offer and by how much, counting 300 working days to the year and estimating board at $3 per week for 52 weeks?

A good mixed dry feed for poultry can be made in any desired quantity from the following formula:

Formula

200 lb. wheat bran
100 lb. middlings
100 lb. corn meal

Refer to formula in answering questions 46–50.

46. What part of the mixture is corn meal?

47. What part of the mixture is wheat bran?

48. How many pounds of each ingredient are there in a ton of the mixture?

49. How many pounds of each ingredient are there in 100 lb. of the mixture?

50. How many pounds of each ingredient are there in 50 lb. of the mixture?

51. Fill in the missing amounts in the following:

One Man's Experience Both Ways

Here is the comparative cost of hauling whole milk, as against hauling cream alone, as figured by one dairyman from his actual experience with both methods:

Cost of Hauling Whole Milk	Cost of Hauling Cream Only
20 weeks, 5 deliveries per week, 5 hours per day, with heavy wagon, time of man and team valued at 20 cents per hour . $	20 weeks, 3 deliveries per week, 2½ hours per day, with light wagon, time of man and team valued at 20 cents per hour . $
32 weeks, 3 deliveries per week, 5 hours per day, with heavy wagon, time of man and team valued at 20 cents per hour . $	32 weeks, 2 deliveries per week, 2½ hours per day, with light wagon, time of man and team valued at 20 cents per hour . $
Total cost per year of delivering whole milk . . . . . $	Total cost per year of delivering cream only . . . . . $

YEARLY SAVING THROUGH HAULING CREAM ONLY . . . $

DENOMINATE AMOUNTS

DEFINITIONS

153. Denominate units are units established by custom or law to measure *capacity, weight, time, length, surface, solids,* etc.

Thus, *a gallon, a pound, a day, a yard,* etc., are denominate units.

154. A simple denominate amount is any number of like denominate units.

Thus, 6 *bushels,* $\frac{1}{2}$ *of a yard,* and $2\frac{1}{2}$ *pounds* are simple denominate amounts.

MEASURES OF CAPACITY

Liquid Measure

155. Table

4 gills (gi.)	**= 1 pint (pt.)**
2 pints	**= 1 quart (qt.)**
4 quarts	**= 1 gallon (gal.)**
$31\frac{1}{2}$ gallons	**= 1 barrel (bbl.)**
63 gallons	**= 1 hogshead (hhd.)**

Barrels are of various sizes, but $31\frac{1}{2}$ gal. is usually taken as the capacity for a barrel unless otherwise stated.

Exercise 77

1. How many pints are there in 4 qt.?
2. One pint is what part of a quart?

3. How many quarts are there in $\frac{1}{2}$ gal.?
4. How many quarts are there in $2\frac{1}{2}$ gal.?
5. How many pints are there in a gallon?
6. How many gills are there in a quart?
7. $\frac{1}{2}$ gal. of alcohol will fill how many gill bottles?
8. What part of a gallon is 2 qt.? 3 qt.?
9. How many quarts are there in 8 pt.? 18 pt.? 24 pt.?
10. How many gallons are there in 24 qt.? 32 qt.? 40 qt.?

156. Dry Measure

Table

2 pints (pt.)	**= 1 quart (qt.)**
8 quarts	**= 1 peck (pk.)**
4 pecks	**= 1 bushel (bu.)**

The heaped bushel, which contains about 5 pk., or $\frac{5}{4}$ of a stricken bushel, is used in measuring corn in the ear, potatoes, apples, lime, turnips, and various other substances.

Two bushels of corn in the ear are considered the equivalent of one bushel of shelled corn.

Exercise 78

1. How many quarts are there in 3 pk.?
2. A quart is what part of a peck?
3. How many quarts are there in $\frac{1}{2}$ pk.? $\frac{1}{4}$ pk.?
4. How many quarts are there in a bushel? $\frac{1}{2}$ bu.?
5. One peck is what part of a bushel?

6. Two pecks are what part of a bushel? 3 pk.?

7. How many quarts are there in $1\frac{1}{2}$ bu.?

8. How many half bushels of grain will fill a 3-bu. bag?

9. A trucker put $\frac{1}{2}$ bu. berries into quart boxes; how many boxes were required?

10. When timothy seed sells for $3 per bushel, how much is that per peck?

d the cost of $2\frac{1}{2}$ bu. of clover seed at $11 per

nen potatoes are selling at 30¢ a peck, how hat per bushel?

5 pk. of seed will fill how many bushel sacks many pecks over?

How many quart boxes will be required to hold cherries?

MEASURES OF WEIGHT

Avoirdupois Weight

Table

16 ounces (oz.)	= 1 pound (lb.)
2000 pounds	= 1 ton (T.)

is sometimes called 1 *hundredweight* (cwt.). Goods are y shipped by the hundredweight.

ng ton, 2240 pounds, is used in custom houses and some- where, especially in selling coal. Anthracite coal is always ie mines, and often retailed, by the long ton. Bituminous equently sold in the same way.

Weights of Grains, Seeds, and Other Produce

158. The following table gives the number of pounds in a bushel of various commodities as used by the Boards of Trade in the principal cities of the United States:

Table

Commodities	Lb.	Commodities	Lb.	Commodities	Lb.
Barley	48	Corn (shelled)	56	Peas	60
Beans	60	Corn (on cob)	70	Rye	56
Buckwheat	48	Clover seed	60	Timothy seed	45
Bran	20	Oats	32	Wheat	60

159. **Weights of Everyday Things**

A barrel of flour weighs	196 lb.
A barrel of salt weighs	280 lb.
A barrel of pork, beef, or fish weighs	200 lb.
A bushel of coarse salt weighs	56 lb.
A keg of nails weighs	100 lb.
A firkin of butter weighs	56 lb.
A keg of powder weighs	25 lb.
A bushel of Portland cement weighs	96 lb.
A bushel of loose lime weighs	70 lb.

Exercise 79

1. What should $\frac{1}{2}$ bu. shelled corn weigh?
2. What should a peck of clover seed weigh?
3. What should $\frac{1}{2}$ peck of oats weigh?

4. How many bushels of bran should there be in 100 lb.?

5. What should $\frac{1}{2}$ bu. of timothy seed weigh?

6. What should 4 oz. of cheese cost at 16¢ per pound?

7. How many more pounds are there in a long ton than in a short ton?

8. What should 1000 lb. of hay cost at $18 per ton?

9. How many 200-lb. sacks of fertilizer will make a ton?

10. How much should a peck of beans weigh?

11. How many bales of hay, each weighing 4 hundredweight, will make a ton?

12. What will $\frac{1}{4}$ lb. of spices cost at 5¢ an ounce?

13. What will 1 lb. 12 oz. of dried beef cost at 24¢ per pound?

Find the cost of 4500 lb. of hay at $18.50 per ton.

Thus, 4500 lb. = 4.500 M lb. = 2.25 T.

$\therefore 2.25 \times \$18.50$ or $\$41.625$ = the cost.

Find the cost of:

14. 1870 lb. bran at $18 per ton.

15. 3880 lb. of straw at $6 per ton.

16. 4660 lb. of fertilizer at $14 per ton.

17. 1840 lb. of hay at $18 per ton.

18. 3250 lb. of mixed feed at $20 per ton.

19. 8560 lb. of phosphate at $18 per ton.

20. 10750 lb. of iron ore at $6 per ton.

21. 270 lb. of bran at $21 per ton.

22. 3420 lb. of old iron at $20 per ton.

23. 4590 lb. of coal at $6 per ton.

24. What should be paid for 1830 lb. of wheat at $ 0.98 per bushel?

25. What should be paid for 2856 lb. of wheat at $ 0.90 per bushel?

26. What should be paid for 3856 lb. of oats at $ 0.50 per bushel?

27. What should be paid for 2900 lb. of oats at $ 0.48 per bushel?

28. What should be paid for 2 kegs of nails at $3\frac{1}{2}¢$ per pound?

29. A ton of flour will make how many barrels and how many pounds over?

MEASURES OF TIME

160. Table

60 seconds (sec.)	= 1 minute (min.)
60 minutes	= 1 hour (hr.)
24 hours	= 1 day (da.)
7 days	= 1 week (wk.)
52 weeks	= 1 year (yr.), nearly
365 days	= 1 common year
366 days	= 1 leap year
100 years	= 1 century

NOTE 1. A centennial year is one whose number is divisible by 100. Thus, 400, 1800, and 1900 were centennial years.

NOTE 2. A centennial year whose number is divisible by 400, or any other year whose number is divisible by 4, is a leap year.

Thus, 1600, 1904, and 1912 were leap years.

NOTE 3. The civil day, or calendar day, is considered to begin at midnight.

Exercise 80

1. The winter months are December, January, and February; how many days are there in the winter months?

2. A family that uses 2 qt. of milk a day spends how much for milk during March, the cost of the milk being 6¢ a quart?

3. Name the 7th day of the week; the 5th; the 2d; the 1st; the 4th; the 3d; the 6th.

4. Name the months that have 30 days.

5. When a pulse beats 18 times in 15 seconds, how many times does it beat in a minute?

6. A certain building was erected in MDCCCXC; how many years ago was that?

7. Name the months that have 31 days.

8. One coin is dated 1880, another 1912; how many years are there between their dates?

9. Name the 11th month; the 4th; the 7th; the 12th; the 8th; the 3d; the 6th; the 1st; the 9th; the 2d; the 5th; the 10th.

10. Name the Roman letters that are used to write the number of the present year.

11. At 20¢ an hour, how much will a motorman earn from 6 A.M. to 3 P.M.?

12. How much rent was paid in 1 yr. 6 mo. for a house that rented for $20 a month?

13. When a hotel is charging 75¢ a meal and $2 for lodging, how much is it charging per week at the same rate?

MEASURES OF LENGTH

Linear Measure

161. The standard unit of length is the **yard**.

162. **Table**

12 inches (in.)	**= 1 foot (ft.)**
3 feet	**= 1 yard (yd.)**
$5\frac{1}{2}$ yards	**= 1 rod (rd.)**
320 rods	**= 1 statute mile (mi.)**

163. **Other Measures of Length**

A fathom = 6 ft., used to measure the depth of the sea.

A hand = 4 in., used in measuring a horse's height.

A geographic mile, or knot = 6086 ft.

A league (England and U.S.) = 3 geographic miles, used to measure distance at sea.

Surveyors' Linear Measure

164. For measuring distances, surveyors generally use a chain (4 rd.) made of 100 links, or a **tape measure**, generally made of steel or cloth, and usually 50 feet or 100 feet long, graduated to feet, tenths, and sometimes hundredths, of a foot.

165. **Table**

7.92 inches	**= 1 link (li.)**
100 links	**= 1 chain (ch.)**
80 chains	**= 1 mile (mi.)**

Exercise 81

1. How many inches are there in 3 ft.? in $\frac{1}{2}$ ft.? in $1\frac{1}{2}$ ft.?

2. How many feet are there in 3 yd.? in $\frac{1}{2}$ yd.? in $1\frac{1}{2}$ yd.?

3. How many 6-in. ropes can be cut from a rope 8 ft. long?

4. 6 inches is what part of a foot?

5. 4 inches is what part of a foot?

6. An 18-in. rule is how many feet long?

7. A horse that is $14\frac{1}{2}$ hands high is how many inches high?

8. What is the distance around a room 24 ft. long and 10 ft. wide?

9. A yardstick was marked off in inches; how many inches?

10. 1 rd. $= 5\frac{1}{2} \times 3$ ft., or —— ft.?

11. The ocean, at a certain place, is 80 fathoms deep; how many feet is that?

12. How many inches are there in $\frac{1}{4}$ yd.? in $\frac{3}{4}$ yd.?

13. What part of a yard is a foot?

14. What part of a yard is 6 in.?

15. What is the distance around a boy's hoop if it turns 4 times in a distance of 20 ft.?

16. In ordinary walking, the pace is $2\frac{1}{2}$ feet; how many feet are there in 10 paces?

17. How many inches are there in a chain?

18. How many feet are there in a chain?

19. How many yards are there in a chain?

20. How many rods are there in a mile? how many chains?

21. How many feet are there in a league?

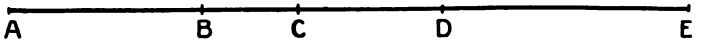
22. A furlong equals 40 rd.; what part of a mile is a furlong?

23. A span equals 9 in.; what part of a fathom is a span?

Measuring to a Scale

Exercise 82

1. Measure the parts of AE and tell how long each is.



If AB represents 1 mi., we say that line AE is drawn to a scale of 1 in. to 1 mi. and we write:

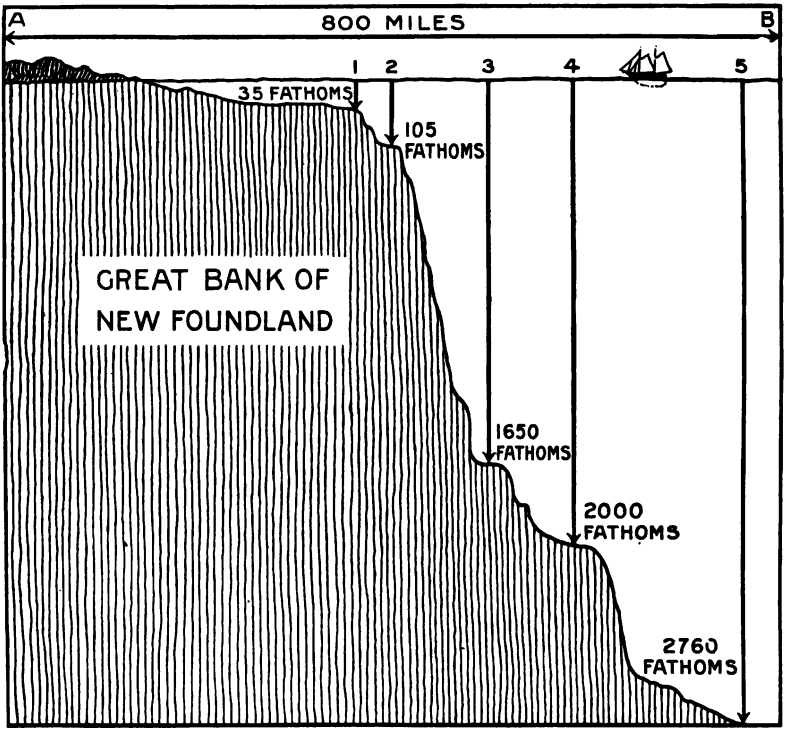
Scale: 1 in. = 1 mi.; *or in practice:* 1″ = 1 mi.

2. When the scale is 1″ = 1 mi., what is the distance from A to C? from B to D? from A to E? from B to E? from C to E?

3. When the scale is 1″ = 100 ft. (′), what is the distance from A to B? from A to C? from B to C? from C to D? from B to D? from D to E?

4. How far are two cities apart if the distance between them on a map is $4\frac{1}{2}$ in., and the map is drawn to the scale of 1 in. to 20 mi.?

5. Measure the distance from A to B on the following diagram and tell how many miles each inch represents.



6. How many miles are represented on this map from arrow 1 to arrow 2? from arrow 2 to arrow 3?

7. How many miles are represented on this map from arrow 3 to arrow 4? from arrow 4 to arrow 5?

8. How many feet deep is the ocean at arrow 1? at arrow 2? at arrow 3? at arrow 4? at arrow 5?

LINES AND ANGLES

166. A **straight line** is a line that has the same direction throughout; as, AB.

167. A **curved line** is a line no part of which is straight; as, CD.

168. The opening between two straight lines drawn from the same point is called an **angle.**

Thus, Fig. 1 is an angle.

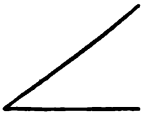


Fig. 1

169. If one straight line meets another straight line so as to form two equal angles, each angle is called a **right angle.**

Thus each angle of Fig. 2 is a right angle.

Fig. 2

170. When one straight line meets another straight line so as to form with it a right angle, the lines are said to be **perpendicular** to each other.

Thus AC and BC, Fig. 3, are perpendicular to each other.

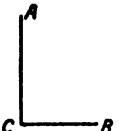


Fig. 3

NOTE. Notice that if BC were extended, AC would form with BC two equal angles.

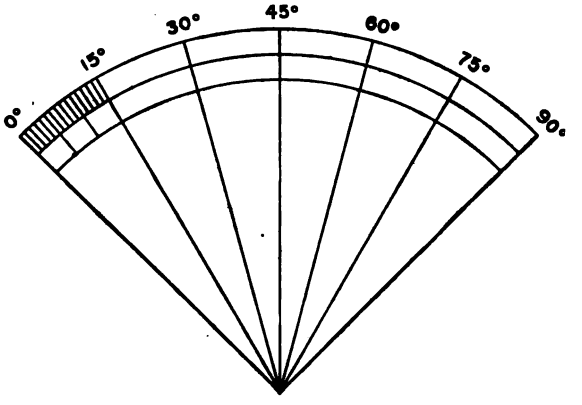
171. An **acute angle** is an angle that is less than a right angle. An **obtuse angle** is an angle that is greater than a right angle.

Acute Angle

Right Angle

Obtuse Angle

172. An angle equal to $\frac{1}{90}$ of a right angle is an angle of 1 **degree**, which is the **unit of angles**; $\frac{1}{60}$ of an angle of



1 degree is an angle of 1 **minute**; and $\frac{1}{60}$ of an angle of 1 minute, an angle of 1 **second**.

173. **Table**

60 seconds (″)	**= 1 minute (′)**
60 minutes	**= 1 degree (deg. or °)**
90 degrees	**= 1 right angle (rt. ∠)**

Exercise 83

Read these angles:

1. 15°; 15° 30′; 15° 30′ 30″.
2. 35°; 10° 25′; 45° 32′ 20″.
3. 47°; 50° 38′; 86° 20′ 15″.

Write, using the proper symbols:

4. 3 degrees; 5 degrees, 6 minutes; 7 degrees, 8 minutes, 10 seconds.

5. 10 degrees; 14 degrees, 30 minutes; 21 degrees, 5 minutes, 4 seconds.

6. 32 degrees; 16 degrees, 25 seconds; 8 degrees, 15 minutes, 2 seconds.

7. Draw an acute angle; an obtuse angle.

8. A right angle will make how many 30° angles? how many 45° angles?

9. How many minutes are there in 2°? 3°?

10. How many minutes are there in 1° 30′?

11. What kind of angle does the hour hand make with the minute hand at 3 o'clock?

12. What kind of angle does the hour hand make with the minute hand at 9 o'clock?

13. If one line runs north and another east, both starting from the same point, what kind of angle do they make with each other?

14. Northeast is halfway between north and east; how many degrees are there between two lines, one running north and the other running northeast, both starting from the same point?

15. How many degrees are there between two lines, one running northeast and the other running east, both starting from the same point?

16. How many degrees are there between two lines, one running northeast and the other running southeast, both starting from the same point?

MEASURES OF SURFACES

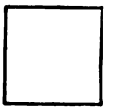
174. Anything that has length and breadth without thickness is called **surface.**

175. A surface on which any number of straight lines can be drawn in different directions is a **plane surface.**

The *number* of square units in a surface is its **area.**

The Square

176. A **square** is a portion of a plane bounded by four equal straight lines and having four right angles.

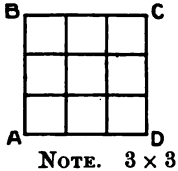


1. A **square inch** is a square 1 inch long.
2. A **square foot** is a square 1 foot long.
3. A **square yard** is a square 1 yard long.
4. A **square rod** is a square 1 rod long.
5. A **square mile** is a square 1 mile long.
6. A **square unit** is a square 1 unit long.

177. The result obtained by using a number twice as a factor is the **square** of the number.

Thus, 3×3, or 9, is the square of 3.

178. $ABCD$ is a square 3 units long.



3 = the number of square units in 1 row.
3 = the number of rows of square units.
$\therefore 3 \times 3$, or 9 = the number of square units in $ABCD$.

NOTE. 3×3 may be written 3^2, read *3 squared.*

179. The area of a square is the square of the number of units in its length.

180. Table of Square Measure

144 square inches (sq. in.)	**= 1 square foot (sq. ft.)**
9 square feet	**= 1 square yard (sq. yd.)**
$30\frac{1}{4}$ square yards	**= 1 square rod (sq. rd.)**

Exercise 84

1. Point out three plane surfaces in this room.
2. Point out three surfaces that are not plane.
3. A square 1 foot long is how many inches long?
4. How many square inches are there in a square 12 in. long?
5. How many feet long is a square a yard long?
6. Represent on the blackboard a square 1 yd. long; scale: $2'' = 1'$. Show how many square feet this square contains.
7. How many square inches are there in a square 2 in. long (a 2-in. square)?
8. How many square inches are there in a 4-in. square?
9. How many square inches are there in a 5-in. square?
10. A square $\frac{1}{2}$ ft. long contains what part of a square foot? How many square inches does it contain?
11. What is the distance around (the **perimeter** of) a square 40 rd. long?
12. What is the perimeter in feet of a square lawn 50 yd. long?
13. Find the cost at 40¢ per rod of a fence that will inclose a square garden 66 ft. long.

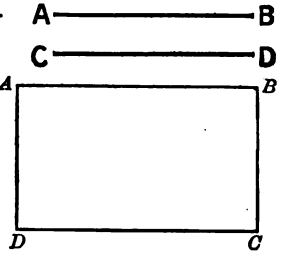
14. How many square feet are there in the walls and ceiling of a bathroom 10 ft. long, 10 ft. wide, and 10 ft. high?

15. How many square feet of paper will cover a box 2 ft. long, 2 ft. wide, and 2 ft. high?

16. Represent on the blackboard a square $5\frac{1}{2}$ yd. long; scale: $2'' = 1'$. Show how many square yards this square contains.

The Rectangle

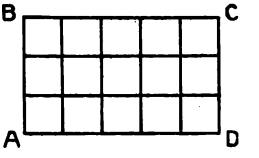
181. **Parallel** lines are lines that lie in the same plane and cannot meet, however far produced.



182. A **rectangle** is a portion of a plane that has four straight sides and four right angles.

183. The opposite sides of a rectangle are equal and parallel.

184. *ABCD* is a rectangle 5 units long and 3 units wide.



5 = the number of square units in each row.

3 = the number of rows of square units.

$\therefore 3 \times 5$, or 15, = the number of square units in *ABCD*.

185. The area of a rectangle is the number of units in its length multiplied by the number of units in its width.

Land Measure

186. The unit of land measure is the **acre**, which may be of any shape; it is equivalent to a rectangular tract 20 rd. long and 8 rd. wide. In cities land is sometimes sold by the square foot.

187. Building lots are usually sold by the **front foot** (per foot). Thus, if a lot extends 60 ft. along a certain street, it is said to have a frontage of 60 ft. on that street, and if it is sold at \$10 per front foot (per foot), the selling price is $60 \times \$10$, or \$600.

NOTE. By the depth of a lot is meant the distance it extends back from the street along which its frontage is measured.

188. Table of Land Measure

10,000 square links (sq. li.) = 16 square rods (sq. rd.) = 1 square chain (sq. ch.)

10 square chains = 160 square rods = 1 acre (A.)

640 acres = 1 square mile (sq. mi.)

NOTE. A square rod is sometimes called a perch.

Exercise 85

1. What is the perimeter of a rectangle 3 ft. long and 2 ft. wide?

2. What is the perimeter of a rectangle 20 rd. long and 8 rd. wide?

3. How many square feet are there in a floor 14 ft. long and 10 ft. wide?

4. How many acres are there in a rectangle 20 rd. long and 8 rd. wide?

5. How many square feet are there in the ceiling of a room 20 ft. long and 18 ft. wide?

6. How many square yards of bunting will make a flag $4\frac{1}{2}$ yd. long and 3 yd. wide?

7. How many square yards of plastering are there in the walls and ceiling of a room 18 ft. long, 12 ft. wide, and 9 ft. high?

8. How many square inches are there in a sheet of paper $12\frac{1}{2}$ in. long and $7\frac{1}{2}$ in. wide?

9. Find the number of square feet in a rug 24 in. by 26 in.

10. Find the number of square inches in a rug 27 in. by 54 in.

11. Find the number of square feet in a rug $4\frac{1}{2}$ ft. by $7\frac{1}{2}$ ft.

12. Find the number of square feet in a rug $7\frac{1}{2}$ ft. by $10\frac{1}{2}$ ft.

13. How many acres are there in a tract of land in the form of a square 80 rd. long?

14. How many square rods are there in a square mile?

15. Divide the number of square rods in a square mile by 160 to find the number of acres in a square mile.

16. Make a drawing representing a square 40 rd. long; scale: $2'' = 5$ rd. How many acres are there in this square?

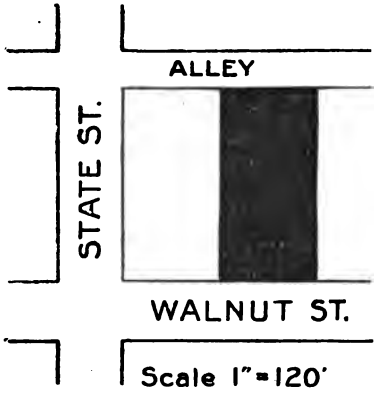
17. Make a drawing representing a square 4 rd. long; scale: $2'' = 1$ rd. What part of an acre does this square contain?

18. How many squares 4 rd. long are required to make an acre?

19. How many square feet are there in a square rod?

20. At $0.90 per rod, find the cost of inclosing a field with wire fence, if the sides of the field measure 6 ch., 8 ch. 50 li., 7 ch. 25 li., 10 ch. 40 li., and 7 ch. 20 li.

21. How many rods of fencing are necessary to inclose a tract of land 1 mile square, and divide it into 4 equal squares?



22. How many feet frontage on Walnut Street has the shaded lot? How many feet deep is this lot? What is it worth at $10 per front foot? How wide is State Street? Walnut Street? the alley?

23. How many 2-inch squares can be cut from a sheet of paper 1 ft. 3 in. long and 9 in. wide?

24. A strip of land 33 ft. wide and $\frac{1}{2}$ mi. long was taken from a farm for a road; how many acres were taken from it?

25. How many acres are there in a rectangular piece of land 16 ch. long and 10 ch. wide?

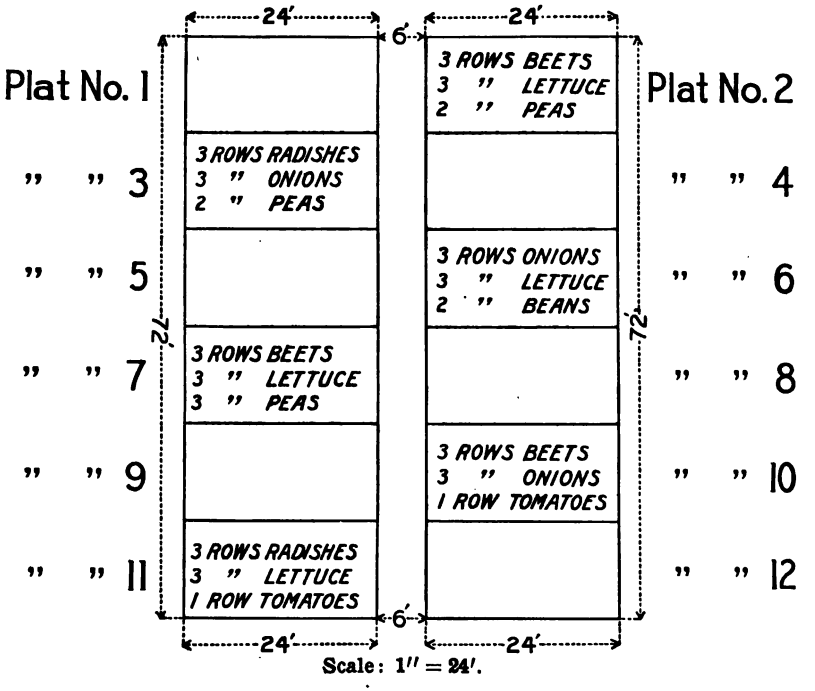
26. How many acres are there in a rectangular piece of land 12 ch. long if the distance around it is 34 chains?

27. What part of an acre is there in a piece of land $2\frac{1}{2}$ ch. long and 1 rd. wide?

Exercise 86

1. How many square feet are there in each plat of the following:

School Garden



Scale: 1″ = 24′.

From 6 to 10 rows, running lengthwise, of vegetables may be planted in each plat, as here shown.

2. Tomatoes are set out 4 ft. apart in a row. The first plant at each end of a row being 2 ft. from the end, how many tomato plants are required to plant the row shown in plat 11?

3. Cabbage plants are set out in rows 3 ft. apart. How many rows running lengthwise can be planted in plat 12, when each outside row is $1\frac{1}{2}$ ft. from the edge?

4. Lettuce plants are set out 1 ft. apart in the row. The plant at each end of a row being 6 in. from the end, how many lettuce plants were required to plant the 3 rows in plat 2?

5. Sweet corn is planted in rows 3 ft. apart, the hills being 2 ft. apart in a row. If the row at each edge is $1\frac{1}{2}$ ft. from the edge and the hill at each end of a row is 1 ft. from the end, how many hills of sweet corn can be planted in plat 5?

6. What is the area of the walk shown between the plats?

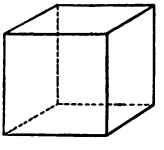
7. How many feet of fence would inclose plat 1?

8. What is the distance around the school garden?

MEASURES OF SOLIDS

189. Anything that has length, breadth, and thickness is a **solid.**

The Cube



190. A **cube** is a solid bounded by six equal squares.

191. The **edges** of a cube are the lines in which the squares meet, and the **faces** are the equal squares.

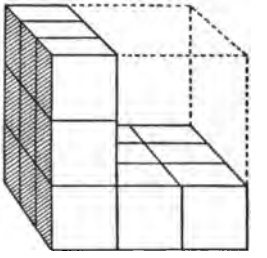
1. A **cubic inch** is a cube an inch long.
2. A **cubic foot** is a cube a foot long.
3. A **cubic yard** is a cube a yard long.

192. A **cubic unit** is a cube a unit long, that is, a cube of any desired length.

NOTE. The result obtained by using a number three times as a factor is the cube of the number.

Thus, $3 \times 3 \times 3$, or 27, is the cube of 3.

193. This figure represents a cube 3 units long.



3 = the number of cubic units in one row.

3 = the number of rows of cubic units.

$\therefore 3 \times 3 =$ the number of cubic units in one layer.

3 = the number of layers.

$\therefore 3 \times 3 \times 3 =$ the number of cubic units in the cube.

NOTE. $3 \times 3 \times 3$ may be written 3^3, read *3 cubed.*

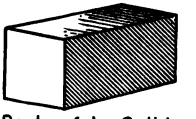
194. The number of cubic units in a cube equals the cube of the number of units in the length.

195. Table of Cubic Measure

1728 cubic inches (cu. in.)	**= 1 cubic foot (cu. ft.)**
27 cubic feet	**= 1 cubic yard (cu. yd.)**

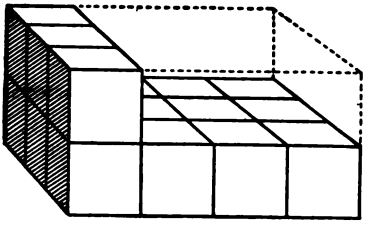
The Rectangular Solid

196. A **rectangular solid** is a solid all of whose faces are rectangles.



Rectangular Solid

197. The figure on page 194 represents a rectangular solid 4 units long, 3 units wide, and 2 units high.



4 = the number of cubic units in 1 row.

3 = the number of rows of cubic units.

∴ 3 × 4 = the number of cubic units in 1 layer.

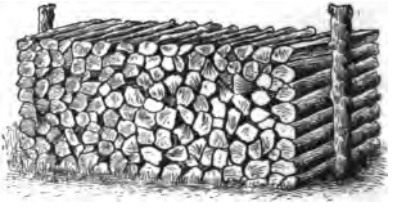
2 = the number of layers.

∴ 2 × 3 × 4, or 24, = the number of cubic units in the rectangular solid.

198. The number of cubic units in a rectangular solid equals the number of units in the height, times the number of units in the width, times the number of units in the length.

Wood Measure

199. A pile of wood 8 ft. long, 4 ft. wide, and 4 ft. high is called a **cord**.



200. **Important Facts**

A cubic yard of earth is called a **load**.
A cubic foot of water weighs $62\frac{1}{2}$ lb.
A stricken bushel = about $\frac{5}{4}$ of a cubic foot.
A heaped bushel = about $\frac{25}{16}$ of a cubic foot.
A cubic foot of anthracite coal broken averages 54 lb.
400 cubic feet of hay in a mow is considered a ton.
231 cubic inches = 1 wine gallon.

Exercise 87

1. How many cubic inches are there in a cube 4 in. long?
2. How many cubic feet are there in a cube 5 ft. long?
3. Two cubic yards are how many cubic feet?
4. How many cubic inches are there in 3 cu. ft.?
5. A cubical box 5 ft. long will hold how many bushels of oats, estimating that 1 bu. equals $1\frac{1}{4}$ cu. ft.?
6. How many bushels of wheat will a cubical box hold that is 3 ft. long, estimating that 1 bu. of wheat equals $1\frac{1}{4}$ cu. ft.?
7. How many cubic inches are there in a rectangular solid 3 in. long, 2 in. wide, and 1 in. high?
8. How many cubic feet of sand will a box hold that is 5 ft. long, 4 ft. wide, and 2 ft. high?
9. How many cubic feet of air are there in a room 15 ft. long, 12 ft. wide, and 9 ft. high?
10. How many cubic yards of earth must be removed in digging a cellar 27 ft. long, 18 ft. wide, and 10 ft. deep?
11. Show that there are 128 cu. ft. in a cord.
12. How many cubic inches are there in a cube 12 in. long?
13. Show that a vessel 11 in. long, 7 in. wide, and 3 in. deep holds 231 cu. in., or 1 gal.
14. How many gallons of water will a cistern hold that is 28 ft. long, 22 ft. wide, and 12 ft. deep?

SUGGESTION. $\frac{28 \times 12 \times 22 \times 12 \times 12 \times 12}{11 \times 7 \times 3}$ = the number of gallons. (Explain why.)

15. One bushel equals about $1\frac{1}{4}$ cu. ft. At this estimate, how many bushels of wheat will a granary hold that is 10 ft. long, 8 ft. wide, and 5 ft. deep?

16. One heaped bushel equals about $1\frac{9}{16}$ cu. ft. At this estimate, how many bushels of potatoes will a bin hold that is 15 ft. long, 10 ft. wide, and 6 ft. deep? How many bushels of apples will the bin hold?

17. Find the combined length of the edges of a cube 2 ft. long.

18. How many cubic feet are there in a cube $2\frac{1}{2}$ ft. long?

19. Show that 27 cu. ft. equals 1 cu. yd.

20. Show that a cubic foot contains 1728 cu. in.

21. What part of a cubic foot does a cube contain that is $\frac{1}{2}$ ft. long?

22. How many cords of wood are there in a pile 256 ft. long, 4 ft. wide, and 4 ft. high?

23. A certain mow of hay is 40 ft. long, 20 ft. wide, and 16 ft. high. Estimating 400 cu. ft. to the ton, how many tons of hay are there in this mow?

24. How many loads of earth, estimating a cubic yard to a load, must be removed in digging a cellar for a house 32 ft. long, 18 ft. wide, and 9 ft. deep? Cancel.

25. What is the weight of the water that a tank 4 ft. long, 2 ft. wide, and 1 ft. deep will hold, water weighing $62\frac{1}{2}$ lb. per cubic foot?

26. Milk is about 1.03 times as heavy as water, which weighs 62.5 lb. per cubic foot. What is the weight of 4 cubic feet of milk?

REVIEW

Exercise 88

Read the following, supplying the missing numbers at sight:

1. 1 pint = —— gills.
2. 1 minute = —— seconds.
3. 1 quart = —— pints.
4. 1 foot = —— inches.
5. 1 hogshead = —— gallons.
6. 1 cubic foot = —— cubic inches.
7. 1 leap year = —— days.
8. 1 short ton = —— pounds.
9. 1 hand = —— inches.
10. 1 square yard = —— square feet.
11. 1 gallon = —— quarts.
12. 1 day = —— hours.
13. 1 barrel = —— gallons.
14. 1 pound = —— ounces.
15. 1 century = —— years.
16. 1 fathom = —— feet.
17. 1 square foot = —— square inches.
18. 1 cord = —— cubic feet.
19. 1 peck = —— quarts.
20. 1 hour = —— minutes.
21. 1 yard = —— feet.
22. 1 cubic yard = —— cubic feet.
23. 1 bushel = —— pecks.
24. 1 common year = —— days.

25. 1 rod = —— yards.
26. 1 week = —— days.
27. 1 acre = —— square rods.
28. 1 chain = —— rods.
29. 1 year = —— months.
30. 1 right angle = —— degrees.
31. October has —— days.
32. 1 gallon = —— cubic inches.
33. August has —— days.
34. 1 hogshead = —— barrels.
35. 1 square rod = —— square yards.
36. 1 year = —— weeks (nearly).

Answer the following at sight:

37. How many pounds are there in a long ton?

38. Name four articles that are bought by the short ton.

39. What is the weight of a barrel of flour?

40. Name the centennial years, beginning with 1500 and ending with 1900.

41. Is the present year a leap year?

42. When was the last leap year?

43. When will the next leap year be?

44. How many hours are there from midnight to noon?

45. How is the third hour of the day written?

46. What is the thirteenth hour of the day called?

47. How many hours are there from 9 P.M. to 3 A.M. of the next day?

48. How is the sixth hour after noon written?

49. How is seven hours and fifteen minutes after midnight written?

50. How deep is the sea at a certain point, if it is 100 fathoms deep?

51. What does 12.01 P.M. mean?

52. How many hours are there from 8.30 A.M. to 12.30 P.M.?

53. Was the year 1800 a leap year?

54. A man's journey began at 10 P.M. and lasted 8 hours; when did it end?

55. Name the months that have 31 days.

56. Which is the longer, and by how many feet, a knot or a statute mile?

57. How many inches are there in 15 hands?

58. How is the area of a square found?

59. What is meant by the perimeter of a square?

60. How is the area of a rectangle found?

61. How is the number of cubic units in a cube found?

62. How many cubic inches are there in a wine gallon?

63. How is the number of cubic units in a rectangular solid found?

64. Tell how long, how wide, and how high a cord of wood is.

65. How many feet are there in a mile?

66. How many cubic feet are there in a rectangular solid 2 ft. long, 1 ft. wide, and $\frac{1}{2}$ ft. high?

REDUCTION

Illustrative Examples

1. Reduce .6 rd. to inches.

Explanation:

.6 rd. = .6 × 5.5 yd., or 3.3 yd.
3.3 yd. = 3.3 × 3 ft., or 9.9 ft.
9.9 ft. = 9.9 × 12 in., or 118.8 in.
∴ .6 rd. = 118.8 in.

The answer may be found thus:

$$\begin{array}{r} 5.5 \\ .6 \\ \hline 3.30 \\ 3 \\ \hline 9.9 \\ 12 \\ \hline 118.8 \end{array}$$

Another Explanation:

1 rd. = $5\frac{1}{2}$ yd.
= $16\frac{1}{2}$ ft.
= 198 in.
∴ .6 rd. = .6 × 198 in., or 118.8 in.

The answer may be found thus:

$$\begin{array}{r} 5\frac{1}{2} \\ 3 \\ \hline 16\frac{1}{2} \\ 12 \\ \hline 198 \\ .6 \\ \hline 118.8 \end{array}$$

2. Reduce 495 in. to rods.

Explanation:

495 = the no. of in.
The no. of in. ÷ 12 = the no. of ft.
∴ $41\frac{1}{4}$ = the no. of ft.
The no. of ft. ÷ 3 = the no. of yd.
∴ $13\frac{3}{4}$ = the no. of yd.
The no. of yd. ÷ $5\frac{1}{2}$ = the no. of rd.
∴ $2\frac{1}{2}$ = the no. of rd.
∴ 495 in. = $2\frac{1}{2}$ rd.

The answer may be found thus:

12)495, the no. of in.
3) $41\frac{1}{4}$, the no. of ft.
$5\frac{1}{2}$) $13\frac{3}{4}$, the no. of yd.
4 4
22) 55
$2\frac{1}{2}$, the no. of rd.

Another Explanation:

1 rd. $= 5\frac{1}{2}$ yd.
$= 16\frac{1}{2}$ ft.
$= 198$ in.
$\therefore$ $495 \div 198$, or $2\frac{1}{2}$ = the no. of rd. required.

The answer may be found thus:

$$\begin{array}{r} 5\frac{1}{2} \\ 3 \\ \hline 16\frac{1}{2} \\ 12 \\ \hline 198)495(2\frac{1}{2} \\ 396 \\ \hline 99 \end{array} \qquad \frac{99}{198} = \frac{1}{2}$$

Exercise 89

1. Reduce 4 gal. to pints.
2. Reduce 12 bu. to quarts.
3. Reduce 6 hr. to seconds.
4. Reduce 8 rd. to feet.
5. Reduce 56 pt. to gallons.
6. Reduce 66 pt. to gallons.
7. Reduce 128 pt. to pecks.
8. Reduce 840 hr. to weeks.
9. Reduce 180 in. to yards.
10. Reduce 3° to seconds.
11. Reduce 5 A. to square yards.
12. Reduce 4 sq. yd. to square inches.
13. Reduce $4\frac{1}{2}$ gal. to pints.
14. Reduce 3.5 yd. to inches.
15. Reduce 2 long tons to pounds.
16. Reduce 2.75 rd. to feet.
17. Reduce 3.25 sq. yd. to square inches.
18. Reduce 3 reams to sheets.
19. Reduce 100 qt. to bushels.

20. Reduce 288 in. to yards.
21. Reduce 3600 min. to days.
22. Reduce 15,730 sq. yd. to acres.
23. Reduce 55,440 ft. to miles.
24. Reduce 61,560 sq. in. to square yards.
25. Reduce 540,000″ to degrees.
26. Reduce 858 in. to rods.

MISCELLANEOUS PROBLEMS

Exercise 90

(Solve as many as possible at sight.)

1. I paid $2.25 for 15 doz. bananas; how much did 3 doz. cost at the same rate?

2. If I buy potatoes at $1 a bushel and retail them at 20¢ a half peck, how much profit do I make on a bushel?

3. At $22 a ton how much should 1000 lb. of hay cost?

4. Find the cost of 2 qt. of sirup at 50¢ a gallon.

5. If a mow of hay is 30 ft. long, 20 ft. wide, and 10 ft. high, how many tons should there be in it, counting 400 cubic feet to 1 ton?

6. What part of the week has passed by midnight Thursday?

7. How many square inches are there in a $2\frac{1}{2}$-in. square?

8. How many yards are there in the distance around a square whose length is 1 rod?

9. Find the number of square inches in a sheet of paper 10 in. long and $8\frac{1}{2}$ in. wide.

10. Find the cost of 500 lb. of bran at $22 per ton.

11. The distance around the earth is about 25,000 miles; how long would it require a train to run this distance, running 50 miles an hour?

12. In one common year there are how many weeks and how many days over?

13. A dealer bought $2\frac{1}{2}$ doz. hats for $72; what should he sell them for apiece to gain 60¢ on each hat?

14. Find the cost at 30¢ a square yard of a piece of oilcloth 4 yd. long and $1\frac{1}{4}$ yd. wide.

15. Wheat weighs 60 lb. to the bushel and corn 56 lb.; what is the weight of the grain a farmer is hauling, if his load consists of 20 bu. of wheat and 18 bu. of corn?

16. When potatoes are selling at 15¢ per $\frac{1}{2}$ peck, how much is that a bushel?

17. How much would 12 oz. of cheese cost at 20¢ a pound?

18. How many 500-lb. bales of cotton are required to weigh 10 tons?

19. Find the amount of this thrashing bill:

350 bu. of wheat at 4¢ a bushel;
320 bu. of oats at 2¢ a bushel;
4 men at $1.50 per day, for a day and a half.

20. How many acres are there in a field 80 rods long and 50 rods wide?

21. How many days will it take to plow a field 40 rods long and 30 rods wide if a man plows at the rate of $1\frac{1}{2}$ acres per day?

22. The distance around a rectangle is 10 yd.; if its length is 4 yd., what is its width?

23. If A is 10 miles east of a certain town and B is 5 miles west of it, how far are they apart?

24. A dealer sold 4 bushels of potatoes for $6; he made a profit of $1; what did each bushel of potatoes cost him?

25. How many cubic inches are there in a rectangular solid 11 in. long, 7 in. wide, and 3 in. deep? The answer is the number of cubic inches in what measure?

26. If chestnuts were bought at $2 a bushel and retailed at 10¢ a quart, what was the profit per bushel?

27. How many quart boxes will $2\frac{1}{2}$ bu. of berries fill?

28. How many 2-in. squares can be cut from a piece of cardboard 9 in. square?

29. Find the gain on $\frac{1}{4}$ of a ream of water-color paper bought for $24 per ream and sold at 6¢ a sheet.

30. Find the cost of shipping 700 lb. of furniture, if the freight is 40¢ per hundred.

31. Cast iron weighs about 7.2 as much as water; what is the weight of a cubic foot of cast iron, water weighing 62.5 lb. per cubic foot?

32. How many dozen are there in 12 score?

33. Find the cost of 1200 lb. of straw at $8 a ton.

34. A farmer's wheat crop weighed, when sold, 19,230 lb.; what did he receive for it, if he sold it at $1.10 per bushel (60 lb.)?

35. A common brick is 8 in. long, 4 in. wide, and 2 in. thick; how many square inches are there in its surface?

36. Timothy seed weighs 46 lb. to the bushel; what should be the weight of a peck of it?

37. When clover seed is selling for $12 per bushel (60 lb.), what should be paid for 45 lb. of clover seed?

38. How many square inches are there in all the faces of a cube 2 in. long?

39. Name the leap years between 1910 and 1930.

40. How many quarts of ice cream must be bought for a company of 10 persons, if one pint is sufficient for 2 persons?

41. The seventh day from to-day comes on what day of the week?

42. A farmer who wishes to use 400 lb. of fertilizer for an acre should purchase how many pounds for 10 acres? how many tons?

43. A laborer who works 10 hours a day and begins work at 6.30 A.M. should quit work at what time, if he takes one hour for dinner?

44. How many half-gill bottles are required to hold a pint of ink?

45. What should be charged for 700 lb. of feed at $24 a ton?

46. A school that opens at 9.30 A.M. and closes at 2.30 P.M. is how many hours in session, provided there are no intermissions?

47. How many bushels of wheat will fill a bin 5 ft. long, 4 ft. wide, and 3 ft. deep, if each bushel is $\frac{5}{4}$ of a cubic foot?

48. How many cubic feet of air are there in a room 20 ft. long, 15 ft. wide, and 10 ft. high?

49. When matches are selling at 3 boxes for 5¢, how much is that per gross?

50. How many sacks, holding 25 lb. each, can be filled from a ton of sugar?

51. A long ton is how many times a short ton?

52. A ship sank in 30 fathoms of water; how many feet deep is the water?

53. A horse whose height is $15\frac{1}{2}$ hands is how many feet high?

54. If a horse eats 1 pk. of oats (standard weight) at a meal, in how many days, eating 3 meals a day, will he eat a ton of oats?

55. A farmer stepped the length of his field and found it to be 160 paces (3.3 ft.); find the length of the field in rods.

56. Show that a mile equals 1760 yd. or 5280 ft.

57. Show that a bushel equals 32 qt.

58. How many square inches are there in a square yard?

59. A peck of chestnuts was retailed at 5¢ a pint; how much was received for them?

60. A school that opens at 8.30 A.M. and closes at 2 P.M. is in session how many hours, if an intermission of 30 minutes is allowed at noon?

61. At 7¢ an ounce what should be charged for $\frac{1}{4}$ lb. of cloves?

62. Find a boy's gain on $\frac{1}{4}$ of a gross of matches which he bought at $1 a gross, if he retailed them at 4 boxes for 5¢.

63. What should be charged for 846 lb. of feed at $22 per ton?

64. Find the cost of 1860 lb. of hay at $18 a ton.

65. Find the cost of 2575 lb. of bran at $23 per ton.

66. Find the cost of 2600 lb. of straw at $8 a ton.

67. Find the cost of 2000 lb. of shelled corn at $0.75 per bushel (56 lb.).

68. Find to the nearest hundredth the number of gallons in a cubic yard.

69. A dealer wishes to sell apples that cost $2 a bushel so as to gain $1.20 a bushel; what should he charge for $\frac{1}{2}$ pk.?

70. What should be charged for 16 lemons at 18¢ per dozen?

71. How many pint boxes are required to hold $\frac{1}{2}$ bu. of berries?

72. How many quarts of ice cream must be bought to serve 50 persons at a hotel, if one pint is served to 2 persons?

73. Name instantly the tenth month; the fifth; the eighth; the eleventh; the fourth.

74. Find the cost of 50 eggs at 30¢ a dozen.

75. What should be paid for 50 bu. of oats weighing 28 lb. to the bushel, when the market price is 50¢ per bushel (32 lb.)?

76. What date is 30 days after May 31st?

77. What date is 90 days after May 31st?

78. What date is 2 months after February 28th?

79. What part of a bushel is $2\frac{1}{2}$ pk.?

80. What part of a yard is 9 in.?

81. A man who was born 14 B.C. and died 44 A.D. lived to what age?

SUGGESTION: "14 B.C." means 14 years before Christ, and "44 A.D." means 44 years after Christ.

82. What time elapsed between two events, one of which happened 150 B.C. and the other 200 A.D.?

83. What date is 3 months after September 30th?

84. Find the length of a day on which the sun rises at 4.29 A.M. and sets at 7.39 P.M.

85. How much should be paid for 3248 lb. of wheat at 92 ¢ per bushel (60 lb.)?

86. What part of a mile is 440 ft.?

87. What part of April has passed by noon, April 11th?

88. How many cubic feet are there in 4 cords of wood?

89. A gallon of water weighs 8.36 lb.; what does a quart of water weigh?

PRACTICAL MEASUREMENTS

Carpeting

201. Carpets are usually 1 yard or $\frac{3}{4}$ of a yard wide. The cheaper carpets are 1 yard wide, and the higher grades, such as velvet, Wilton, and Axminster, are $\frac{3}{4}$ of a yard wide. Linoleum and oilcloth are usually sold by the square yard.

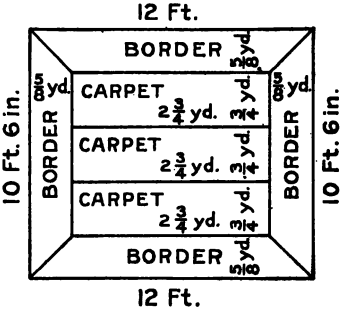
202. In determining the number of yards of carpet required for a floor, the number of strips needed should be found; a fractional part of a strip should be regarded as a full strip.

REMARK. In practice it is seldom possible, because of the great variety of patterns with which carpets are woven, to estimate the exact amount necessary for a room.

203. Sometimes the carpet of a room is surrounded by a border; the length of the border required is then the entire distance around the room. The standard widths of borders are $\frac{3}{8}$ yd., $\frac{5}{8}$ yd., and $\frac{3}{4}$ yd.

Illustrative Example

Find the amount of carpet and of border that must be bought to carpet a room 12 ft. long and 10 ft. 6 in. wide with carpet $\frac{3}{4}$ of a yard wide, the strips running lengthwise and the carpet surrounded by a border $\frac{5}{8}$ of a yard wide.



Explanation:

12 ft. = 4 yd.; $10\frac{1}{2}$ ft. = $3\frac{1}{2}$ yd.

4 yd. − ($2 \times \frac{5}{8}$ yd.), or $2\frac{3}{4}$ yd. = the length of each strip.

$3\frac{1}{2}$ yd. − ($2 \times \frac{5}{8}$ yd.), or $2\frac{1}{4}$ yd. = the total width of carpet.

$2\frac{1}{4} \div \frac{3}{4}$, or 3 = the number of strips required.

$3 \times 2\frac{3}{4}$ yd., or $8\frac{1}{4}$ yd. = the amount of carpet required.

$2 \times (4 \text{ yd.} + 3\frac{1}{2} \text{ yd.})$, or 15 yd. = the amount of border required.

Exercise 91

1. How many strips of carpet, 1 yd. wide, must be bought to carpet a room 12 ft. wide, the strips to be laid lengthwise?

2. How many strips of carpet, $\frac{3}{4}$ yd. wide, must be bought to carpet a room 18 ft. wide, the strips to be laid lengthwise?

3. How many strips of carpet, $\frac{3}{4}$ yd. wide, must be bought to carpet a room 14 ft. wide, the strips to be laid lengthwise?

4. How many strips of carpet, $\frac{3}{4}$ yd. wide, must be bought to carpet a room 27 ft. long, the strips to be laid crosswise?

5. How many strips of carpet, $\frac{3}{4}$ yd. wide, must be bought to carpet a room 20 ft. long and 18 ft. wide:

1st, if the strips are to laid lengthwise?

2d, if the strips are to be laid crosswise?

6. How many yards of carpet, 1 yd. wide, must be bought to carpet a floor 15 ft. long and 12 ft. wide?

7. How many yards of carpet, $\frac{3}{4}$ yd. wide, must be bought to carpet a floor 18 ft. long and $13\frac{1}{2}$ ft. wide?

8. Find the cost of the linoleum required to cover the floor of a kitchen 18 ft. long and 12 ft. wide, the linoleum to cost 54¢ a square yard.

9. At 40¢ a square yard find the cost of the oilcloth required for a kitchen 12 ft. square.

10. How many strips of carpet $\frac{3}{4}$ yd. wide must be bought to carpet a room 13 ft. 6 in. wide, the strips to be laid lengthwise?

11. How many strips of carpet $\frac{3}{4}$ yd. wide must be bought to carpet a room 12 ft. 9 in. wide, the strips to be laid lengthwise?

12. How many yards of carpet $\frac{3}{4}$ yd. wide must be bought to carpet a room 24 ft. long and 18 ft. wide, the strips to be laid lengthwise, and $4\frac{1}{2}$ yd. to be allowed for waste in matching?

13. How many yards of carpet $\frac{3}{4}$ yd. wide must be bought for a floor 18 ft. long and 16 ft. wide, if the strips are laid lengthwise, 3 yards being allowed for waste in matching?

14. Find the cost of carpeting a room 16 ft. long, 14 ft. wide, with carpet $\frac{3}{4}$ yd. wide, at \$1.60 a yard, if the strips run lengthwise, $\frac{1}{2}$ yd. being allowed on each strip for matching the pattern.

15. What is linoleum selling for per square yard, if it is 2 yd. wide and quoted at \$2 per running yard?

16. At \$0.85 per square yard, find the cost of the linoleum that must be bought to cover a kitchen floor 12 ft. long and $10\frac{1}{2}$ ft. wide.

17. Make a drawing of a floor 18 ft. long, 16 ft. wide; scale: $1'' = 1'$; showing strips of carpet $\frac{3}{4}$ yd. wide running lengthwise on this floor, and surrounded by a border $\frac{3}{4}$ yd. wide. How many yards of carpet and of border are required, no allowance being made for waste?

18. Make a drawing on the blackboard; scale: 2 in. = 1 ft.; showing 4 strips of carpet $\frac{3}{4}$ yd. wide and 15 ft. long surrounded by a border $\frac{5}{8}$ yd. wide. If this carpet and border exactly cover a floor, how long is the floor? how wide?

Lumber Measure

204. Lumber is usually bought and sold by **board measure**. The unit of board measure is the **board foot**, which is equivalent to the contents of a board a foot square and an inch thick.

205. The number of board feet in a board 16 ft. long, 8 in. wide, and 1 in. thick ($16' \times 8'' \times 1''$) may be found thus:

16 = the number of feet in the length.

$\frac{8}{12}$ = the number of feet in the width.

$\frac{8}{12} \times 16$, or $\frac{8 \times 16}{12}$ = the number of board feet in the board.

From the above explanation we have the following:

206. The number of board feet in a board one inch thick equals the number of inches in the width times the number of feet in the length, divided by 12.

207. All boards less than one inch thick are bought and sold by surface measure, the thickness not being considered. The number of board feet in a board one inch or less in thickness is therefore the number of square feet of surface in one side of the board.

208. The number of board feet in a plank 16 ft. long, 6 in. wide, and 2 in. thick ($2'' \times 6'' \times 16'$) may be found thus:

$\frac{6 \times 16}{12}$ = the number of board feet in the plank if it were 1 in. thick.

$\frac{2 \times 6 \times 16}{12}$ = the number of board feet in the plank 2 in. thick.

From the above statement we have the following:

209. The number of board feet in a piece of timber more than an inch thick equals the number of inches in the thickness, times the number of inches in the width, times the number of feet in the length, divided by 12.

210. Usually a piece of lumber is spoken of as containing so many feet, meaning board feet.

Thus, 200 feet of lumber means 200 board feet.

NOTE 1. Lumber is usually quoted by the 1000 feet, meaning board feet.

NOTE 2. Roofing laths are usually sold by the hundred linear (lin.) feet.

Exercise 92

Find the number of feet (board feet) of inch boards in each of the following:

	Number of Pieces	Size	Number of Feet
1.	6	16′ × 12″	
2.	12	12′ × 8″	
3.	36	10′ × 6″	
4.	48	14′ × 6″	
5.	50	12′ × 12″	
6.	38	14′ × 8″	
7.	56	12′ × 16″	
8.	100	12′ × 8″	
9.	10	16′ × 9″	
10.	20	12′ × 12″	
11.	28	12′ × 9″	
12.	18	2″ × 6″ × 14′	
13.	12	2″ × 4″ × 16′	
14.	10	2″ × 4″ × 14′	
15.	6	5″ × 5″ × 18′	
16.	17	3″ × 6″ × 18′	
17.	18	2″ × 8″ × 12′	
18.	20	3″ × 4″ × 9′	
19.	25	3″ × 9″ × 8′	
20.	10	4″ × 4″ × 12′	
21.	8	5″ × 6″ × 12′	
22.	75	2″ × 5″ × 18′	
23.	24	2″ × 8″ × $12\frac{1}{2}$′	
24.	24	3″ × 5″ × 16′	

	Number of Pieces	Size	Number of Feet
25.	2	$2'' \times 6'' \times 18'$	
26.	2	$2'' \times 6'' \times 14'$	
27.	2	$2'' \times 4'' \times 8'$	
28.	12	$4'' \times 6'' \times 12'$	
29.	12	$\frac{7}{8}'' \times 3'' \times 18'$	
30.	12	$\frac{7}{8}'' \times 9'' \times 3'\ 6''$	
31.	13	$\frac{7}{8}'' \times 8'' \times 10'\ 6''$	

32. Find the cost of 10 boards $16' \times 9''$ at \$ 30 per M.

33. Find the cost of 40 boards $12' \times 6''$ at \$ 32 per M.

34. Find the cost of 60 pieces of scantling $3'' \times 4'' \times 12'$ at \$ 28 per M.

35. Find the cost of a load of lumber consisting of:

20 planks $2'' \times 6'' \times 12'$ at \$ 24 per M,

42 boards $16'' \times 12'$ at \$ 36 per M.

36. At \$ 24 per thousand feet find the cost of 80 fence boards $16' \times 6''$.

37. Find the cost of 12 posts $6'' \times 6'' \times 8'$ at \$ 40 per M.

38. Find the cost of 60 flooring boards $16' \times 3\frac{1}{2}''$ at \$ 32 per M.

39. Find the cost of 12 planks $1\frac{1}{2}'' \times 6'' \times 14'$ at \$ 35 per M.

40. Find the cost of a stick of timber 50 ft. long, 18 in. wide, and 10 in. thick at \$ 40 per M.

41. At \$ 24 per M, what is the value of a pile of lumber 16 ft. long, 8 ft. wide, and 6 ft. high?

42. Find the cost of 112 roofing laths averaging 12 ft. long at \$ 1.15 per 100 linear feet.

43. Find the cost of the following bill of lumber:

275 lin. ft. of roofing laths at $1.15 per 100 lin. ft.
300 ft. of white pine siding at $50 per M.
12 boards 1″ × 12″ × 16′ at $50 per M.
2200 plastering laths at $0.60 per C.
24 pieces 3″ × 5″ × 12′ at $30 per M.

Papering

211. Wall paper is generally 18 inches wide and is sold by the roll. A single roll is 8 yards long, and a double roll is 16 yards long. **Border paper** is sold by the linear yard and is of various widths. **Molding** is sold by the foot.

NOTE. Imported papers vary in width and in length of roll.

Wall paper is generally put up in double rolls, but prices quoted are generally for the single roll.

212. No definite rule can be given for determining the exact amount of paper required for a room, because of the allowance that must be made for windows, doors, matching of patterns, and width of border. Usually, however, one of three methods is followed by paper hangers in determining the amount:

First Method. The areas of the doors and windows are deducted from the area of the entire surface of the side walls above the baseboard, and the number of square feet remaining divided by the number of square feet in a double roll, thus determining the number of double rolls required for the side walls.

Second Method. The width of the doors and windows is deducted from the perimeter of the room, and the number of feet remaining is divided by the number of feet ($1\frac{1}{2}$) in the width of a strip of paper; the result is the number of strips required for the side walls. The

number of double rolls required is then ascertained by dividing the number of strips determined by the number of strips of required length that may be cut from a double roll.

NOTE. This rule is sometimes modified by counting two strips for each yard in the perimeter after the width of the doors and windows is deducted.

Third Method. The number of square feet in the side walls is divided by the number of square feet in a double roll, and then half of a single roll is deducted for each door and window, thus determining the number of double rolls required for the side walls.

213. Paper for the ceiling being of a pattern different from that used on the side walls, the quantity required should be calculated separately.

NOTE 1. Usually no deduction is made for border, thereby making sufficient allowance for waste and matching.

NOTE 2. A fractional part of a roll is considered a whole roll.

Illustrative Example

Find by the first method the cost of the paper and the border required for the walls of a room 18 ft. long, 12 ft. wide, and 9 ft. high above the baseboard, if the room contains two windows 6 ft. by 3 ft. and a door $7\frac{1}{2}$ ft. by 4 ft., the paper to cost \$ 0.25 a roll, and the border \$ 0.10 a yard.

Explanation:

$2 \times (18 \text{ ft.} + 12 \text{ ft.})$, or 60 ft. = the perimeter of the room.

9 ft. = the estimated height of the room.

9×60, or 540, = the number of square feet in the surface of the walls.

$2 \times (3 \times 6) + (4 \times 7\frac{1}{2})$, or 66 = the number of square feet to be deducted.

$540 - 66$, or 474 = the number of square feet to be papered.

$48 \times 1\frac{1}{2}$, or 72 is the number of square feet in a double roll.

$474 \div 72 = 6\frac{7}{12}$.
7 double rolls, or 14 single rolls are required.
$14 \times \$0.25$, or $\$3.50$ is the cost of paper.
60 ft., or 20 yd. = the length of border required.
$20 \times \$0.10$, or $\$2.00$ = the cost of the border.
$\$3.50 + \$2$, or $\$5.50$ = the required cost.

Exercise 93

1. How many square yards are there in a single roll of paper?

2. How many square yards are there in a double roll of paper?

3. How many strips of paper are necessary to cover a wall 18 ft. long?

4. How many strips of paper are necessary to cover a ceiling 24 ft. long? (Strips running crosswise.)

5. How many strips 9 ft. long can be cut from a single roll of paper, and how much waste will there be?

6. How many yards of border are required for a room 24 ft. long and 18 ft. wide?

7. At 6¢ per foot, find the cost of the molding for a room 24 ft. long and $22\frac{1}{2}$ ft. wide.

8. How many strips of paper are required to paper the walls of a room 24 ft. long and 18 ft. wide, making no allowance for doors or windows?

9. How many whole strips of paper can be cut from a double roll, if the strips are 16 ft. long? 12 ft. long? 10 ft. long?

10. How many strips of paper are required to cover a ceiling 24 ft. long and 21 ft. wide, when the strips run

lengthwise? How many are required when the strips run crosswise?

11. How many strips of paper are required for a ceiling 20 ft. long and 16 ft. wide, when the strips run lengthwise? How many are required when the strips run crosswise?

SUGGESTION. Consider a fractional part of a strip a whole strip.

12. How many double rolls of paper must be bought for a ceiling 18 ft. long and 15 ft. wide, if the strips run crosswise? How much waste will there be in each double roll?

13. Estimate the number of double rolls of paper that must be bought for the four walls of a room 24 ft. long and 16 ft. wide, allowing for two doors, each 4 ft. wide, and four windows, each $3\frac{1}{2}$ ft. wide, the walls being 9 ft. high above the baseboard. (Use second method.)

14. Find the cost of the paper required for the walls of a room 18 ft. long, 14 ft. wide, and 9 ft. high above the baseboard, allowing for 1 door $7\frac{1}{2}$ ft. $\times$ 4 ft., and 4 windows, each 6 ft. $\times$ 4 ft., the paper to cost \$0.35 a roll and the border \$ 0.20 per yard. (Use first method.)

15. Find the cost of the paper required for the walls and ceiling of a room 20 ft. long, 16 ft. wide, and 9 ft. high above the baseboard, allowing for 2 doors and 6 windows, for each of which $\frac{1}{2}$ of a single roll is deducted; the paper to cost \$ 0.22 a roll and the border \$ 0.15 a yard.

16. Find the cost of the paper required for the walls and ceiling of a room 18 ft. long, 16 ft. wide, and 10 ft. high, allowing for 2 doors, each $7\frac{1}{2}$ ft. $\times$ 4 ft., 4 windows, each 6 ft. $\times$ 4 ft., and a baseboard 9 in. wide; the paper

to cost $0.18 a roll and the border $0.10 a yard. (Use first method.)

Plastering, Paving, and Painting

214. In making estimates for the cost of labor, the square yard or the square foot is taken as a unit in plastering, painting, and paving.

215. No definite rule is followed in plastering and painting in making allowances for doors, windows, and other openings. In some localities half the areas of all openings is deducted; in others no deductions are made.

216. The approximate amount of ready-mixed paint that is required for the outside of a building may be found thus:

Measure in feet the height of the building at one corner, and the perimeter; find their product and divide by 250. The result is the approximate number of gallons required for two coats.

Thus, if a building is 40 ft. long and 35 ft. wide and 20 ft. high at one corner, the number of square feet estimated is $20 \times (2 \times 40 + 2 \times 35)$, or 3000, and $3000 \div 250$, or 12 is the number of gallons of paint required for the two coats.

NOTE. The area of the gables is usually ignored in the calculations. The quantity as found by the above method is sufficient to paint the gables also, unless the building has more than two gables.

Exercise 94

1. How many square yards of plastering are there in a wall 18 ft. long and 9 ft. high?

2. How many square yards of plastering are there in the ceiling of a room 24 ft. long and 18 ft. wide?

3. Find the cost, at 25¢ per sq. yd., of plastering a wall 24 ft. long and 12 ft. high, deducting one half the area of an opening $7\frac{1}{2}$ ft. by 6 ft.

4. Find the cost, at 25 ¢ per square yard, of plastering a wall 30 ft. long and 12 ft. high, deducting one half the area of one opening $7\frac{1}{2}$ ft. by 6 ft., and of two openings each 6 ft. by 3 ft.

5. How many square yards of plastering are there in the walls of a room 21 ft. long, 18 ft. wide, and 9 ft. high, making no allowance for openings?

6. How many square yards of plastering are there in the walls and ceiling of a room 18 ft. long, 14 ft. wide, and 8 ft. high, making no allowance for openings?

7. At $ 0.15 per square foot, find the cost of a cement pavement 60 ft. long and $4\frac{1}{2}$ ft. wide.

8. How many gallons of ready-mixed paint are required to give two coats to a building 50 ft. long, 36 ft. wide, and 18 ft. high at one corner? [See Sec. 216.]

9. How many gallons of ready-mixed paint are required to give two coats of paint to a building 45 ft. long, 32 ft. wide, and 18 ft. high at one corner? [See Sec. 216.]

10. About 1300 laths are required for every 100 sq. yd. of plastering; how many laths are required for 250 sq. yd.?

11. 9 lb. of 3-penny nails are required for the laths for 100 sq. yd. of plastering. At $3\frac{1}{2}$ ¢ per pound, find the cost of the lath nails that must be bought for 240 sq. yd. of plastering.

12. Estimating plastering laths at $ 0.60 per hundred, find the cost of the plastering laths that must be bought for 350 sq. yd. of plastering, 1300 plastering laths being required for 100 sq. yd. of plastering.

13. 10 bu. of lime and 10 lb. of hair are required for 100 sq. yd. of plastering. Estimating the lime at $0.30 a bushel and the hair at $5\frac{1}{2}¢$ a pound, find the cost of the lime and the hair necessary for 180 sq. yd. of plastering.

14. Estimating the cost of labor at $12 for each 100 sq. yd. of plastering (two coats), find the cost of the labor for plastering the walls and ceiling of a room 20 ft. long, 16 ft. wide, and 10 ft. high.

15. Some contractors estimate the cost of labor and material for each square yard of plastering at $0.35. At this estimate, find the cost of plastering the walls and the ceiling of a room 20 ft. long, 18 ft. wide, and 9 ft. high.

16. At $20¢$ per square yard, find the cost of plastering the walls and ceiling of a room 20 ft. by 18 ft. and 10 ft. high, there being one door $7\frac{1}{2}$ ft. by 4 ft., and 3 windows each 6 ft. by 4 ft., one half the areas of the doors and windows being deducted.

17. A plasterer made the following estimate for 100 sq. yd. of plastering; find the total cost:

Material	Quantity			Cost
Laths	1300	@	$0.60 per C	?
Nails	9 lb.	@	0.03	?
Hair	10 lb.	@	$0.05\frac{1}{2}$	?
Lime	10 bu.	@	0.30	?
Sand	2 T.	@	1.00	?
	Total cost of material			?
	Labor			$18.00
	Total			?

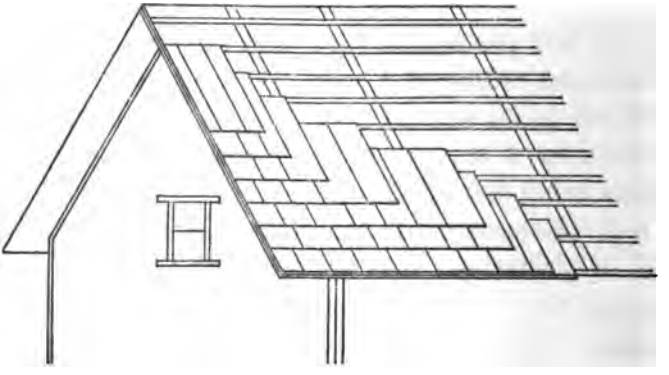
Roofing

217. Various materials are used for roofing, some of the most common of which are shingles, slate, and tin.

218. Shingles are of various lengths, as 24 inches, 20 inches, 18 inches, and 16 inches; they are usually estimated to be $3\frac{1}{2}$ inches, 4 inches, $4\frac{1}{2}$ inches, 5 inches, or 6 inches wide, and are put up in bales of 25 and 50 of uniform widths; also in bales of 200 and 167 of varying widths, 6 bales of the last size being estimated to contain a thousand.

219. Shingles, whether in bales or loose, are quoted by the thousand.

NOTE. "*A thousand* four-inch" shingles means a quantity of shingles equivalent to 1000 uniformly 4 inches wide.



220. Shingles are laid in courses as shown in the accompanying figure, the first course usually being double. If shingles average 4 inches wide and are *exposed 6 inches to the weather*, then each shingle covers 24 square inches of roof surface.

221. Slate is usually sold by the **square**, which is a quantity sufficient to roof 100 square feet of surface. Slates usually range in

size from 6 inches by 12 inches to 14 inches by 24 inches, and are roofed so that they are exposed to the weather $1\frac{1}{2}$ inches less than one half of their length.

Thus, if a building is roofed with slate $9'' \times 18''$ the portion of the slate exposed to the weather is $7\frac{1}{2}''$ long; that is, $67\frac{1}{2}$ sq. in. of each slate is exposed.

222. Roofing tin is usually sold in boxes of 112 sheets, 20 inches by 28 inches. Making proper allowance for waste and seams, a box of tin is estimated to cover 360 square feet of roof.

Exercise 95

1. How many shingles averaging 4 in. wide are required for a course 40 ft. long?

2. Roof shingles should not exceed in exposure to the weather the length of the shingle less $1\frac{1}{2}$ in. divided by 3. At this estimate, how much of the length of a 24-in. shingle should be exposed to the weather?

3. Estimating shingles 6 in. wide and laid $7\frac{1}{2}$ in. to the weather, how many shingles are required for 100 sq. ft. of roof?

4. Estimating shingles 5 in. wide and laid 6 in. to the weather, how many shingles are required for 100 sq. ft. of roof?

5. Estimating shingles 4 in. wide and laid 6 in. to the weather, how many square feet of roof will a thousand shingles cover?

6. Find the number of shingles, averaging 4 in. wide and laid 6 in. to the weather, required for a roof 30 ft. long and extending 21 ft. from ridge to eaves, the first

course on each side of the roof being double, and an allowance of 100 shingles being made for waste.

7. An architect made the following estimate of the cost of labor and the quantity and cost of the material required for a roof 40 ft. long and extending 24 ft. from ridge to eaves, the first course of shingles on each side being double, and an ample number being allowed for waste; find the cost of each item and the total cost:

ITEM	QUANTITY	KIND	PRICE	COST
Roofing Laths	3200 lin. ft.	1″ × 2½″	$1.15 per 100 lin. ft.	?
Shingles	9700	4″ × 24″	$12.00 per M.	?
Nails	64 lb.	10 penny	$2.55 per cwt.	?
Nails	68 lb.	4 penny	$2.85 per cwt.	?
Labor	72 hr.	Carpentry	$0.47½ per hour	?

8. Estimate the number of slates (correct to the nearest unit) required for 100 sq. ft. of roofing, if the slates are 24 in. long and 14 in. wide, and $1\frac{1}{2}$ in. less than one half the length of each slate is exposed to the weather.

9. Find the cost, at $4 per square, of the slate necessary for $3\frac{1}{2}$ squares of roofing. If the slates are 18 in. × 9 in., and $1\frac{1}{2}$ in. less than half the length of each slate is exposed to the weather, estimate the number of slates required, correct to the nearest slate.

10. Find the cost of the slate necessary for a roof 36 ft. long and 24 ft. from ridge to eaves, estimating that each slate will roof $67\frac{1}{2}$ sq. in., and that 154 slates are required for the double courses and breakage, the slate costing $5.50 per square.

11. Estimating that a box of tin will cover 360 sq. ft of roof and that tin is selling for $20 a box, find the cost of the tin necessary for a square of roofing.

12. With tin costing $16 a box, a plumber estimates the cost of the items necessary for 100 sq. ft. of tin roofing as follows; find the total cost:

Tin	$4.50
Solder	0.80
Nails and cleats	0.18
Painting tin one coat on each side	1.60
Labor putting tin together	0.50
Labor putting tin on roof	1.25

13. Estimating the cost of a square of tin roofing at $8.90, find the cost of covering with tin a roof 36 ft. long and 24 ft. from ridge to eaves.

Masonry, Concrete Work, and Brickwork

223. The unit of measure for ordinary stone walls is the **perch.** It is a wall $16\frac{1}{2}$ feet long, $1\frac{1}{2}$ feet wide, and 1 foot high, and therefore contains $24\frac{3}{4}$ cubic feet.

224. When stone is laid in mortar, 22 cubic feet, or $\frac{8}{9}$ of a perch, of stone are required for a perch of wall, the remainder, $2\frac{3}{4}$ cubic feet, being allowed for mortar.

225. In making estimates for the cost of labor for rough stone walls, "mason's measure" is generally used; that is, the distance around the wall on the outside is measured, corners being thus counted twice, and no allowance is made for openings, such as doors and windows, unless they are unusually large. In making estimates for material, "quarry measurement" is generally used; that is, deductions are made for corners and all openings.

226. Concrete work is estimated by the cubic yard, except such as is less than 6 inches in thickness, which is usually estimated by the square foot.

227. In making estimates for the cost of labor and material for walls built of concrete, the same rules are followed as those used in making estimates for walls built of rough stone. This is frequently modified by special contract.

228. The unit of measure for walls built of cut stone is usually the **square foot.**

229. Brickwork is usually estimated by the **thousand bricks.**

230. In making estimates for the cost of labor and material for walls built of brick, allowance is made for corners and all openings, if no special contract is made to the contrary.

The dimensions of a common brick are 8 in. × 4 in. × 2 in.

Bricks are usually laid in walls as indicated in the diagram, the courses of mortar being $\frac{1}{4}$ of an inch thick.

231. It requires for each square foot in the estimated outside surface of a wall:

7 bricks, if the wall is 1 brick, or 4 in., thick.
14 bricks, if the wall is 2 bricks, or 9 in., thick.
21 bricks, if the wall is 3 bricks, or 13 in., thick
28 bricks, if the wall is 4 bricks, or 18 in., thick.

From the above we have the following:

232. To find the number of bricks required for a wall, multiply the number of square feet by 7, 14, 21, or 28 according as the wall is 1, 2, 3, or 4 bricks thick.

Illustrative Examples

1. Estimate for cost of labor, the masonry in the walls of a cellar of a house $42\frac{1}{2}$ ft. long, 40 ft. wide, the walls being $1\frac{1}{2}$ ft. thick and 9 ft. high.

$2 \times (42\frac{1}{2}$ ft. $+ 40$ ft.), or 165 ft. = the estimated perimeter of the walls.

9 ft. = the height of the walls.

$1\frac{1}{2}$ ft. = the thickness of the walls.

$\therefore 1\frac{1}{2} \times 9 \times 165$, or $2227\frac{1}{2}$ = the estimated number of cubic feet in the walls.

$2227\frac{1}{2} \div 24\frac{3}{4}$, or 90 = the number of perches required.

2. Find the number of bricks required to build to the height of 18 ft. the four walls of a house 40 ft. long, 36 ft. wide, outside measurement, the walls being two bricks, or 9 in., thick, and an allowance of 600 sq. ft. being made for doors and windows.

$2 \times (40 + 36) \times 18$, or 2736 = the estimated number of square feet of exposed surface in the walls.

$4 \times \frac{3}{4} \times 18$, or 54 = the number of square feet to be deducted for corners.

600 = the number of square feet to be deducted for openings.

$\therefore 2736 - (54 + 600)$, or 2082 = the estimated number of square feet to be built.

14 = the number of bricks required for each square foot.

$\therefore 2082 \times 14$, or 29,148 = the number of bricks required.

Exercise 96

1. How many perches of stone are there in a pile 33 ft. long, 4 ft. wide, and 6 ft. high?

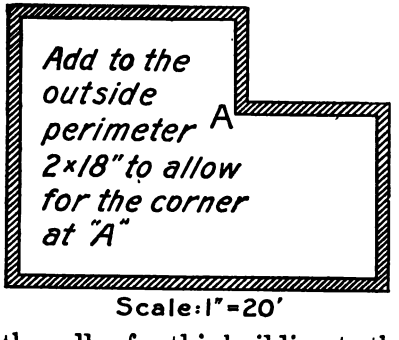
2. At \$1.20 a perch, find the cost of building the walls of a cellar 40 ft. long and 36 ft. wide, if the walls are 18 in. thick and are built to a height of 10 ft.

3. A factory 80 ft. long and 60 ft. wide is built on a concrete foundation 8 ft. deep and 3 ft. thick; find the cost of the concrete work at $6 per cubic yard.

4. Concrete for walls and pavements may be made by mixing 1 part of cement with 2 parts of sand and 4 parts of stone. If one barrel of cement contains 4 cu. ft., how many cubic feet of stone and how many of sand must be used with one barrel of cement?

5. Find the number of bricks required to build to the height of 16 ft. the four walls of a house 36 ft. long and 32 ft. wide, outside measurement, the walls being 2 bricks, or 9 in., thick, making allowance for 3 doors $7\frac{1}{2}$ ft. by 4 ft., and 16 windows 6 ft. by 4 ft.

6. Estimating the cost of the material and labor for each 1000 bricks in a wall to be $16, find the cost of building to the height of 24 ft. the four walls of a house 36 ft. long, 30 ft. wide, outside measurement, the walls being 2 bricks thick, and there being 4 doors $7\frac{1}{2}$ ft. by 4 ft., and 24 windows 6 ft. by 4 ft.



7. At $7.50 a cubic yard, find the cost of a concrete floor 50′ × 30′ × 9″.

8. At $1.20 a perch, find the cost of building on the foundation here shown a wall 18 in. thick, to the height of 9 ft. Find the cost of excavating the cellar for this building to the depth of $7\frac{1}{2}$ ft. at 25¢ a load.

9. Find the cost of building with concrete the walls for the cellar of a house 36 ft. long and 32 ft. wide, the walls being 9 in. thick and 9 ft. high, if the cost is $7.50 a cu. yd. and if no deduction is made for corners or openings.

The following table gives the quantity of cement, sand, and crushed stone or gravel that may be used to make a certain quantity of a standard concrete. The table may be used to advantage in making small batches:

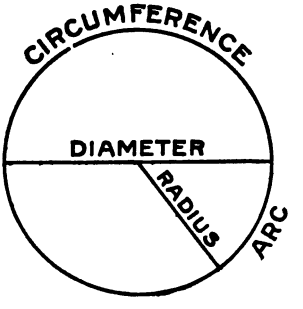
Portland Cement	Sand	Crushed Stone or Gravel	Concrete Obtained
2 bags	3 bu. (stricken)	6 bu. (stricken)	8½ cu. ft.

10. Find the quantity of each of the materials given in the above table required to make 85 cu. ft. of concrete.

Remark. The mixture given above is a 1:2:4 mixture; that is, a mixture of 1 part by measure of cement with two parts of sand and 4 parts of stone or gravel. A bag of cement is estimated to be 1 cu. ft.

The Circle

233. A **circle** is a plane figure bounded by a curved line all points of which are equally distant from a point within called the **center.**



234. The boundary of a circle is called the **circumference.**

235. Any portion of the circumference is called an **arc.**

236. A straight line drawn from the center of a circle to any point in the circumference is called a **radius**.

237. A straight line drawn through the center of a circle and terminating in the circumference is called a **diameter**.

238. Two perpendicular diameters divide the circumference of a circle into four equal parts called **quadrants**.

$\frac{1}{90}$ of a quadrant = an arc of 1 degree (°)
$\frac{1}{60}$ of an arc of 1° = an arc of 1 minute (′)
$\frac{1}{60}$ of an arc of 1′ = an arc of 1 second (″)

239. **Table**

60″ = 1′	90° = 1 quadrant
60′ = 1°	360° = 1 circumference

Important Facts

240. 1. *The diameter equals two times the radius.*

2. *The circumference equals about 3.1416 times the diameter.*

3. *The area of a circle equals one half the number of units in the radius times the number of units in the circumference.*

4. *The area of a circle equals 3.1416 times the square of the number of units in the radius.*

NOTE 1. Results obtained by using 3.1416 are more nearly accurate than those obtained by using $\frac{22}{7}$, which is often used.

NOTE 2. For fuller treatment of the circle see page 406.

Exercise 97

Express as minutes:

1. 25° 2. 32° 3. 45° 4. 90° 5. 120°

Express as seconds:

6. 12′ 7. 24′ 8. 30′ 9. 50′ 10. 58′

Express as minutes:

11. 300″ 12. 420″ 13. 540″ 14. 720″ 15. 1080″

Express as degrees:

16. 240′ 17. 360′ 18. 840′ 19. 1170′ 20. 1260′

21. What part of a circumference is an arc of 90°?

22. What part of a circumference is an arc of 6°?

23. What part of a circumference is an arc of 45°?

24. If an arc of 30° is 6 in. long, what is the circumference of the circle?

25. How many minutes are there in $2\frac{1}{2}$°?

26. How many minutes are there in a circumference?

27. What part of a circumference does the point of the minute hand of a watch pass over in 25 minutes?

Using 3.1416, find the circumference of a circle whose diameter is:

28. 15 in. 29. 40 ft. 30. 65 ft. 31. 7.2 ft.
32. 1.75 rd. 33. 1.25 yd. 34. 4.6 ch. 35. 7.46 ch.

Using 3.1416, find the circumference of a circle whose radius is:

36. 9 in. 37. 24 ft. 38. 32.5 ft. 39. 16.25 yd.
40. 12 rd. 41. 16 ch. 42. 12.6 rd. 43. 14.25 ch.

Using 3.1416, find the diameter of a circle whose circumference is:

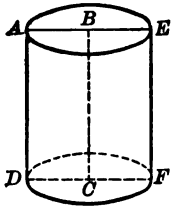
44. 15.7080 in. 45. 21.9912 ft. 46. 31.416 yd.
47. 47.1240 rd.

Using 3.1416, find the radius of a circle whose circumference is:

48. 12.5664 in. 49. 25.1328 ft. 50. 62.8320 yd.
51. 94.2480 rd.
52. Find the area of a circle whose radius is 2 in.
53. Find the area of a circle whose radius is 6 ft.
54. Find the area of a circle whose radius is 8 yd.
55. At \$0.50 per square foot, find the cost of cementing the bottom of a cistern, the bottom being a circle whose radius is 4 ft.

The Right Circular Cylinder

241. The solid formed by revolving a rectangle about one of its sides as an **axis** is called a **right circular cylinder**, or a **cylinder of revolution**.



Thus, the solid formed by revolving the rectangle $ABCD$ about the side BC as an axis is a right circular cylinder. The surface formed by the side AD is the **lateral surface** and its area is the **lateral area**. The circles formed by the sides AB and DC are the **bases**. The axis BC is the **altitude**.

REMARK. In this book cylinders of revolution only are treated. They will be called merely cylinders.

Important Facts

242. 1. **The lateral area of a cylinder equals the number of units in the altitude times the number of units in the circumference of the base.**

2. **The number of cubic units in a cylinder equals the number of units in the altitude times the number of square units in the base.**

Exercise 98

1. Find the lateral area of a cylinder the diameter of whose base is 8 in. and whose altitude is 10 in.

SUGGESTION. 3.1416×8 in. = the circumference of the base.
$10 \times 3.1416 \times 8$ = the number of square inches required.

2. Find the lateral area of a cylindrical vessel the diameter of whose base is 6 in. and whose altitude is 12 in.

3. How many square yards of metal sheeting, correct to the nearest hundredth of a square yard, are required for a cylindrical tank, inclusive of ends, if the circumference of the tank is 60 ft. and its altitude is 12 ft.?

4. Find the area of the entire surface of a cylinder the radius of whose base is 4 ft. and whose altitude is 6 ft.

5. Find the number of cubic feet in a cylinder the radius of whose base is 6 ft. and whose altitude is 5 ft.

SUGGESTION. 3.1416×6^2 = the number of square feet in the base.
$5 \times 3.1416 \times 6^2$ = the number of cubic feet required.

6. How many cubic feet will a silo hold that is 10 ft. in diameter and 20 ft. deep, if the silo is in the form of a cylinder?

7. The standard bushel of the United States is in the form of a cylinder $18\frac{1}{2}$ in. in diameter and 8 in. deep; how many cubic inches does it contain?

8. Find, correct to the nearest gallon, the capacity of a tank in the form of a cylinder 8 ft. in diameter and 12 ft. high.

9. Find, correct to the nearest gallon, the capacity of a cistern 6 ft. in diameter and 20 ft. deep.

COMPOUND DENOMINATE AMOUNTS

243. A **compound denominate amount** is a combination of two or more simple denominate amounts of the same kind of quantity, but of different denominations.

REMARK. Compound amounts are rapidly going out of use, except in the measurement of *time*, *angles*, and *arcs*.

Reduction

Illustrative Examples

1. Reduce 9 rd. 4 yd. 2 ft. to ft.

Explanation:

1 rd. $= 5\frac{1}{2}$ yd.
9 rd. $= 9 \times 5\frac{1}{2}$ yd., or $49\frac{1}{2}$ yd.
9 rd. 4 yd. $= 49\frac{1}{2}$ yd. $+ 4$ yd., or $53\frac{1}{2}$ yd.
1 yd. $= 3$ ft.
$53\frac{1}{2}$ yd. $= 53\frac{1}{2} \times 3$ ft., or $160\frac{1}{2}$ ft.
9 rd. 4 yd. 2 ft. $= 160\frac{1}{2}$ ft. $+ 2$ ft., or $162\frac{1}{2}$ ft.

The answer may be found thus:

$9 - 4 - 2$
$5\frac{1}{2}$
$49\frac{1}{2}$
4
$53\frac{1}{2}$
3
$160\frac{1}{2}$
2
$162\frac{1}{2}$

2. Reduce 256 feet to a compound amount.

Explanation:

256 = the number of feet.
The number of feet ÷ 3 = the number of yards.
∴ 256 ft. = 85 yd. 1 ft.
The number of yards ÷ $5\frac{1}{2}$ = the number of rods.
Or the number of half yards ÷ 11 = the number of rods.
∴ 85 yd. 1 ft., or 256 ft. = 15 rd. $2\frac{1}{2}$ yd. 1 ft.
= 15 rd. 2 yd. $2\frac{1}{2}$ ft.
= 15 rd. 2 yd. 2 ft. 6 in.

The answer may be found thus:

3	256	
$5\frac{1}{2}$	$85\frac{1}{2}$	(read *85 and 1 over*)
2	2	
11	170	
	$15\frac{2}{2}\frac{1}{1}$	

15 rd. $2\frac{1}{2}$ yd. 1 ft. = 15 rd. 2 yd. $2\frac{1}{2}$ ft.
= 15 rd. 2 yd. 2 ft. 6 in.

3. Reduce 4 T. 12 cwt. 25 lb. to hundredweight.

1 lb. = .01 cwt.
∴ 25 lb. = .25 cwt.
1 T. = 20 cwt.
∴ 4 T. = 80 cwt.
∴ 4 T. 12 cwt. 25 lb. = 80 cwt. + 12 cwt. + .25 cwt., or 92.25 cwt.

4. Reduce $\frac{39}{64}$ bu. to a compound amount.

Explanation:

1 bu. = 4 pk.
∴ $\frac{39}{64}$ bu. = $\frac{39}{64}$ of 4 pk., or $2\frac{7}{16}$ pk.
1 pk. = 8 qt.
∴ $\frac{7}{16}$ pk. = $\frac{7}{16}$ of 8 qt., or $3\frac{1}{2}$ qt.
1 qt. = 2 pt.
∴ $\frac{1}{2}$ qt. = $\frac{1}{2}$ of 2 pt., or 1 pt.
∴ $\frac{39}{64}$ bu. = 2 pk. 3 qt. 1 pt.

5. Reduce .7 rd. to a compound amount.

Explanation:

1 rd. = 5.5 yd.
∴ .7 rd. = .7 × 5.5 yd., or 3.85 yd.
1 yd. = 3 ft.
∴ .85 yd. = .85 × 3 ft., or 2.55 ft.
1 ft. = 12 in.
∴ .55 ft. = .55 × 12 in. or 6.6 in.
∴ .7 rd. = 3 yd. 2 ft. 6.6 in.

Exercise 99

Express:

1. 3 gal. 2 qt. 1 pt. as pints.
2. 4 T. 8 cwt. 5 lb. as pounds.
3. 4 bu. 3 pk. 2 qt. as quarts.
4. 4° 25′ 12″ as seconds.
5. 2 rd. 5 yd. 2 ft. as feet.
6. 2 sq. rd. 25 sq. yd. 2 sq. ft. as square feet.
7. 81 pt. as gallons, etc.
8. 150 qt. as bushels, etc.
9. 4565 lb. as short tons, etc.
10. 265 ft. as rods, etc.
11. 5000″ as degrees, etc.
12. 8740 sq. yd. as acres, etc.
13. 12 hhd. 20 gal. 2 qt. as gallons.
14. 2 rd. 12 ft. 6 in. as feet.
15. 4 bu. 3 pk. 2 qt. as pecks.
16. 2 A. 20 sq. rd. 11 sq. yd. as square rods.
17. 3 T. 4 cwt. 30 lb. as hundredweight.
18. $\frac{3}{4}$ rd. as a compound amount.

19. $\frac{3}{5}$ da. as a compound amount.

20. $\frac{3}{4}$ sq. rd. as a compound amount.

21. $\frac{2}{11}$ mi. as a compound amount.

Addition and Subtraction

Illustrative Examples

1. Find the sum of 5 rd. 1 yd. 2 ft., 8 rd. 5 yd. 1 ft., and 7 rd. 2 ft.

Explanation:

	rd.	yd.	ft.
	5	1	2
	8	5	1
	7	0	2
Sum	= 21	$1\frac{1}{2}$	2
	= 21	1	$3\frac{1}{2}$
	= 21	2	$0\frac{1}{2}$

The sum of the column of feet is 5 ft., which equals 1 yd. 2 ft. Write 2 under the column of feet and add 1 yd. to the yards. The sum of the column of yards is 7 yd., which equals 1 rd. $1\frac{1}{2}$ yd. Write $1\frac{1}{2}$ under the column of yards and add 1 rd. to the rods. The sum of the column of rods is 21 rd.; write 21 under the column of rods. $1\frac{1}{2}$ yd. 2 ft. = 1 yd. $3\frac{1}{2}$ ft. = 2 yd. $0\frac{1}{2}$ ft.

∴ the sum is 21 rd. 2 yd. $\frac{1}{2}$ ft. = 21 rd. 2 yd. 6 in.

2. From 4 rd. 2 yd. 1 ft. take 1 rd. 4 yd. 2 ft.

Explanation:

	rd.	yd.	ft.
	4	2	1
	1	4	2
Diff.	= 2	$2\frac{1}{2}$	2
	= 2	2	$3\frac{1}{2}$
	= 2	3	$0\frac{1}{2}$

We cannot take 2 ft. from 1 ft., so we take 1 yd. from 2 yd. in the minuend, leaving 1 yd., and add it to the 1 ft., making 1 yd. 1 ft., or 4 ft. 2 ft. from 4 ft. leaves 2 ft. We cannot take 4 yd. from 1 yd., so we take 1 rd. from the 4 rd. in the minuend, leaving 3 rd., and add it to the 1 yd., making 1 rd. 1 yd., or $6\frac{1}{2}$ yd. 4 yd. from $6\frac{1}{2}$ yd. leaves $2\frac{1}{2}$ yd. 1 rd. from 3 rd. leaves 2 rd. $2\frac{1}{2}$ yd. 2 ft. = 2 yd. $3\frac{1}{2}$ ft. = 3 yd. $0\frac{1}{2}$ ft.

∴ The difference is 2 rd. 3 yd. $\frac{1}{2}$ ft. = 2 rd. 3 yd. 6 in.

Exercise 100

Add:

	yr.	mo.	da.
1.	4	2	10
	27	3	12
	16	1	15

	bu.	pk.	qt.
2.	2	1	4
	5	3	2
	1	2	7

	rd.	yd.	ft.	in.
3.	103	4	1	8
	75	3	2	5
	43	1	1	3

	T.	cwt.	lb.
4.	4	14	63
	12	9	21
	7	18	42

	A.	sq. rd.	sq. yd.
5.	16	128	0
	4	10	16
	4	8	20

	°	′	″
6.	24	15	40
	75	35	18
	80	18	45

Subtract:

	yr.	mo.	da.
7.	1914	6	12
	1888	9	27

	rd.	yd.	ft.
8.	18	3	1
	6	4	2

	°	′	″
9.	180	0	0
	75	30	45

	A.	sq. rd.	sq. yd.
10.	10	40	4
	2	80	5

	T.	cwt.	lb.
11.	15	6	0
	6	17	75

	yr.	mo.	da.
12.	1915	3	15
	63	7	29

Multiplication and Division

Illustrative Examples

1. Multiply 6 rd. 3 yd. 2 ft. by 5.

rd.	yd.	ft.
6	3	2
		5
33	$1\frac{1}{2}$	1

Explanation:

5×2 ft. $= 10$ ft., or 3 yd. 1 ft.

5×3 yd. $= 15$ yd.

15 yd. $+ 3$ yd. $= 18$ yd., or 3 rd. $1\frac{1}{2}$ yd.

5×6 rd. $= 30$ rd.

30 rd. $+ 3$ rd. $= 33$ rd.

$\therefore$ the product $= 33$ rd. $1\frac{1}{2}$ yd. 1 ft.

$= 33$ rd. 1 yd. $2\frac{1}{2}$ ft.

$= 33$ rd. 1 yd. 2 ft. 6 in.

2. Divide 18 bu. 3 pk. 4 qt. by 4.

bu.	pk.	qt.
4)18	3	4
4	2	7

Explanation:

18 bu. ÷ 4 = 4 bu. and 2 bu. over.
2 bu. 3 pk. = 11 pk.
11 pk. ÷ 4 = 2 pk. and 3 pk. over.
3 pk. 4 qt. = 28 qt.
28 qt. ÷ 4 = 7 qt.
∴ the quotient = 4 bu. 2 pk. 7 qt.

Exercise 101

Multiply:

	bu.	pk.	qt.		yr.	mo.	da.		°	′	″
1.	12	3	5	2.	27	10	15	3.	135	40	45
			4				7				3
	rd.	yd.	ft.		T.	cwt.	lb.		A.	sq. rd.	sq. yd.
4.	120	5	2	5.	20	15	75	6.	6	30	3
			9				6				5

Divide:

	yr.	mo.	da.		bu.	pk.	qt.		°	′	″		T.	cwt.	lb.
7.	4)123	11	16	8.	3)65	3	1	9.	9)175	32	21	10.	7)97	13	87

	A.	sq. rd.	sq. yd.		rd.	yd.	ft.
11.	8)11	143	5	12.	6)21	2	2

Exercise 102

1. Washington was born Feb. 22, 1732, and died at the age of 67 yr. 9 mo. 22 da. Find the date of his death.

2. How long was it from sunrise to sunset on a day on which the sun rose at 6.02 A.M. and set at 5.36 P.M.?

3. Find the cost of shipping 6 cwt. 25 lb. of merchandise at $0.85 per cwt.

4. A wagon loaded with grain weighed 32 cwt. 36 lb. The wagon weighed 12 cwt. 25 lb. Find the weight of the grain.

5. A dealer bought a tract of land containing 12 A. 120 sq. rd. He divided it into building lots each containing 24 sq. rd. How many building lots did the tract make?

6. Find the circumference of a circle, if an arc of 60° measures 3 yd. 2 ft. 1 in.

7. A merchant shipped four invoices of goods. The first weighed 2 T. 17 cwt. 25 lb., the second 1 T. 18 cwt. 15 lb., the third 2 T. 1 cwt. 60 lb., and the fourth 18 cwt. 40 lb. Find the total weight of the invoices.

8. What time elapses from 6.20 P.M. Monday to 5.15 A.M. Thursday?

9. What was the duration of an eclipse that began at 7 hr. 37 min. 12 sec. A.M., and ended at 10 hr. 11 min. 42 sec. A.M.?

10. If an arc of 1° measures 4 yd. 2 ft. 6 in., what is the length of an arc of 8°?

11. If 15 times an arc is 62° 2′ 30″, what is the arc?

12. At 15¢ per hour, what did a laborer earn during the 26 working days of March of a certain year, if he began work at 7.00 A.M. and quit at 6.30 P.M., taking each day 30 min. for lunch?

13. The sides of a certain field measure 12 ch. 20 li., 9 ch. 42 li., 18 ch. 68 li., 10 ch. 56 li., and 14 ch. 9 li.; find the distance around the field.

DENOMINATE AMOUNTS OF MINOR IMPORTANCE

Foreign Money

244. The standard units of value in some of the most important European countries and the equivalents in U. S. money are as follows:

COUNTRY	STANDARD	APPROXIMATE EQUIVALENT	EXACT EQUIVALENT
Great Britain	Pound	$5.00	$4.8665
Germany	Mark	0.25	0.238
France	Franc	0.20	0.193
Russia	Ruble	0.50	0.515
Austria-Hungary	Crown	0.20	0.203
Italy	Lira	0.20	0.193
Spain	Peseta	0.20	0.193
Netherlands	Florin	0.40	0.402
Switzerland	Franc	0.20	0.193

245. **Table of English Money**

4 farthings (far.)	= 1 penny (d.)
12 pence	= 1 shilling (s.)
20 shillings	= 1 pound (£)
21 shillings	= 1 guinea
5 shillings	= 1 crown
2 shillings	= 1 florin

NOTE. In Canada the units of money are the same as in the United States.

Troy Weight

246. Troy weight is used in weighing gold, silver, platinum, and jewelry, except gems and precious stones. The standard unit is the **Troy pound.**

247. **Table**

24 grains (gr.)	**= 1 pennyweight (pwt.)**
20 pennyweights	**= 1 ounce (oz.)**
12 ounces	**= 1 pound (lb.)**

NOTE. The unit of weight for gems and precious stones is the **carat,** which equals 200 milligrams,* or about 3.08 Troy grains. The term *carat* is also used to denote the purity, or fineness, of gold, 24 carats denoting **pure gold.**

Apothecaries' Weight and Measure

248. Medicines are sold in prescriptions by apothecaries' weight, which is as follows:

1 pound (℔) = 12 ounces (℥)
= 96 drams (ʒ)
= 288 scruples (℈)
= 5760 grains (gr.)

249. In mixing liquid medicines druggists use apothecaries' fluid measure, which is as follows:

60 minims (♏) = 1 fluid dram ($f\mathfrak{z}$)
8 fluid drams = 1 fluid ounce ($f\mathfrak{z}$)
16 fluid ounces = 1 pint (O)
8 pints = 1 gallon (Cong.)

NOTE 1. A minim is about a drop.

NOTE 2. An ounce bottle holds a fluid ounce, or $\frac{1}{16}$ of a pint; a 2-ounce bottle, $\frac{1}{8}$ of a pint; a 3-ounce bottle, $\frac{3}{16}$ of a pint, and so on.

NOTE 3. 45 drops of water equal about a teaspoonful; 1 teaspoonful equals 1 fluid dram; 1 dessert-spoonful equals 2 teaspoonfuls; and 1 tablespoonful equals 2 dessert-spoonfuls.

* See Supplement, Section 35.

Exercise 103

1. Express £3 as pence.
2. Express £2 4*s.* as pence.
3. Express 240*s.* as pounds.
4. Express 2758*d.* as pounds, etc.
5. What is the exact value in United States money of 100 francs?
6. What is the exact value in United States money of 1000 marks?
7. What is the exact value in United States money of 200 pesetas?
8. What is the approximate value in marks of $1? in francs? in crowns? in lire? in pesetas?
9. What is the exact value in United States money of 200 rubles?
10. An English florin has a value nearly equal to what piece of United States money?
11. One crown is worth approximately how much United States money?
12. Express, correct to the nearest hundredth of a pound, the exact value in English money of $1000.
13. Find the exact value in French money, correct to the nearest franc, of $100.
14. Express, correct to the nearest mark, the value in German money of $100.
15. How many grains are there in a pound Troy?
16. The standard gold dollar of the United States weighs 25.8 gr.; .9 of it is pure gold and the remainder

copper; what is the weight of the copper in a $5 gold piece?

17. The standard gold dollar weighs 25.8 gr. and is .9 pure gold; what is the weight of the pure gold in 5 eagles?

18. The United States silver dollar weighs 412.5 gr. and is .9 pure silver. 1 pound Troy of pure silver will furnish silver enough to make how many silver dollars and how many grains over?

19. About what is the weight in Troy grains of a diamond weighing 12 carats?

20. How many 2-grain quinine pills can be made from an ounce of quinine?

21. How many grains of gold are there in a piece of jewelry 18 carats fine and weighing 4 pwt. 16 gr.?

22. A pint of medicine will fill how many 2-ounce bottles?

23. What is the weight in Troy pounds of a thousand gold dollars?

24. 1 pwt. 10 gr. of pure gold will be sufficient for how many grains of jewelry 12 carats fine?

25. A pearl weighing $9\frac{1}{2}$ carats will weigh about how many Troy grains?

REVIEW OF DENOMINATE AMOUNTS

Exercise 104

(Solve as many as possible at sight.)

1. Find the cost of $8\frac{1}{2}$ doz. plants at 3¢ each.

2. Porcelain nest eggs can be bought for 5¢ each or 25¢ per dozen; how much less will a dozen cost, if bought in one quantity rather than separately?

3. On a certain scale of measurement $\frac{3}{4}$ in. is taken to represent 10 ft. What should represent 25 ft.?

4. What is the cost of a load of hay weighing 2765 lb. at \$16 per ton?

5. Find the cost of shipping 3650 lb. of potatoes at 6¢ per 100 lb.

6. A dealer bought a bushel of chestnuts for \$2.50. They shrunk 1 qt. before they were sold; what was his profit on the bushel, if he retailed them at 12¢ a quart?

7. What part of an acre does a square contain if it can be inclosed by 32 rd. of fencing?

8. How much is received for a bushel of plums sold at the rate of 2 qt. for 25¢?

9. Estimating the weight of a gallon of water to be $8\frac{1}{3}$ lb., what weight of water will a tank hold whose capacity is 12 hogsheads (63 gal.)?

10. What date is 90 days prior to December 15?

11. A family that uses 16 eggs a day uses how many dozen in 9 days?

12. Hay well dried in the field loses about $\frac{1}{3}$ of its weight if allowed to dry out and pack in the mow; what weight of dry hay from the field will weigh 10 tons (2000 lb.) when settled in the mow?

13. A huckster had 20 doz. eggs. If he sold $2\frac{1}{2}$ doz. to one woman, 6 eggs to another, and $1\frac{1}{2}$ doz. to another, what did he receive for the remainder of them, if he sold them at 30¢ per dozen?

14. A barrel of good lime is used in laying 1000 bricks; how many barrels of lime are needed to lay 8750 bricks?

15. The average yield of hay per acre in Michigan is 1.46 T.; how many pounds is this?

16. Sand weighs about 133 lb. per cubic foot; estimating a bushel to be $\frac{5}{4}$ of a cubic foot, what should a bushel of sand weigh?

17. Find the cost of cementing the floor of a cellar 36 ft. long and 24 ft. wide at $1.50 per square yard.

18. How many seconds are there in a day?

19. What should be charged for 720 lb. of coal at $7.50 per short ton?

20. 90 days after July 15 comes on what date?

21. Ten eggs are estimated to weigh a pound; at this estimate what should be the weight of $7\frac{1}{2}$ doz. eggs?

22. What time elapses from 7.30 P.M. Thursday till 1 P.M. Friday?

23. What is the value of 4 loads of hay weighing, respectively, 1800 lb., 1650 lb., 1760 lb., and 1850 lb., at $18 per ton?

24. How many cubic yards are there in a cube 6 ft. long?

25. How many cubic inches are there in a cube 2 ft. long?

26. Find the value of 61 lb. 4 oz. of butter at 40¢ a pound.

27. The average yield per acre of hay in Wisconsin is 3600 lb. How many tons should a 10-acre field yield?

28. Find the cost of a piece of linoleum 4 yd. long and 12 ft. wide at 39¢ per square yard.

29. How many cords of wood are there in a pile 36 ft. long, 4 ft. wide, and 4 ft. high?

30. Find the cost of two hams at 16¢ a pound, one weighing 12 lb. 9 oz. and the other 14 lb. 11 oz.

31. A dealer paid $240 for a ton of cheese, and retailed it at 16¢ per pound. Find his profit.

32. How many cards 4 in. × 2 in. can be cut from a cardboard 3 ft. × 2 ft.?

33. If the perimeter of a rectangular piece of land 12 rd. long is 40 rd., what part of an acre does the piece of land contain?

34. How many gallons does a tank hold, if it is 7 ft. long, $5\frac{1}{2}$ ft. wide, and 6 ft. deep?

35. There are 2150.42 cu. in. in a bushel and 231 cu. in. in a gallon. Estimate to the nearest hundredth the number of bushels in a barrel of 40 gal.

36. How many acres are there in a tract of land 2 mi. square?

37. How many perches of masonry are there in a wall 99 ft. long, $1\frac{1}{2}$ ft. thick, and 10 ft. high?

38. A man has a lot 80 ft. × 60 ft. on which he intends to build a house 40 ft. × 32 ft.; how many square feet of the lot will he have left for the yard?

39. Under ordinary conditions of weather and soil, cabbage will yield 15000 lb. to the acre; how many acres should yield 20 T.?

40. A farmer wishes to sow fertilizer on his land to the amount of 400 lb. per acre. How much should he sow on a piece of land 55 rd. long and 16 rd. wide?

41. Find the cost of 8 planks, 16 ft. × 8 in. × 3 in. at $20 per 1000 feet.

42. What time elapsed between two events, one of which occurred July 4, 1776, and the other Jan. 1, 1863?

43. Estimating that a load of earth contains 1 cu. yd., how many loads of earth must be removed in digging a cellar for a house 36 ft. long and 24 ft. wide, if the cellar is dug to the depth of 9 ft.?

44. The following is a recipe for muffins, the quantities here given being sufficient to make five. How much of each of these substances is necessary to make 5 doz. muffins?

1 cup of milk
$1\frac{1}{2}$ cups of flour
2 eggs
$\frac{1}{2}$ teaspoonful of salt
$1\frac{1}{2}$ teaspoonfuls of baking powder

45. Estimating a cubic foot of water at $62\frac{1}{2}$ lb., how many tons (2000 lb.) of water will a tank hold that is 10 ft. long, 4 ft. wide, and 5 ft. deep?

46. When coal is being retailed from house to house at 8¢ per bucket, 20 lb., how much more is being paid for it per ton (2000 lb.) than if it were bought for $6.75 per ton?

47. If a miller buys bran at $24 per ton and retails it at 28¢ per bushel (20 lb.), what is his profit per ton?

48. The three ingredients of concrete are cement, sand, and broken stone; for general use these may be mixed in the proportion of 1 part of cement with $2\frac{1}{2}$ parts of sand and 5 parts of broken stone. How many cubic yards of sand and how many of broken stone must be used to make concrete in which are used 30 bags Portland cement? Consider 1 bag Portland cement = 1 cu. ft.

49. The output of hops in Oregon during a recent year was 93,000 bales of 185 lb.; what was received for them at $12\frac{1}{2}$¢ per pound?

50. Slates that are 8 in. by 12 in. will cover about 36 sq. in. of exposed roof surface; how many slates of these dimensions will cover a roof surface 18 ft. long and 12 ft. wide?

51. Find the number of square inches in the entire outside surface of a box 6 ft. by 4 ft. by 2 ft.

52. A bill of goods that cost 500 marks cost how much in United States money?

53. How many leap years are there in the twentieth century?

54. Wheat bran weighs 20 lb. to the bushel; how many bushels of wheat bran are there in a ton of 2000 lb.?

55. A dealer by selling drawing paper at 10¢ per dozen sheets gained $\frac{1}{4}$¢ per sheet; what did he pay for the paper per ream?

56. On what date does a debt fall due that is payable 60 days after December 31, 1913?

57. The annual cut of timber in Oregon is estimated at 2,000,000,000 feet, valued at $30,000,000; what is the estimated value per 1000 feet?

58. There are 11,000 sq. ft. in the exposed surface of the walls of a certain building. These walls were built of cut stone at a cost of 50¢ per square foot for labor and material; find the entire cost.

59. Find the cost of 68 roofing laths averaging 12 ft. long at 65¢ per 100 linear feet.

60. How many bricks laid on the broadest face are necessary to make a pavement 100 yd. long and 8 ft. wide?

61. A debt that is due two calendar months after Dec. 31, 1913, is due on what date?

62. How many acres are there in a tract of land in the form of a square one side of which measures 80 rd.?

63. How many feet of boards 1 in. thick can be sawed from a log 16 ft. long by 18 in. by 18 in., making no allowance for waste in sawing?

64. Find the weight of a block of Susquehanna granite 8 ft. long, 2 ft. 6 in. wide, and 1 ft. thick, the granite weighing 2.704 times as much as an equal bulk of water.

65. A dealer bought 800 tons (2240 lb.) of coal at $ 5 per long ton and retailed it at $ 6.75 per short ton; what was his profit?

66. A dealer who buys thread at 55¢ per dozen spools and retails it at 5¢ a spool gains how much on a gross of thread?

67. How many acres are there in a tract of land $\frac{1}{2}$ mile square?

68. How many dust cloths 14 in. square can be made from 7 yd. of material 28 in. wide?

69. Find the cost of 2856 lb. of corn at $0.65 per bushel (56 lb.).

70. Allowing 400 cu. ft. to the ton, estimate the weight of a mow of hay 24 ft. long, 16 ft. wide, and 12 ft. deep.

71. How many square inches of pasteboard are necessary to make a cube 6 in. long?

72. It is estimated that 1300 plastering laths are required for 100 sq. yd. of plastering; at this estimate, how many laths are required to plaster a room 24 ft. long, 18 ft. wide, and 10 ft. high?

73. An avoirdupois pound contains 7000 grains; how many grains are there in an avoirdupois ounce?

74. How many feet long is a line that is 8 chains long?

75. The total rainfall during the year in Minnesota is about 4 ft. Estimate, correct to the nearest gallon, the rainfall to each square yard.

76. Find the cost of 25,680 lb. of hay at $15 per ton.

77. How much does a man gain who buys 5 T. (2240 lb.) of coal at $6 per ton and retails it at 8¢ per bucket, 20 lb. to the bucket?

78. At $2.50 a cord what is the value of a pile of wood 48 ft. long, 6 ft. high, and 4 ft. wide?

79. Estimating a barrel ($31\frac{1}{2}$ gal.) at $4\frac{1}{4}$ cu. ft. is making the estimate too high by how many cubic inches?

80. How many feet of 2-inch planks are required to lay a barn floor 42 ft. long and 16 ft. wide?

81. What is the sum of the angles of a square? of a rectangle?

82. Estimating a bushel of wheat at $\frac{5}{4}$ of a cubic foot, how many bushels of wheat will a bin hold that is 6 ft. × 6 ft. × 4 ft. 6 in.?

83. Over how many minute spaces does the minute hand of a clock pass in a day?

84. How many shingles are required for 240 sq. ft. of roofing, if they are 4 in. wide and laid 6 in. to the weather?

85. A farmer sold 125 bu. of oats which weighed 3750 lb.; the standard weight of a bushel of oats being 32 lb., find the loss in weight per bushel.

86. A rectangular piece of land 120 rd. long requires 320 rd. of fencing to inclose it; how many acres does the tract contain?

87. Find the cost of shipping farm machinery weighing 2648 lb., if the rate charged was 48¢ per 100 lb.

88. When shingles are selling at $12.50 per 1000, what should be paid for 16 bales of shingles, 200 to the bale?

89. How many perches of masonry are there in a wall 33 ft. long, 4 ft. high, and 18 in. thick?

90. How many yards of carpet $\frac{3}{4}$ yd. wide must be bought for a floor 24 ft. long and 16 ft. wide, if the strips are to be laid lengthwise and $\frac{1}{2}$ yd. allowed on each strip for waste in matching?

91. When hay is bought at $16 per ton (2000 lb.) and retailed at $1 per bale of 100 lb., what is the profit per ton?

92. A schoolroom should have not less than 15 sq. ft. of floor space for each pupil; at this estimate, a room 40 ft. long and 33 ft. wide will provide sufficient floor space for how many pupils?

93. The annual yield of butter fat from 550 cows in Minnesota during a recent year was 102,569.50 lb.; what was the average annual yield per cow?

94. What is the value of 2,722,528 lb. of skimmed milk at 15¢ per hundredweight?

95. Allowing 1 T. of clover hay, 10 bu. of corn, and 10 bu. of oats as the ration for a cow for 120 da., find the cost of keeping a herd of 20 cows 6 mo. with hay at $15 per ton, corn $0.60 per bushel, and oats $0.42 per bushel.

96. What is the income for June from a cow that gives an average of 22 lb. of milk daily, if this milk tests .035 butter fat, which is sold at $0.32 per pound?

97. Including land rental, labor, and machinery value consumed, the cost of producing wheat in Minnesota was recently shown to be $7.249 per acre. Estimate a farmer's profit on 100 A. of wheat averaging 20.6 bu. per acre and sold at $0.90 a bushel.

98. It is estimated that 2 bu. of corn in the ear (measuring $\frac{2}{1}\frac{5}{6}$ cu. ft. per bushel) will make 1 bu. of shelled corn. A quantity of corn in the ear which fills a box 10 ft. long, 3 ft. wide, and 25 in. deep will make how many bushels of shelled corn?

99. A plasterer estimates that 5 bu. of lime will be necessary for each coat in 100 sq. yd. of plastering. At this estimate, how many bushels of lime are required to give 2 coats of plaster to a room 24 ft. long, 18 ft. wide, and 9 ft. high?

100. Steers should be fed each day hay or its equivalent to the amount of 2.84 lb. for each 100 lb. of their live weight. How much hay or its equivalent should be fed per day to an 860-lb. steer?

GENERAL REVIEW

Exercise 105

(Solve as many as possible at sight.)

1. The several items entering into the cost of maintaining a farm work-horse for a year were recently shown to be as follows; find the total:

Interest on investment	$6.74
Depreciation	4.35
Shoeing	1.45
Feed	75.03
Labor	11.88
Miscellaneous Expenses	0.29
Total	

2. What postage must be paid on a book weighing 18 oz. mailed for delivery within the third zone?

3. A farmer who sows 500 lb. of fertilizer to the acre must purchase how many tons for 12 A.?

4. Find how many bushels of wheat a bin 6 ft. $\times$ 4 ft. $\times$ 4 ft. will hold, estimating that a bushel of wheat occupies about $\frac{5}{4}$ of a cubic foot.

5. Good wheat will produce about .42 of its weight of finest flour, and in addition .18 of its weight of seconds and .09 of its weight of biscuit flour; how many pounds of each of these grades of flour are produced from a bushel of wheat weighing 60 pounds?

6. A certain horse is $15\frac{3}{4}$ hands high; express this height in feet and inches.

7. How many cords of wood are there in a pile of wood 80 ft. long, 4 ft. wide, and 4 ft. high?

8. Find the cost of sending a telegram of 30 words, if the rate is 40 ¢ for the first ten words and 3 ¢ for each word over ten.

9. A consignment of 500 bbl. of beef, each barrel weighing 200 lb, weighs how many tons?

10. If $\frac{3}{4}$ in. on a map represents 10 mi., what is the distance between two towns, if the space between them on the map is $4\frac{1}{8}$ in.?

11. Find the total weight of a piece of linoleum $3\frac{1}{2}$ yd. long and $2\frac{1}{2}$ yd. wide, which weighs $3\frac{1}{2}$ lb. to the square yard.

12. How many cakes can be bought for 25 ¢ at the rate of 10 ¢ per dozen?

13. The time consumed by a man in doing $\frac{5}{6}$ of a piece of work was $12\frac{1}{2}$ da.; how long should it take him to finish it?

14. Find the cost of 3374 lb. of shelled corn at 80 ¢ per bushel (56 lb.).

15. How many leap years are there between 1900 and 1950?

16. Find the cost of 15 pieces of scantling $2'' \times 3'' \times 18'$ at $ 20 per thousand feet.

17. Find the value of 31 lb. 12 oz. of beef at 16 ¢ per pound.

18. If 2 is added to both terms of the fraction $\frac{3}{4}$, is the resulting fraction greater or less than $\frac{3}{4}$, and how much?

19. If 2 is added to both terms of the fraction $\frac{4}{5}$, is the resulting fraction greater or less than $\frac{4}{5}$, and how much?

20. A drover sold a steer for $63. If it weighed 1050 lb., what did the drover receive per pound for it?

21. What part of June has passed by midnight, June 20?

22. How many 2-inch cubes can be cut from a block of wood in the form of a rectangular solid 8 in. × 6 in. × 4 in.?

23. What part of the day has passed by 7.30 A.M.?

24. A miller that returns 42 lb. of flour for every bushel (60 lb.) of wheat ground, should receive how many bushels of wheat, that weighs only 56 lb. per bushel, in return for 1 barrel of flour (196 lb.)?

25. Two loads of corn contained together $75\frac{1}{2}$ bushels. They were sold at $\$\frac{3}{4}$ a bushel. $\$4\frac{1}{2}$ more was received for one load than for the other; how many bushels did each contain?

26. How many cubical blocks, each 1 in. long, can be cut from a block 2 ft. long, 6 in. wide, and 1 in. thick?

27. How many cubical blocks 3 in. long can be cut from a block 1 ft. 6 in. long, 9 in. wide, and 7 in. thick?

28. A common brick weighs about $4\frac{1}{2}$ lb.; find correct to the nearest brick the number that a teamster should load in order that the weight may be a ton.

29. If one stove consumes $\frac{3}{4}$ gal. of oil in a day and another $\frac{5}{8}$ gal., what part of a day will a gallon last both stoves?

30. The sum of two numbers is 26, their difference is 6; what are the numbers?

SUGGESTION. $2 \times$ the smaller number $+ 6 = 26$.
$2 \times$ the smaller number $= 20$.

31. A and B together purchased 150 acres of land, of which A took $30\frac{1}{2}$ acres more than B. How many acres did each take?

SUGGESTION. $2 \times$ B's $+ 30\frac{1}{2}$ acres $= 150$ acres.

$2 \times$ B's $= 150$ acres $- 30\frac{1}{2}$ acres.

32. A dealer gained twice the cost of spices by selling them at 5¢ an ounce; what did they cost him per pound?

33. The sum of two numbers is 98, and $\frac{4}{5}$ of the larger is $\frac{5}{6}$ of the smaller. What are the numbers?

34. What is the weight of the water in a tank 6 ft. long and 4 ft. wide if the depth of the water is 1.5 ft., water weighing 62.5 lb. per cubic foot?

35. Make a drawing on paper of a floor $16\frac{1}{2}$ ft. long and $13\frac{1}{2}$ ft. wide; scale: 1 in. = 2 ft.; showing strips of carpet $\frac{3}{4}$ yd. wide running lengthwise, and surrounded by a border $\frac{3}{4}$ yd. wide. How many yards of carpet and how many yards of border must be bought to carpet this floor?

36. A farmer sold 2116 lb. of wheat at \$0.96 per bushel (60 lb.); how much did he receive for it?

37. Find the total length of the edges of a box $3\frac{1}{2}$ ft. long, $2\frac{1}{2}$ ft. wide, and $1\frac{3}{4}$ ft. high.

38. Simplify: $\dfrac{.3\frac{1}{3} \div .66\frac{2}{3}}{.37\frac{1}{2} \div 1.5}$.

39. A farmer is offered 60¢ a bushel for 200 bu. of potatoes. He refuses the offer and later sells them for 70¢ a bushel, at which time they have shrunk $\frac{1}{10}$. Did he gain or lose by holding them, and how much?

40. If a piece of land 80 yd. long and 22 yd. wide produced 22 bu. of oats, what was the yield per acre?

41. A thousand shingles require about $3\frac{1}{2}$ lb. of four-penny nails. At this estimate, what weight of nails would 25,000 shingles require?

42. Find the cost of plastering the walls and ceiling of a room 24 ft. long, 18 ft. wide, and 10 ft. high at 25 ¢ per square yard, allowing 420 sq. ft. for doors and windows.

43. If granite is 2.704 times as heavy as water, which weighs 62.5 lb. per cubic foot, find the weight of a block of granite 6 ft. by $1\frac{1}{2}$ ft. by $1\frac{1}{2}$ ft.

44. A man and his son earned \$ $25\frac{1}{2}$ in a week. The man earned \$ $5\frac{1}{2}$ more than the son. How much did each earn?

45. How much will it cost at \$ 0.20 per cubic yard to excavate a cellar for a house 28 ft. long, 18 ft. wide, with an L attached, 18 ft. by 16 ft., the average depth of the cellar being 10 ft.?

46. A dealer bought 200 bu. of clover seed at \$ $7\frac{1}{2}$ a bushel and sold it at a profit of \$ 200; what did he receive for it per bushel?

47. If a rectangular piece of land 44 yd. long and 40 yd. wide produced 312 lb. of wheat, what was the yield per acre at this rate, wheat weighing 60 lb. per bushel?

48. How many square feet of zinc are required to line the sides and bottom of a box 4 ft. long, 4 ft. wide, and 3 ft. deep?

49. If 9 bu. of apples of average quality will make a barrel of $31\frac{1}{2}$ gal. of cider, how many bushels of apples of the same quality will make a barrel of 42 gal. of cider?

50. Estimating that ice is $\frac{10}{11}$ as heavy as water and

that water weighs 62.5 lb. per cubic foot, find the weight of a block of ice $5\frac{1}{2}$ ft. $\times$ 2 ft. $\times$ 18 in.

It is estimated that 100 lb. of good hay will possess the feeding value of:

$44\frac{1}{2}$ lb. of wheat	59 lb. of oats
$62\frac{1}{2}$ lb. of corn	$53\frac{1}{2}$ lb. of rye
669 lb. of beets	360 lb. of potatoes
28 lb. of beans	347 lb. of oat straw
58 lb. of barley	429 lb. of rye straw

Refer to the above table in answering questions 51–58.

51. One ton of hay has the same feeding value as how many pounds of wheat?

52. One ton of hay has the same feeding value as how many pounds of oat straw?

53. How much rye straw has the same feeding value as 500 lb. of hay?

54. How much corn has the same feeding value as one ton of hay?

55. How much barley has the same feeding value as one ton of hay?

56. Fifty pounds of hay have the same feeding value as how many pounds of beans?

57. Thirty-six pounds of potatoes have as much feeding value as how many pounds of hay?

58. Ten pounds of hay have as much feeding value as how many pounds of beets? how many pounds of oats? how many pounds of rye?

59. If $2\frac{1}{2}$ in. on a certain drawing represent 10 mi., what should $\frac{3}{4}$ of an inch represent on the same drawing?

60. How many yards of carpet must be bought to carpet a room 20 ft. long and 14 ft. wide, if the strips, which are $\frac{3}{4}$ of a yard wide, are to be laid lengthwise, and $\frac{1}{2}$ yd. is to be allowed on each strip for waste in matching?

61. Make a drawing of a floor 18 ft. long and 16 ft. wide; scale: $2'' = 1'$; showing strips of carpet $\frac{3}{4}$ yd. wide running lengthwise on the floor; find the cost of the carpet required, at \$1.20 per yard.

62. A ten-acre field of wheat yielded 12,000 lb. of wheat, and 20,000 lb. of straw; how many bushels (60 lb.) of wheat did it yield, and how many tons (2000 lb.) of straw? How many bushels of wheat did this field yield per acre, and how many tons of straw?

63. Hay well dried in the field loses about one third of its weight, if it is allowed to dry out and is then packed in the mow. Which is cheaper and how much, to buy 6 tons of hay from the field at \$15 a ton, or to buy from the mow at \$20 a ton?

64. If one machine can skim 75 gallons of milk in $1\frac{3}{4}$ hours and another 60 gallons per hour, how long must both run to skim 300 gallons of milk?

65. When clover seed is selling at \$9 per bushel, find the cost of sufficient seed for a 15-acre field, allowing 10 lb. of seed for an acre, clover seed weighing 60 lb. per bushel.

66. If the length of an arc of $22° 30'$ is 6 in., what is the circumference of a circle?

67. Show that there are 4 rods in one chain.

68. During a certain year which began on Sunday, a rural mail carrier covered his route of 30 miles each week day except February 22 and July 4. How many miles did he travel during that year in delivering his mail?

69. At $2.50 per cord, what is the value of a pile of wood 40 ft. long, 6 ft. high, and 4 ft. wide?

70. Find the cost of 20 planks, each 12 ft. by 6 in. by $1\frac{1}{2}$ in., at $30 per 1000 ft.

71. A tract of land 1 mile square contains how many acres?

72. Every public schoolroom built in Pennsylvania after May 18, 1911, must have not less than 200 cu. ft. of air space per pupil. A schoolroom 40 ft. long, 30 ft. wide, and 10 ft. high contains sufficient air space for how many pupils, if the room is built in accordance with the regulation stated?

73. In a certain experiment it was found that the cost of maintaining a herd of 261 cows was $44.67 each per year, and that the value of the product per cow was $60.05. Find the net profit from the herd.

74. When a Mexican dollar is quoted at $0.4875 in United States money, what are 1000 Mexican dollars worth in United States money?

75. Show why each common year begins 1 day later in the week than the preceding common year.

76. In measuring distances, a pace is considered $\frac{1}{5}$ of a rod; how many feet is this?

77. Estimating £1 to equal $4.8665, what is the value in United States money of £5 4*s*.?

78. Simplify:

$$116 \times 4 + 8 - 84 \div 12 \times 3 + 63 \div (9 \times 7).$$

79. If one machine can grind 10 bu. of grain in $2\frac{1}{2}$ hr. and another can grind 10 bu. in $1\frac{2}{3}$ hr., how long will it take both machines to grind 100 bu. of grain?

80. Express each of the following as a decimal part of a dollar and find their sum: 12.5 cents, $18\frac{3}{4}$ mills, $\frac{7}{8}$ of a dollar, 1.75 cents, $\frac{7}{8}$ of a dime, and 14 cents 6.5 mills.

81. How many 2-in. cubes can be cut from a block of wood in the form of a rectangular solid 6 in. by 5 in. by 4 in.?

82. If a lot having a frontage of 40 ft. and a depth of 60 ft. was sold for $25 per foot front, what was the price per acre?

83. Find, correct to the nearest pound, the amount of flour a miller should give in exchange for 2 bu. of wheat weighing 62 lb. to the bushel, if he gives 38 lb. of flour in exchange for 60 lb. of wheat.

The following is the standard daily ration for a 1000-pound horse:

10 lb. hay (timothy) 12 lb. oats

Refer to Ration given above in answering questions 84–87.

84. How much hay and how much oats will the horse be given in a week?

85. How much hay and how much oats will the horse consume during the month of October?

86. How many bushels of oats would be required to keep this horse a year, provided he was fed no other grain than oats? (Count 32 lb. to the bushel.)

87. Find the cost of the hay this horse would consume in one year, the hay costing $25 a ton.

88. A knot is 6086 ft. and a statute mile is 5280 ft.; estimate, correct to the nearest hundredth, the number of statute miles in a knot.

89. How much is saved by using a 100-ride ticket between two stations instead of tickets bought at the rate of 10 for $4, if the 100-ride ticket cost $22.80?

90. Express 648 units as gross.

91. Express 540,000″ as degrees.

92. Estimating that a pound of pure ground white lead is required to give three coats of paint to 2 sq. yd. of surface, how many pounds of this lead are required in giving three coats to a building 36 ft. long, 24 ft. wide, and 15 ft. high at one corner? (Disregard gables.)

93. Estimating house rent at $10 per month and board at $3 per week, what yearly wages without board or house rent are equivalent to $15 per month with board and house rent free? Count 12 months to the year in reckoning wages and house rent, and 52 weeks to the year in reckoning board.

94. A tract of land 10 ch. long and 3.75 ch. wide was bought at $75 per acre. The purchaser had it inclosed by a fence at a cost of $0.62 per rod. Find the total cost.

95. Express: 56,760 ft. as miles.

96. A dairyman recently reported an average of 420 pounds of butter fat per year per cow from a herd of 29 cows. The average price of butter fat at the local creamery was $0.32 a pound. What was the average income from each cow? What was the income from the whole herd? The cost of keeping this herd was estimated at $50 per cow. What was the average net income from each cow and the net income from the whole herd?

97. Fill in the missing averages:

Daily Average of Hours of Labor of Regular Farm Workers, by Months, for One Year

1910	1. — New England Dairy Farm		2. — Iowa Grain and Stock Farm	
	Week days	Sundays	Week days	Sundays
	Hours	*Hours*	*Hours*	*Hours*
January	11.4	7.2	10.8	5.9
February	11.7	7.3	11.3	4.9
March	11.4	5.1	12.3	4.8
April	12.2	5.3	12.2	4.7
May	11.9	4.4	12.3	3.6
June	11.9	3.7	12.7	3.3
July	11.8	3.6	12.6	3.1
August	11.3	3.3	12.6	3.2
September	11.2	4.0	11.8	2.7
October	11.5	3.5	12.3	2.9
November	10.3	4.9	12.2	3.3
December	9.7	4.3	12.4	8.2
Yearly average				

98. A crib 24 ft. long, 4 ft. wide, and 10 ft. high is filled with corn on the cob; estimating that each bushel of corn on the cob is $\frac{2}{1}\frac{5}{6}$ of a cubic foot and that 2 bu. of corn on the cob will make one bushel of shelled corn, find how many bushels of shelled corn this crib of corn will make.

99. The following table shows the quantity of corn by weight that remained after shrinkage at the close of each month from each 100 bu. by weight placed in the crib, as determined in a recent trial at an Experimental Station. In the column headed "Market Price" fill in the market price of corn prevailing in your community at the close of each month of the current year, and determine in what month the farmer would have received the greatest amount for the 100 bu. placed in the crib in November:

Date	Quantity	Market Price	Value
Nov. 30	100 bu.		
Dec. 31	97.9 bu.		
Jan. 31	97.6 bu.		
Feb. 28	97 bu.		
Mar. 31	97.8 bu.		
Apr. 30	96.9 bu.		
May 31	93.1 bu.		
June 30	92 bu.		
July 31	89.4 bu.		
Aug. 31	86.7 bu.		
Sept. 30	87.7 bu.		
Oct. 31	89.8 bu.		

THE EQUATION

Introductory Exercises, Definitions, and Principles

250. 1. If a stands for 3, what is the value of $2 \times a$?

$2 \times a$ may be written $2a$.

2. If a stands for 4, what is the value of $5a$? $a + 2a$? $3a - a$?

3. If a stands for 4 and b stands for 2, what is the value of $a + b$? $a - b$? ab? $a \div b$?

Any letter may stand for any number.

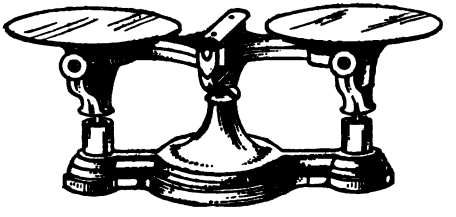
4. If x stands for 6, what is the value of $2x$? $x \div 3$? $3x + 2x$? $3x - x$?

251. When x stands for 4, $2x = 8$.

The expression $2x = 8$ is called an *equation*. $2x$ is the *first member* of the equation and 8 the *second member*.

252. An **equation** is a statement that two numbers or expressions are equal.

253. These scales are true; hence, they will balance when 6 lb. are placed on each plate. They will still balance if,



(1) 2 lb. additional are placed on each plate; or,
(2) 2 lb. are taken from each plate; or,
(3) 2×6 lb. are placed on each plate; or,
(4) 6 lb. $\div$ 2 are placed on each plate.

254. Section 253 illustrates the following:

(1) *If equals are added to equals, the results are equal.*
(2) *If equals are subtracted from equals, the results are equal.*
(3) *If equals are multiplied by equals, the results are equal.*
(4) *If equals are divided by equals, the results are equal.*

255. If x stands for 4,

$$(1)\quad 6x - 2 = 3x + 10.$$
$$(2)\quad 6x - 10 = 3x + 2.$$

In equations (1) and (2) observe that the 2 and also the 10 are in different members, and that the signs before each of these numbers are different in the two equations; hence,

256. A term may be transposed from one member of an equation to the other, if the sign before it is changed.

Exercise 106

1. If $x + 6 = 8$, for what number does x stand; that is, *what is the value of x?*

$$x + 6 = 8$$
$$x = 8 - 6 \quad [\S\ 256]$$
$$x = 2$$

In each of the following find the value of x:

2. $x+8=10$	3. $x-3=2$	4. $3=x+2$
5. $x-9=1$	6. $x-1=4$	7. $5=x-2$
8. $3+x=4$	9. $8+x=12$	10. $1=x-1$
11. $7+x=12$	12. $3+x=7$	13. $5=x+3$

14. If $5x=40$, what is the value of x?

$$5x=40$$

Dividing by 5, $x=8$ [§ 254, 4]

15. If $\frac{1}{3}x=4$, what is the value of x?

$$\tfrac{1}{3}x=4$$

Multiplying by 3, $x=12$ [§ 254, 3]

16. If $\frac{2}{3}x=4$, what is the value of x?

$$\tfrac{2}{3}x=4$$

Multiplying by 3, $2x=12$ [§ 254, 3]

Dividing by 2, $x=6$ [§ 254, 4]

In each of the following find the value of x:

17. $2x=16$	18. $3x=7$	19. $4x=15$
20. $5x=21$	21. $2x=\frac{1}{2}$	22. $3x=1\frac{1}{2}$
23. $\frac{1}{2}x=4$	24. $\frac{1}{3}x=5$	25. $\frac{1}{4}x=\frac{1}{2}$
26. $\frac{1}{2}x=\frac{2}{3}$	27. $\frac{1}{5}x=2\frac{1}{2}$	28. $\frac{1}{8}x=12\frac{1}{2}$
29. $\frac{2}{3}x=6$	30. $\frac{3}{4}x=9$	31. $\frac{4}{5}x=12$
32. $\frac{2}{3}x=7$	33. $\frac{3}{4}x=10$	34. $\frac{4}{5}x=15$
35. $\frac{3}{4}x=\frac{1}{2}$	36. $\frac{2}{3}x=\frac{3}{4}$	37. $\frac{4}{5}x=2\frac{1}{2}$
38. $\frac{4}{5}x=3\frac{1}{3}$	39. $\frac{4}{3}x=12$	40. $2\frac{1}{2}x=15$

41. If $4x-3=2x+9$, find the value of x.

$$4x-3=2x+9$$

Transposing, $4x-2x=9+3$

Uniting terms, $2x=12$

Dividing by 2, $x=6$

In each of the following find the value of x:

42. $4x-2=x+7$

43. $7x+2=5x+6$

44. $5x-1=3x+7$

45. $7x+8=4x+11$

46. $10x+5=4x+8$

47. $12x+1=9x+3\frac{1}{4}$

48. $x+4=\frac{1}{2}x+5$

49. $\frac{3}{4}x+2=\frac{1}{2}x+6$

50. $2\frac{1}{2}x-5=1\frac{1}{4}x+15$

Illustrative Example

The sum of two numbers is 25, and the larger is 4 times the smaller; what are the numbers?

Explanation:

Let x = the smaller number.

Then $4x$ = the larger number,

and $5x$ = the sum.

25 = the sum.

$\therefore\ 5x=25$.

$x=5$, the smaller.

$4x=20$, the larger.

Check: $20+5=25$; $20=4\times 5$.

Exercise 107.

1. The sum of two numbers is 30, and the larger is 4 times the smaller; what are the numbers?

2. The sum of two numbers is 15, and the smaller is $\frac{1}{2}$ of the larger; what are the numbers?

3. Two girls have 50¢; if one of them has 4 times as much as the other, how much has each?

4. A woman bought a certain number of yards of lining and twice as many yards of dress goods; if she bought 30 yd. of both, how many yards did she buy of each?

5. A and B together own 350 acres of woodland; if A owns 4 times as much as B, how much does each own?

6. An article was sold at a gain of \$12, which was $\frac{1}{4}$ of the cost; what was the cost?

7. An article was sold at a loss of \$2.50, which was $\frac{1}{5}$ of the cost; what was the cost?

8. A dealer sold an article for \$300, which was at a gain of $\frac{1}{2}$ of the cost; what was the cost?

9. An article was sold for \$75, which was at a loss of $\frac{1}{4}$ of the cost; what was the cost?

10. Two men together bought 65 head of cattle; if one of them bought 5 head more than the other, how many did each buy?

11. After a man had sold $\frac{1}{4}$ of his land to one man and $\frac{1}{5}$ of it to another, he had left 220 acres; how much had he at first?

PERCENTAGE

257. Per cent means *hundredths.*

Thus, 1 per cent of a number is .01, or $\frac{1}{100}$, of it.
10 per cent of a number is .10, or $\frac{10}{100}$, or $\frac{1}{10}$, of it.
$12\frac{1}{2}$ per cent of a number is .12$\frac{1}{2}$, or $\frac{12\frac{1}{2}}{100}$, or $\frac{1}{8}$, of it.

The sign % stands for per cent.

Thus, *6 per cent of 25* may be expressed *6 % of 25.*

258. The number or amount of which a given per cent is taken is called the **base.** The number of hundredths of the base taken is called the **rate per cent,** or the **rate,** and the result obtained by taking a given per cent of the base is called the **percentage.**

Exercise 108

What part of a number is:

1. 2 % of it?
2. 5 % of it?
3. 20 % of it?
4. 25 % of it?
5. 50 % of it?
6. 75 % of it?
7. 30 % of it?
8. 40 % of it?
9. 80 % of it?
10. 60 % of it?
11. 90 % of it?
12. 70 % of it?
13. $12\frac{1}{2}$ % of it?
14. $62\frac{1}{2}$ % of it?
15. $37\frac{1}{2}$ % of it?
16. $87\frac{1}{2}$ % of it?
17. $6\frac{1}{4}$ % of it?
18. $18\frac{3}{4}$ % of it?
19. $33\frac{1}{3}$ % of it?
20. $16\frac{2}{3}$ % of it?
21. $8\frac{1}{3}$ % of it?

How many times a number is:

22. 100 % of it? 23. 600 % of it? 24. 500 % of it?

25. 300 % of it? 26. 800 % of it? 27. 900 % of it?

28. 250 % of it? 29. 750 % of it? 30. 1000 % of it?

What per cent of a number is:

31. $\frac{1}{2}$ of it? Thus, $\frac{1}{2}$ of a number = $\frac{1}{2}$ of 100 % of it, or 50 % of it.

32. $\frac{1}{4}$ of it? 33. $\frac{1}{5}$ of it? 34. $\frac{1}{20}$ of it?

35. $\frac{3}{4}$ of it? 36. $\frac{4}{5}$ of it? 37. $\frac{2}{25}$ of it?

38. $\frac{2}{5}$ of it? 39. $\frac{7}{10}$ of it? 40. $\frac{7}{100}$ of it?

41. $\frac{1}{8}$ of it? 42. $\frac{3}{8}$ of it? 43. $\frac{5}{8}$ of it?

44. $\frac{7}{8}$ of it? 45. $\frac{1}{3}$ of it? 46. $\frac{2}{3}$ of it?

What per cent of a number is:

47. 2 times the number? 48. 6 times the number?

49. 5 times the number? 50. 8 times the number?

51. $2\frac{1}{2}$ times the number? 52. $1\frac{1}{2}$ times the number?

What per cent of a number is:

53. .2 of it? Answer thus: .2 of a number = .20 of it, or 20 % of it.

54. .7 of it? 55. .8 of it? 56. .5 of it?

57. .1 of it? 58. .9 of it? 59. .3 of it?

60. 1.2 times the number? 61. 2.56 times the number?

62. State instantly the per cent equivalent of each of the following:

.25;	.35;	.65;	.75;	.95;
.81;	.12;	.17;	.92;	.99;
1.50;	1.25;	2.05;	2.75;	3.45.

SUGGESTION. 1.50 is *one hundred fifty hundredths*, or 150 %.

7.1;	8.3;	2.4;	2.7;	3.2;
$.12\frac{1}{2}$;	$.37\frac{1}{2}$;	$.62\frac{1}{2}$;	$.87\frac{1}{2}$;	$.33\frac{1}{3}$;
$\frac{1}{2}$;	$\frac{1}{4}$;	$\frac{1}{5}$;	$\frac{1}{3}$;	$\frac{1}{8}$;
$1\frac{3}{4}$;	$1\frac{2}{5}$;	$1\frac{1}{2}$;	$1\frac{3}{5}$;	2.

What per. cent is:

63. 2 of 4?

Answer thus: $2 = \frac{1}{2}$ of 4, or 50% of 4.

64. 1 of 5?	65. 4 of 8?	66. 3 of 12?
67. 4 of 5?	68. 4 of 10?	69. 2 of 3?
70. 6 of 8?	71. 1 of 8?	72. 6 of 16?
73. 5 of 8?	74. 5 of 15?	75. 4 of 12?
76. 2 of 1?	77. 6 of 2?	78. 9 of 3?

TO THE TEACHER. Numerous simple exercises involving small numbers should be given in percentage, in order that pupils may become thoroughly grounded in reckonings involving per cent, before they are required to take up the more difficult problems of the subject or to study the applications of percentage. In the following exercise they should be able to name the result in each example instantly, and if they are not, they should receive more drill in these types of problems.

Exercise 109

Answer at sight:

1. What is 1% of $100?
2. What is 2% of $100?
3. What is $2\frac{1}{2}$% of $100?
4. 50% of a gallon is how many quarts?
5. 25% of a bushel is how many pecks?
6. How many are 5% of a score?
7. What is 10% of $1000?
8. $12\frac{1}{2}$% of a peck is how many quarts?
9. 25% of a day is how many hours?
10. 20% of a dime is how many cents?
11. 200% of one cent is what?
12. $37\frac{1}{2}$% of a pound (avoir.) is how many ounces?
13. 400% of $2 is what sum?
14. $66\frac{2}{3}$% of a yard is how many feet?
15. 100% of 2 quarts equals how many quarts?
16. 150% of 2 is what number?
17. 250% of 4 is what number?
18. $1 is 50% of what sum?
19. 5 yd. is 25% of what length?
20. 2 lb. is 20% of what weight?
21. 9 ft. is 300% of what length?
22. 25¢ is $33\frac{1}{3}$% of what sum?
23. $18 is $66\frac{2}{3}$% of what sum?

24. 5 min. is $12\frac{1}{2}\%$ of what time?

25. 80 words is 40% of how many words?

26. 15 lb. is 200% of what weight?

27. \$1.25 is $33\frac{1}{3}\%$ of what sum?

28. \$1.50 is 50% of what sum?

29. 6 lb. is $37\frac{1}{2}\%$ of what weight?

30. 14 is $87\frac{1}{2}\%$ of what number?

31. $\frac{1}{4}$ is 50% of what fraction?

32. What per cent is \$1 of \$100?

33. What per cent is \$2 of \$100?

34. What per cent is \$15 of \$100?

35. What per cent is \$7.50 of \$100?

SUGGESTION. The number of units estimated on 100 *of the same kind* is the rate; here 7.5 is the number estimated on 100.

36. What per cent is \$12.50 of \$100?

37. What per cent is \$1.25 of \$100?

38. What per cent is \$2 of \$200?

39. What per cent is \$0.50 of \$100?

40. What per cent is \$25 of \$200?

41. What per cent is 1¢ of 2¢?

42. What per cent is 5¢ of 2¢?

43. 1 qt. is what per cent of a peck?

44. 5 times any number is what per cent of it?

Illustrative Examples

1. Find $37\frac{1}{2}\%$ of $\$24.80$.

$$\begin{array}{r} \$24.80 \\ .375 \\ \hline 12400 \\ 17360 \\ 7440 \\ \hline \$9.30000 \end{array}$$

Explanation:

$37\frac{1}{2}\%$ of $\$24.80 = .375 \times \$24.80$, or $\$9.30$.

The result may also be found by taking $\frac{3}{8}$ of $\$24.80$.

2. $\$74.76$ is 42% of what sum?

$$\begin{array}{r} \$178 \\ .42\overline{)\$74.76} \\ 42 \\ \hline 327 \\ 294 \\ \hline 336 \\ 336 \\ \hline \end{array}$$

Explanation:

$.42 \times$ the sum $= \$74.76$.

$\therefore \$74.76 \div .42$, or $\$178 =$ the sum.

3. $\$240.20$ is $66\frac{2}{3}\%$ of what sum?

Explanation:

$66\frac{2}{3}\%$, or $\frac{2}{3}$, of the sum $= \$240.20$.

$\therefore \frac{1}{3}$ of the sum $= \frac{1}{2}$ of $\$240.20$, or $\$120.10$.

$\therefore \frac{3}{3}$ of the sum $= 3 \times \$120.10$, or $\$360.30$.

4. What per cent is $\$12.50$ of $\$200$?

Explanation:

$\$12.50$ is the same per cent of $\$200$ that $\$6.25$ is of $\$100$; that is, 6.25%.

5. What per cent is $\$200$ of $\$1600$?

Explanation:

$\$200$ is $\frac{1}{8}$ of $\$1600$.

$\therefore \frac{1}{8}$ of 100%, or $12\frac{1}{2}\%$ = the required rate.

6. What per cent is $\frac{3}{4}$ of $\frac{4}{5}$?

Explanation:

$\frac{3}{4} = \frac{15}{20}$; $\frac{4}{5} = \frac{16}{20}$.

$\therefore$ $\frac{3}{4}$ is $\frac{15}{16}$ of $\frac{4}{5}$.

$\therefore$ $\frac{15}{16}$ of 100 %, or $93\frac{3}{4}$ % = the required rate.

7. What per cent of \$ 56.875 is \$ 3, correct to the nearest .01 per cent?

Explanation:

$3 \div .56875 = 56875\overline{)300000.000}$ 5.274+

284375

etc.

1 % of \$ 56.875 = .01 of \$ 56.875 = \$ 0.56875.

$3 \div .56875 = 5.274+$.

$\therefore$ 5.27 % = the required rate.

NOTE. The nature of the problem should determine the method used in solving problems of percentage, especially those in which the rate per cent is to be found. Unless the relation between the numbers involved is a simple one, the method given in Example 7 will be found the most practical in finding the rate per cent. The equation having already been explained, those teachers desiring to use x and the equation to find the missing term will be able to do so.

Exercise 110

Find:

1. 6 % of 275.
2. 8 % of 390.
3. 12 % of 296.
4. $12\frac{1}{2}$ % of 650.
5. 15 % of \$ 785.
6. $12\frac{1}{2}$ % of \$ 460.
7. 36 % of 475 bu.
8. $66\frac{2}{3}$ % of 861 ft.
9. 75 % of \$ 3200.
10. $37\frac{1}{2}$ % of \$ 185.
11. $62\frac{1}{2}$ % of 864 pupils.
12. $16\frac{2}{3}$ % of \$ 600.
13. $6\frac{1}{4}$ % of 800 A.
14. 125 % of \$ 280.
15. 2.5 % of \$ 1000.
16. 4.7 % of \$ 2000.

Find the number or amount of which:

17. 54 is 12%.
18. 135 is 18%.
19. $175 is 20%.
20. $700 is 25%.
21. $240 is $33\frac{1}{3}$%.
22. $75 is $12\frac{1}{2}$%.
23. $240 is $66\frac{2}{3}$%.
24. 162 bu. is $37\frac{1}{2}$%.
25. 550 A. is $62\frac{1}{2}$%.
26. $210 is $87\frac{1}{2}$%.
27. 105 is 2.1%.
28. 288 is 4.8%.

What per cent is:

29. 90 of 360?
30. 126 of 280?
31. 63 of 350?
32. 150 of 750?
33. $130 of $390?
34. $68 of $425?
35. $494 of $741?
36. $324 of $5400?
37. 288 bu. of 3600 bu.?
38. $182 of $4550?
39. $95 of $4750?
40. $500 of $2000?

What per cent (correct to the nearest .01 per cent) is:

41. 4 of 15?
42. 7 of 75?
43. 14 of 115?
44. 35 of 237?
45. 89 of 589?
46. 97 of 971?
47. 128 of 4271?
48. 357 of 3764?
49. 25 of 5176?
50. 19 of 4768?

Illustrative Examples

1. An agent collected $87\frac{1}{2}\%$ of a debt of $832 and charged 5% of the sum collected; how much did he charge?

The answer may be found thus:

$832
.875
4160
5824
6656
$728.000
.05
$36.40 *Ans.*

Explanation:

$832 = the debt.

.875 × $832, or $728 = the amount collected.

.05 × the amount collected = the charges.

∴ .05 × $728, or $36.40 = the charges.

NOTE. $87\frac{1}{2}\%$ of $832 could also be found by taking $\frac{7}{8}$ of $832.

2. Milk yields butter to the amount of about 4% of its weight; what weight of milk will yield 240 lb. of butter?

Explanation:

.04 × the weight of the milk = the weight of the butter.

240 lb. = the weight of the butter.

∴ 240 lb. = .04 × the weight of the milk.

∴ 240 lb ÷ .04, or 6000 lb. = the required weight.

The answer may be found thus:

.04)240.00 lb.
6000 lb. *Ans.*

3. When an investment of $65.75 earns $4.50, what is the rate per cent of earning, correct to the nearest .01 %?

The answer may be found thus:

```
        6.844
.6575)4.50000000
      39450
       55500
       52600
        29000
        26300
         27000
         26300
```

6.84 % *Ans.*

Explanation:

$65.75 = the investment.
$4.50 = the earnings.
$0.6575 = the earnings if they were 1 % of the investment.
4.50 ÷ .6575 = 6.844.
∴ 6.84 % = the required rate.

Exercise 111

(Solve as many of these as possible orally.)

1. 80 % of a class of 40 pupils were promoted; how many were promoted?

2. A farmer sold 60 % of his crop of 40 tons of hay; how many tons did he sell?

3. How many words does a boy miss out of 50, if he spells 90 % of them correctly?

4. A family that uses 2 % of a ton (2000 lb.) of coal a day uses how many pounds a day?

5. What per cent should a boy receive who spells correctly 40 words out of the 50 given?

6. When cloth is advanced from $1 to $1.10 per yard, what is the per cent of advance?

7. If poultry, in dressing, lose 10% of their live weight, what should a dressed turkey weigh whose live weight is 15 lb.?

8. A boy who buys newspapers at $\frac{1}{2}$¢ each and sells them at 1¢ each is selling them at what per cent of cost?

9. A pupil who attends school 18 days out of 20 during a certain month should receive what per cent for attendance?

10. When $6 is paid for the use of $100, what rate per cent is paid?

11. $12\frac{1}{2}$% of a pole 24 ft. long is in the ground; how far is the top of the pole from the ground?

12. When 2 bu. of wheat, sowed, yield 30 bu., what is the per cent of yield?

13. How much did an agent collect during a certain month if he received 5% of the sum collected, and his earnings were $100?

14. After a farmer had sold 60% of his crop of corn he had remaining 280 bu.; what was the amount of his crop?

SUGGESTION. 280 bu. = 40% of the required crop.

15. A newsboy sold papers at 5¢ a copy, which was 200% of what he paid for them; what did he pay for them?

16. 30% of a class failed in an examination; if 14 of the class passed, how many were in the class?

17. How many words did a boy spell correctly if he missed 10 words and was given an average of 75%?

18. Calves, in dressing, lose about 40 % of their weight; what was the live weight of a calf whose dressed weight is 80 lb.?

19. When 10 lb. of green tea are mixed with 15 lb. of black tea, what per cent of the mixture is black tea and what per cent is green tea?

20. In a crate of eggs containing 6 doz., 9 eggs were found broken; what was the loss per cent from breakage?

21. All school buildings hereafter built or rebuilt in Pennsylvania must have the total light area equal to at least 20 % of the floor space. What is the least light area a school building may have whose floor space is 1200 sq. ft.?

22. If $37\frac{1}{2}$ % of the pupils in a school are boys and there are 40 girls in this school, how many pupils are there?

23. The standard gold coin of the United States is 9 parts pure gold and 1 part alloy. What per cent is alloy? What per cent is pure gold?

24. Pure gold is designated as 24 carats fine; what is the per cent of gold in jewelry that is 18 carats fine?

25. A man bought a horse for $275 and sold it for 20 % more than he paid for it; what was the selling price?

26. 10 % of all U. S. silver coin is copper alloy; find the weight of the copper and also of the silver in a silver dollar, the weight of which is 412.5 gr.

27. How much does a merchant lose of a bill of $275, if he is able to collect only 75 % of it?

28. Water expands 10 % of its volume in freezing; how many cubic feet of ice can be made from 100 cubic feet of water?

29. Wood ashes contain about 5 % of potash, 1.5 % of phosphoric acid, and 32.5 % of lime; how many pounds of each of these substances are in a ton of wood ashes?

30. If a miller takes 10 % of each bushel of corn ground for toll, and there is also a loss of about 1 % in grinding, how many pounds of feed should be returned to a farmer for 30 bu. (56 lb.) of corn?

31. Potatoes are about 75 % water, and of the remainder about 64.8 % is starch; how many pounds of starch are there in 200 lb. of potatoes?

32. After a piece of cloth had shrunk 5 % of its length, it measured 38 yd.; how many yards did it measure before shrinking?

33. After a laborer's wages had been increased 20 %, they were $54 a month; what were they before the increase?

34. A merchant's sales for a certain day were $714.35, which was 87½ % of his sales the preceding day. What were his sales for both days?

35. Corn is about 5.4 % fat; how many pounds of fat are there in 20 bu. (56 lb.) of corn?

36. A horse was sold for 12½ % less than it cost. If it was sold for $175, what was the cost?

37. Last year a farmer's wheat crop yielded 20 bu. to the acre. This year it yielded $21\frac{1}{2}$ bu. to the acre. What was the per cent of increase?

38. If a teacher's salary is advanced from \$60 to \$65 per month, what is the per cent of increase?

39. If 1 cu. ft. of cement be mixed with 2 cu. ft. of sand and 4 cu. ft. of broken stone to make concrete, what per cent of the mixture is cement? What per cent of the mixture is sand? What per cent of the mixture is broken stone?

40. In the 8th grade of the schools of a certain city 210 minutes are assigned per week to the subject of arithmetic. If the school is in session 1500 minutes per week, what per cent of the total time is assigned to arithmetic?

41. When tomatoes are advanced in price from 10¢ to 12¢ per can, what is the per cent of advance?

42. What is 7% of \$600?

SUGGESTION. Move the decimal point two places to the left, and multiply the result by 7.

43. The amount of land surface in New Jersey is 7455 sq. mi., and the amount of water surface 360 sq. mi.; what per cent of the area of the state is land and what per cent water? (Answer correct to the nearest .01%.)

44. On an investment of \$85.38 a man gained \$4.75; find the rate per cent of gain, correct to the nearest .01%.

45. In ten years, recently, the population of Oklahoma increased from 398,331 to 1,637,155; find the per cent of the increase, correct to the nearest .01%.

46. In ten years, recently, the population of Indianapolis increased from 169,164 to 233,650. Find the per cent of increase, correct to the nearest .1%.

47. In ten years, recently, the population of Lansing, Mich., increased from 16,485 to 31,489. What was the per cent of increase, correct to the nearest .1 % ?

48. During ten years, recently, the population of a certain township decreased from 875 to 854. What was the per cent of decrease, correct to the nearest .1 % ?

49. A long ton is how many per cent more than a short ton ?

50. A man's investment earned him, during a certain year, $135. This was 4.5 % of the investment; what was the investment ?

COMMERCIAL DISCOUNT

259. Commercial Discount is any deduction made from the *list price*, the *time price*, or the *marked price* of goods; the **net price** is the remainder.

260. If but one discount is made on any price, a certain per cent of that price is deducted; if two or more discounts are made, the first discount is reckoned on the price, the second on the remainder, the third on the next remainder, and so on.

Thus, if an article is marked $ 0.80 and sold at a discount of 25 %, the discount is 25 % of $ 0.80, or $ 0.20. If it is sold at a discount of 25 % and 10 %, the second discount is 10 % of $ 0.60, or $ 0.06, and the total discount is $ 0.26.

Illustrative Example

What is the net cost of a bill of goods listed at \$240 and bought at 20 % and $12\frac{1}{2}$ % off?

Explanation:

20 %, or $\frac{1}{5}$, of \$ 240, or \$ 48 = the first discount.
$\therefore$ \$ 240 − \$ 48, or \$ 192 = the first remainder.
$12\frac{1}{2}$ %, or $\frac{1}{8}$, of \$ 192, or \$ 24 = the second discount.
And, \$ 192 − \$ 24, or \$ 168 = the net cost.

261. Business men generally convert two successive rates of discount into an equivalent single rate before they reckon the discount; to do so they use the following:

A single rate of discount equivalent to two successive rates equals their sum diminished by $\frac{1}{100}$ of their product.

Thus, a single rate of discount equivalent to

$$20\ \% \text{ and } 10\ \% = 20\ \% + 10\ \% - \frac{20 \times 10}{100}\ \%$$
$$= 30\ \% - 2\ \%, \text{ or } 28\ \%.$$

Exercise 112

State at sight the missing discounts in Examples 1–10:

	Marked Price	Rate of Discount	Discount
1.	\$ 2.00	25 %	?
2.	\$ 1.60	25 %	?
3.	\$ 1.20	$33\frac{1}{3}$ %	?
4.	\$ 4.00	$12\frac{1}{2}$ %	?
5.	\$ 0.80	$37\frac{1}{2}$ %	?

	Marked Price	Rate of Discount	Discount
6.	$ 5.00	25 %	?
7.	$ 0.50	40 %	?
8.	$ 3.00	25 %	?
9.	$ 0.05	20 %	?
10.	$ 0.32	$6\frac{1}{4}$ %	?

11. On a bill of goods amounting to $ 410.20 a discount of 5 % was allowed. Find the cost.

12. Find the cost of a piano catalogued at $ 600, bought subject to a discount of 20 %.

13. A merchant reduced gingham aprons, marked 25 ¢, 40 %. What was then the asking price?

14. Find the asking price of a robe that was marked $ 4, but was reduced $18\frac{3}{4}$ %.

15. A dealer bought a bill of dry goods amounting to $ 275, subject to a discount of 20 % and 10 %. Find the cost.

16. What was paid for a bill of goods amounting to $ 250, and bought subject to a discount of 20 % and 5 %?

17. A wholesaler sold wall paper to a retailer, subject to a discount of 20 %, $12\frac{1}{2}$ %, and 5 %. The total bill amounted to $ 160. How much should the retailer pay?

18. March 1, a dealer bought a bill of goods amounting to $ 350, payable in 60 da., subject to a discount of 1 % if paid within 30 da., or 3 % if paid within 10 da. How much would settle this bill Apr. 30? Mar. 8? Mar. 28?

19. After deducting a discount of 20 %, a dealer sent $ 12.60 in payment of a bill of hardware. What was the bill?

SUGGESTION. 100 % of the bill − 20 % of the bill, or 80 % of the bill = $ 12.60.

20. A dealer paid $ 275.45 for a bill of shoes which he bought subject to a discount of $12\frac{1}{2}$ %. What was the bill?

21. Find the single rate of discount equivalent to 50% and 20%.

22. Find the single rate of discount equivalent to 30 % and 20 %.

23. Find the single rate of discount equivalent to 40 % and 10 %.

24. Find the single rate of discount equivalent to 20 % and 20 %.

25. What single rate of discount is equivalent to 10 % and 10 %?

26. What single rate of discount is equivalent to 20 % and 15 %?

27. What single rate of discount is equivalent to 40 %, 20 %, and 10 %?

Find the rate of discount on:

28. A china dinner set marked down from $ 25 to $ 17.

29. Vests marked down from $ 2 to $ 0.50.

30. Trousers marked down from $ 10 to $ 7.50 a pair.

31. Afternoon dresses marked down from $25 to $11.50.

32. Muffs marked down from $35 to $28.

33. Serge suit marked down from $30 to $18.

In marking goods, a merchant generally uses a *key* consisting of a word or words. He writes the asking price above a horizontal line and the cost below it. If *bankruptcy* is the key chosen, the letters *b-a-n-k-r-u-p-t-c-y*, in order, would stand for 1, 2, 3, 4, 5, 6, 7, 8, 9, 0.

Thus, if an article cost $1.20 and is marked at $1.48 it would be marked $\frac{\$1.48}{B^{ay}}$, if the dealer wishes to conceal only the cost, and $\frac{B^{kt}}{B^{ay}}$, if he wishes to conceal both the asking price and the cost.

34. Read the costs of the following goods, *bankruptcy* being used as the key word: $\overline{K^{yy}}$; $\overline{B^{ty}}$; $\overline{N^{pr}}$; $\overline{U^{cy}}$.

Mark each at an advance of 25 %.

PROFIT AND LOSS

262. The **cost** of an article is the amount paid for it. The **selling price** is the amount received for it. The **profit** is the amount that the selling price exceeds the cost and the expenses. The **loss** equals the amount that the cost and the expenses exceed the selling price. When there are no expenses, the profit equals the selling price less the cost, and the loss equals the cost less the selling price.

263. *Gain or loss per cent is always reckoned on the cost.*

Illustrative Examples

1. A dealer bought shoes at $2.50 a pair and sold them at a profit of 20 %; what was the selling price?

The answer may be found thus:

$2.50
.20
.50
2.50
$3.00 *Ans.*

Explanation:

$2.50 = cost.
20 %, or $\frac{1}{5}$, of the cost = gain.
$\frac{1}{5}$ of $2.50, or $0.50 = gain.
$2.50 + $0.50, or $3 = selling price (s. p.).

REMARK. The selling price may be found directly by taking $1.20 \times \$2.50$.

2. Coffee was sold at 40¢ a pound, which was at a profit of 25 %; find the cost.

The answer may be found thus:

1.25) $0.4000 | $0.32 *Ans.*
375
250
250

Explanation:

.25 × cost = gain.
∴ 1.25 × cost = selling price.
$0.40 = selling price.
∴ $0.40 = 1.25 × cost.
∴ $0.40 ÷ 1.25, or $0.32 = cost.

REMARK. Example 2 may also be explained as follows, this method being preferable in many examples:

.25, or $\frac{1}{4}$, of cost = gain.
∴ $\frac{5}{4}$ of cost = selling price.
40¢ = selling price.
∴ $\frac{5}{4}$ of cost = 40¢.
cost = 32¢.

3. What was the gain per cent on an office desk bought for $40 and sold for $55?

Explanation:

$55 = selling price.
$40 = cost.
∴ $15 = gain.
∴ $\frac{3}{8}$ of cost = gain.
∴ $\frac{3}{8}$ of 100 %, or $37\frac{1}{2}$ % = the rate.

Another Explanation:

$55 = selling price.
$40 = cost.
∴ $15 = gain.
.01 × $40 or $0.40 = gain at 1 %.
$15 \div .40 = 37\frac{1}{2}$.
∴ $37\frac{1}{2}$ % = the rate.

REMARK. The second explanation is preferable if the gain or loss does not bear some simple relation to the cost.

Exercise 113

TO THE TEACHER. Encourage pupils to solve orally as many examples as possible. Much drill should be given in exercises involving simple numbers.

1. Name at sight each missing gain or loss:

Cost	Gain Per Cent	Loss Per Cent	Gain	Loss
$25	20 %		?	
$6		25 %		?
$100	$12\frac{1}{2}$ %		?	
$200		1 %		?
$0.02	200 %		?	

2. Name at sight each missing rate per cent of gain:

Cost	Gain	Rate Per Cent of Gain
$0.05	$0.01	?
$0.01	$0.03	?
$1	$0.05	?
$100	$2.50	?
$1	$0.50	?

3. Name at sight each missing rate per cent of loss:

Cost	Selling Price	Rate Per Cent of Loss
$0.02	$0.01	?
$0.05	$0.01	?
$0.05	$0.04	?
$1	$0.25	?
$300	$100	?

Find the loss and the selling price:

4. If the cost is $10.50, and the rate of loss, $33\frac{1}{3}\%$.

5. If the cost is $2.40, and the rate of loss, $12\frac{1}{2}\%$.

6. If the cost is $0.75, and the rate of loss, 4%.

7. If the cost is $2000, and the rate of loss, $12\frac{1}{2}\%$.

Find the gain and the selling price:

8. If the cost is $3.60, and the rate of gain is $66\frac{2}{3}\%$

9. If the cost is $45, and the rate of gain is $2\frac{1}{2}\%$.

10. If the cost is $150, and the rate of gain is 75%.

11. If the cost is $2000, and the rate of gain is $7\frac{1}{2}\%$.

Find the cost and the gain or loss:

12. If the selling price is $60, and the rate of gain is 25%.

13. If the selling price is $72, and the rate of gain is $12\frac{1}{2}\%$.

14. If the selling price is $6.30, and the rate of gain is 200%.

15. If the selling price is $3.20, and the rate of loss is 20%.

16. If the selling price is $2.50, and the rate of loss is 50%.

Find the selling price when:

17. The gain is $2.10, and the rate of gain is 10%.

18. The gain is $6.10, and the rate of gain is $66\frac{2}{3}$%.

19. The loss is $3,60, and the rate of loss is $37\frac{1}{2}$%.

20. The loss is $4.80, and the rate of loss is 40%.

Find the rate per cent of gain or loss when the cost is:

21. $5, and the selling price is $7.

22. $75, and the selling price is $25.

23. $200, and the selling price is $205.

24. $400, and the selling price is $392.

25. $1000, and the selling price is $750.

26. $100, and the selling price is $112.50.

27. Find the gain per cent on goods listed at $5, bought at a discount of 20%, and sold at $4.80.

28. Find the gain per cent on goods listed at $4.50, bought at a discount of 20%, and sold for $4.20.

29. Find the gain per cent on goods listed at $0.40, bought at a discount of 10%, and sold for $0.48.

30. Find the gain per cent on goods listed at $25, bought at a discount of 25%, and sold for $20.

31. Find the gain on goods listed at $6, bought at a discount of 10%, and sold at a gain of 20%.

32. Find the gain on goods listed at $48, bought at a discount of $12\frac{1}{2}$%, and sold at a profit of 8%.

33. Find the gain on goods listed at $200, bought at a discount of 20% and 10%, and sold at a gain of 5%.

34. Find the loss on an article listed at $250, bought at a discount of 20% and $12\frac{1}{2}$%, and sold at a loss of 5%.

35. What is the gain per cent on goods that are sold for 3 times their cost?

36. What per cent is lost on goods that are sold for half their cost?

37. Find the gain per cent on 12 doz. spools of thread, bought for 50¢ per dozen spools, and sold for 5¢ per spool.

38. By selling a horse for $220, a dealer gained 10%; what was the gain?

39. Shoes were sold at $3 a pair, which was at a profit of 20%; what was their cost?

40. Crochet hooks are sold at 2¢ each, which is at a gain of 44%; what did they cost per gross?

41. Belts that cost $9 per dozen were retailed at a profit of 20%; what was the selling price of each belt?

42. A sweater that cost $1.50 was marked to gain $33\frac{1}{3}$%, but it was sold out of season at a discount of 25%; what was the gain or loss?

43. Serge dresses that cost $7.50 each were marked

to gain 20 %, but they were sold at a discount of $33\frac{1}{3}$ %; find the gain or loss per cent.

44. Men's silk shirts that cost \$ 2 each were marked to gain 75 %, but they were sold at a discount of 33 %; find the gain or loss per cent.

45. A dealer bought carpet at \$ 1.20 a yard, subject to a discount of $16\frac{2}{3}$ %, and sold it for \$ 1.50 a yard; find the gain per cent.

46. If chestnuts were bought for \$ 2 a bushel and retailed at 10 ¢ a quart, what was the gain per cent if they shrunk one quart before they were sold?

47. $33\frac{1}{3}$ % was gained by selling 20 doz. eggs for \$ 6.40; what did they cost a dozen?

48. A woman gains 50 % by selling 6 boxes of matches for 10 ¢; what per cent would she have gained by selling 2 boxes for 5 ¢?

49. The loss on an article sold for \$ 9 was 25 %; what would have been the gain per cent had it been sold for \$ 15?

50. A dealer bought English cheese at \$ 0.20 a pound; how many ounces should he give for 10 ¢ in order to gain 20 %?

51. A dealer bought oranges at 20 ¢ a dozen; what should he charge for three of them in order that his rate of gain may be 50 %?

52. A chiffonier was sold at 120 % of its cost; if the gain on it was \$ 8, what was the selling price?

53. A dealer bought hams at 15¢ a pound; at what price must he sell them to gain $33\frac{1}{3}$ %, if they lose 20 % of their weight in drying?

54. I invest a sum of money and sell the investment at a loss of 30 %; I invest the proceeds and sell at a gain of 50 %. Do I gain or lose, and what per cent of the original investment?

55. A dealer bought goods at a discount of 20 % and $12\frac{1}{2}$ % from the list price; he sold them at a discount of 16 % from the list price. Find his gain per cent.

56. A dealer marked a hat to gain 25 %, but he sold it for $1.75, which was at a loss of $12\frac{1}{2}$ %; find the marked price.

57. A dealer made a profit of $87\frac{1}{2}$ % by selling oranges at 30¢ a dozen. What per cent would he have gained had he sold them at 20¢ a dozen?

58. A dealer gained $33\frac{1}{3}$ % when he sold herring at $1 per hundred; what per cent did he gain on those that he sold at 15¢ per dozen?

59. What per cent profit does a dealer make on goods that he buys at 20 % and $12\frac{1}{2}$ % from the list price and sells at the list price?

60. Goods are marked to gain 20 %, but having become damaged, they are sold at a discount of 10 % from the marked price; find the gain per cent.

COMMISSION

264. An **agent** is a person who transacts business for another called the **principal.**

An agent that receives into his possession the property of the principal, and transacts in his own name the business relating to such property, is called a **commission merchant.** An agent that does business in the name of his principal, and without receiving the property of the principal into his possession, is called a **broker.**

265. Commission or **brokerage** is the sum charged by an agent for transacting business for a principal, and is usually a certain per cent of the amount involved in the transaction.

When a sale is made, commission is reckoned on the amount of the sales, and when a purchase is made, on the amount of the purchase.

266. A **consignment** is the goods sent by one person to another for sale. The party who sends the goods is the **consignor,** and the party who receives them is the **consignee.** The **net proceeds** of a consignment is the sum that is left after all expenses are paid.

Illustrative Examples

1. An agent sold fertilizer to the amount of $3450. He paid storage and other expenses amounting to $35; what were the net proceeds, if he deducted a commission of 1.5%?

The answer may be found thus:

$3450
.015
17250
3450
51.750
35
$86.75

Explanation:

$3450 = amount of sales.
.015 × $3450, or $51.75 = commission.
$35 = storage.
∴ $51.75 + $35, or $86.75 = total charges,
and $3450 − $86.75, or $3363.25 = net proceeds.

$3450 − $86.75 = $3363.25 *Ans.*

2. My agent purchased for me 500 bu. of corn at 75¢ per bushel, prepaid freight amounting to $25, and charged me a commission of 2%; find the entire cost of the corn.

The answer may be found thus:

$0.75
500
375.00
.02
7.50
25.00
375.00
$407.50 *Ans.*

Explanation:

500 × $0.75, or $375 = cost of the corn.
2% of $375, or $7.50 = commission.
$25 = freight charges.
∴ $407.50 = entire cost.

Exercise 114

In the following examples supply at sight the missing amounts:

	Sales	Rate of Commission	Commission	Net Proceeds
1.	$80	5%	?	?
2.	$100	$12\frac{1}{2}$%	?	?
3.	$40	$37\frac{1}{2}$%	?	?

	Sales	Rate of Commission	Commission	Net Proceeds
4.	$10	5%	?	?
5.	$500	1%	?	?
6.	$100	$\frac{1}{2}$%	?	?

In the following examples supply at sight the missing amounts:

	Amount of Purchase	Rate of Commission	Commission	Entire Cost
7.	$1	5%	?	?
8.	$100	$2\frac{1}{2}$%	?	?
9.	$40	$62\frac{1}{2}$%	?	?
10.	$5	100%	?	?
11.	$6.50	50%	?	?
12.	$30	8%	?	?

In the following examples supply at sight the missing quantity or per cent:

	Amount of Sales	Rate of Commission	Commission	Other Expenses	Net Proceeds
13.	$50	10%	?	$5	?
14.	$100	?	?	$5	$80
15.	?	5%	$50	$100	?
16.	$400	?	$50	$50	?
17.	?	10%	?	$30	$150
18.	?	5%	$20	?	$370

In the following examples supply the missing terms:

	Amount of Purchase	Rate of Commission	Commission	Other Expenses	Entire Cost
19.	$100	12½ %	?	$5	?
20.	?	5 %	$20	$10	?
21.	$200	?	?	$10	$226
22.	?	4 %	?	$30	$550

23. An agent was paid 12½ % commission on his sales and $50 a month. How much did he earn during a certain month in which his sales amounted to $1280?

24. A commission merchant sold 650 lb. of dressed turkey at $0.18 a pound. He charged a commission of 5 %, and his other expenses amounted to $2.50. Find the net proceeds of the sale.

25. Find the entire cost of 1020 bu. of wheat at $0.92 a bushel, if the purchasing agent charged a commission of 2 % and the freight charges amounted to $48.50.

26. A commission merchant sold 650 doz. eggs at $0.32 a dozen. After deducting a commission of 4 % and other expenses amounting to $1.75, how much should he remit to his principal?

27. How much money should I send my agent to pay for 50 T. of bran at $22 a ton, and pay his commission amounting to 2½ %?

28. An agent sold a consignment of 8 wagons at $85 each. He charged a commission of 15 %, and paid delivery charges amounting to $12.50. Find the net proceeds.

29. A real estate agent sold a tract of 216 A. of land at $40 an acre. He charged the owner a commission of 2 %, and the transfer cost $35.50 additional; how much should be paid to the owner?

30. My agent purchased oats for me at $0.50 a bushel, charging a commission of 2 %; how many bushels did he buy if the total cost was $510?

SUGGESTION. Find the commission on the cost of 1 bu.

31. An administrator is usually paid a commission of 5 % on all personal property sold, and $2\frac{1}{2}$ % on all real estate; what commission should be paid an administrator of an estate the personal property of which sold for $625, and the real estate for $3020?

32. Find the commission charged on a bill of goods the entire cost of which was $220, if the rate of commission was 10 %.

SUGGESTION. 10 % of the cost of goods = commission.
∴ 110 % of the cost of the goods = the entire cost, or $220.

33. An agent received $2000 with which to buy flour at $4.50 a barrel. If he charged a commission of 5 %, how many barrels did he buy and what was the surplus?

34. A lawyer collected for his client 80 % of a debt of $650; if the lawyer charged a commission of 25 %, how much should he pay over to his client?

35. A dealer in farm implements sent his principal $578 as the net proceeds of a sale after he had deducted his commission of $102. What rate of commission was charged?

36. I sent my agent $512.50 with which to purchase leaf tobacco after deducting his commission of $2\frac{1}{2}\%$; how much commission should he deduct?

37. The entire cost of a carload of 100 sheep, including the agent's commission and $150 freight charges, was $1500. The sheep cost $12 a head; what rate of commission was charged?

38. A lawyer charged me a commission of 10 % for collecting a debt. If he remitted to me $585, what commission did he charge?

39. I paid $483 for 500 bu. of wheat purchased through an agent at $0.92 a bushel. What rate of commission did the agent charge?

40. The cost of a bill of goods including a commission of $\frac{3}{4}\%$ and $50.70 other expenses was $1300; find the commission charged.

SIMPLE INTEREST

267. Interest is money paid for the use of money.

268. The **principal** is the money for the use of which interest is paid.

269. The **rate of interest** is the rate per cent of the principal paid as interest for a certain time, usually a year.

REMARK. The rate of interest is understood to mean the rate per cent of the principal paid as interest for *one year*, unless otherwise specified.

Thus, the interest on $200 at 5 % means that 5 % of $200 is one year's interest; but the interest on $200 at 2 % per quarter means of $200 is 3 months' interest.

270. The **amount** is the sum of the principal and the interest.

271. **Simple interest** is interest charged on the principal only.

Each state has established a **legal rate** of interest for the settlement of interest accounts when no rate of interest has been agreed upon by the parties concerned. Many of the states have also a **maximum contract rate.**

272. **Usury** is the amount of interest charged above what is legally allowed; it is unlawful interest, and there are various penalties in the different states for receiving it.

273. The time for which interest is paid is usually reckoned in *years and days when it exceeds a year, and in days only when it is less than a year, the exact number of days being counted.*

Thus, if a sum of money is on interest from April 1, 1913, to June 20, 1914, the time is counted as follows: from April 1, 1913, to April 1, 1914, is 1 yr., and from April 1, 1914, to June 20, 1914, is 80 da., making in all 1 yr. and 80 da. Again, if a sum of money is on interest from March 1, 1913, to July 1, 1913, the time is 122 da.

REMARK. Settlements of interest accounts are usually made at intervals of not more than a year, so that in practice it seldom becomes necessary to reckon interest on a sum of money for a time greater than a year.

274. Interest for a time less than a year is usually reckoned on a basis of 360 days to a year; that is, 12 months of 30 days each, in which case the interest is called **common interest.**

275. Interest for a time less than a year reckoned on a basis of 365 days to the year is called **exact interest.**

NOTE. In this book, unless otherwise specified, interest will be reckoned on the basis of 30 da. to the month; that is, 1 da. will be considered $\frac{1}{360}$ of a year.

276. The following table shows the number of days from any day of one month to the same day of any other month in the same year except when Feb. 29 intervenes:

Table

FROM ANY DAY OF	TO THE SAME DAY OF											
	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
January . .	365	31	59	90	120	151	181	212	243	273	304	334
February . .	334	365	28	59	89	120	150	181	212	242	273	303
March . . .	306	337	365	31	61	92	122	153	184	214	245	275
April	275	306	334	365	30	61	91	122	153	183	214	244
May	245	276	304	335	365	31	61	92	123	153	184	214
June	214	245	273	304	334	365	30	61	92	122	153	183
July	184	215	243	274	304	335	365	31	62	92	123	153
August . . .	153	184	212	243	273	304	334	365	31	61	92	122
September .	122	153	181	212	242	273	303	334	365	30	61	91
October . .	92	123	151	182	212	243	273	304	335	365	31	61
November .	61	92	120	151	181	212	242	273	304	334	365	30
December .	31	62	90	121	151	182	212	243	274	304	335	365

Exercise 115

State at sight how much is the interest on:

1. \$100 for 1 yr. at 5%.
2. \$100 for 2 yr. at 6%.
3. \$200 for 3 yr. at $4\frac{1}{2}$%.

4. $200 for $1\frac{1}{2}$ yr. at 6%.
5. $400 for 6 mo. at 4%.
6. $600 for 1 yr. 6 mo. at 6%.
7. $1000 for 9 mo. at 4%.
8. $200 for 3 mo. at $4\frac{1}{2}$%.
9. $1000 for 2 yr. 3 mo. at 6%.
10. $100 for 2 yr. at 3% semiannually.
11. $200 for 1 yr. 6 mo. at 2% semiannually.
12. $500 for 9 mo. at 1% quarterly.
13. $600 for 1 yr. 3 mo. at 2% quarterly.
14. $1000 for 1 yr. 9 mo. at 1% quarterly.

Methods in Use Explained and Applied

The General Method

Illustrative Example

Find the interest on $360 from Jan. 6, 1913, to Sept. 3, 1915, at 6%.

(1) *Solution with the time reckoned in years and days.*

$360	The time from Jan. 6, 1913, to Jan. 6, 1915 = 2 yr.
.06	The time from Jan. 6, 1915, to Sept. 3, 1915 =
$21.60	(25 + 28 + 31 + 30 + 31 + 30 + 31 + 31 + 3) da.,
$2\frac{2}{3}$	or 240 da.; the principal is on interest 2 yr. 240
43.20	da., or $2\frac{2}{3}$ yr.
14.40	.06 × $360, or $21.60 = the interest for 1 yr.
$57.60	$2\frac{2}{3}$ × $21.60, or $57.60 = the required interest.

(2) *Solution with the time reckoned in years, months, and days.*

	yr.	mo.	da.	
\$ 360				
.06	1915	9	3	
\$ 21.60	1913	1	6	
$2\frac{237}{360}$	2	7	27	= 2 yr. 237 da., or $2\frac{237}{360}$ yr.
43.20				.06 × \$ 360, or \$ 21.60 = the interest for 1 yr.
14.22				$2\frac{237}{360}$ × \$ 21.60, or \$ 57.42 = the required interest.
\$ 57.42				

The Six Per Cent Method

277. (1) \$ 0.06 = the interest on \$ 1 for 1 yr. (12 mo.) at 6 %.

(2) \$ $0.00\frac{1}{2}$ = the interest on \$ 1 for 1 mo. (30 da.) at 6 %.

(3) \$ $0.000\frac{1}{6}$ = the interest on \$ 1 for 1 da. at 6 %.

Observe that in (1), (2), and (3) the principal is \$ 1, and the rate is 6 %; that in (2) $\frac{1}{2}$ of the number of months is the number of cents in the interest; that in (3) $\frac{1}{6}$ of the number of days is the number of mills in the interest. Therefore:

278. To find the interest on \$1 at 6 %, reckon \$0.06 for each year in the time; \$0.00$\frac{1}{2}$ for each month in the time; \$0.000$\frac{1}{6}$ for each day in the time.

Illustrative Example

Find the interest on \$ 360 from Jan. 6, 1913, to Sept. 3, 1915, at 6 %.

(1) *Solution with the time reckoned in years and days.*

The time from Jan. 6, 1913, to Sept. 3, 1915, is 2 yr. 240 da.

\$ 0.12	= the interest on \$ 1 for 2 yr. at 6 %.
0.04	= the interest on \$ 1 for 240 da. at 6 %.
\$ 0.16	= the interest on \$ 1 for 2 yr. 240 da. at 6 %.
360	
\$ 57.60	= the required interest.

(2) *Solution with the time reckoned in years, months, and days.*

yr.	mo.	da.	
1915	9	3	
1913	1	6	
2	7	27	= the required time.

$0.12 = the interest on $1 for 2 yr. at 6 %.
$0.035 = the interest on $1 for 7 mo. at 6 %.
$0.0045 = the interest on $1 for 27 da. at 6 %.
$0.1595 = the interest on $1 for the required time at 6 %.

360
95700
4785
$57.42 = the required interest.

To find the interest at:

2 %, take $\frac{1}{3}$ of the interest at 6 %.

3 %, take $\frac{1}{2}$ of the interest at 6 %.

4 %, take $\frac{2}{3}$ of the interest at 6 % (subtract $\frac{1}{3}$ of the interest at 6 %).

4$\frac{1}{2}$ %, take $\frac{3}{4}$ of the interest at 6 % (subtract $\frac{1}{4}$ of the interest at 6 %).

5 %, take $\frac{5}{6}$ of the interest at 6 % (subtract $\frac{1}{6}$ of the interest at 6 %).

7 %, take $\frac{7}{6}$ of the interest at 6 % (add $\frac{1}{6}$ of the interest at 6 %).

7$\frac{1}{2}$ %, take $\frac{5}{4}$ of the interest at 6 % (add $\frac{1}{4}$ of the interest at 6 %).

The 60-Day Method

279. (1) .06 of the principal = the interest for 1 yr. (360 da.) at 6 %.

(2) .01 of the principal = the interest for 60 da. at 6 %.

(3) .001 of the principal = the interest for 6 da. at 6 %.

From (2) and (3) we have the following:

280. To find the interest on any principal at 6%, reckon .01 of the principal as the interest for each 60 da. in the time; and .001 of the principal as the interest for each 6 da. in the time.

Illustrative Example

Find the interest on $360 from Jan. 6, 1913, to Sept. 3, 1915, at 6 %.

(1) *Solution with the time reckoned in years and days.*

The time from Jan. 6, 1913, to Sept. 3, 1915, is 2 yr. 240 da.

$3.60 = the interest for 60 da. at 6 %.

16

2160

360

$57.60 = the interest for 2 yr. 240 da. (16 × 60 da.).

(2) *Solution with the time reckoned in years, months, and days.*

yr.	mo.	da.	
1915	9	3	
1913	1	6	
2	7	27	= the required time.

$3.60 = the interest for 60 da. at 6 %.

15

1800

360

$54.00 = the interest for 2 yr. 6 mo. (15 × 60 da.).

1.80 = the interest for 1 mo. ($\frac{1}{2}$ of 60 da.).

1.44 = the interest for 24 da. (4 × 6 da.).

0.18 = the interest for 3 da. ($\frac{1}{2}$ of 6 da.).

$57.42 = the interest for 2 yr. 7 mo. 27 da. at 6 %.

REMARK. Observe that by each of the three methods explained the interest is $57.60, when the time is counted in years and days; and $57.42 when it is counted in years, months, and days.

Method of Exact Interest

281. Exact interest is used by the United States Government, by some state governments, and by many trust companies.

282. As exact interest is based on a reckoning of 365 da. to a year, neither the six per cent method nor the 60-day method can be employed in finding the exact interest, as they are based on a reckoning of 360 da. to a year.

Illustrative Example

Find the interest on \$240 from June 1 to Dec. 3 of the same year, at 6 %.

Explanation:

The time from June 1 to Dec. 3,

$= (29 + 31 + 31 + 30 + 31 + 30 + 3)$ da. $= 185$ da., or $\frac{37}{73}$ yr.

\$240
.06
\$14.40 = the interest for 1 yr.

37
10080
4320
73)\$532.800(\$7.298+ = the interest for 185 da.

That is, \$7.30 = the interest, correct to the nearest \$0.01.

Exercise 116

NOTE. Teachers who desire to do so may use only one method of reckoning common interest throughout this exercise. They should employ the method in use in their state and count time in accordance with the practice prevailing therein. In this book the time is generally reckoned in years and days.

Find by the general method the interest on:

1. \$ 378 from May 1 to Dec. 31, at 4 %.
2. \$ 200 from Feb. 1, 1913, to June 20, 1915, at 6 %.
3. \$ 1000 from Sept. 1, 1913, to Dec. 1, 1914, at $4\frac{1}{2}$ %.
4. \$ 60.25 from July 15, 1913, to Jan. 5, 1915, at $5\frac{1}{2}$ %.
5. \$ 230 from Apr. 1, 1913, to Mar. 1, 1914, at $4\frac{1}{2}$ %.
6. \$ 278 from July 8, 1913, to Oct. 6, 1914, at 3 % semiannually.

SUGGESTION. The time is 2 half years, 90 da., or $2\frac{1}{2}$ semiannual periods.

7. \$ 500 from July 15, 1913, to Aug. 20, 1914, at $2\frac{1}{2}$ % semiannually.
8. \$ 200 from May 1 to Dec. 18, at $1\frac{1}{2}$ % per quarter.
9. \$ 285 from June 1, 1913, to Dec. 31, 1913, at $4\frac{1}{2}$ %.
10. \$ 1000 from June 1, 1913, to Sept. 15, 1914, at $2\frac{1}{2}$ % semiannually.

Find by the six per cent method the interest on:

11. \$ 450 from Mar. 15 to Dec. 31, at 6 %.
12. \$ 1000 from Mar. 1, 1913, to Oct. 16, 1914, at 6 %.
13. \$ 1250 from July 1 to Dec. 31, at $4\frac{1}{2}$ %.
14. \$ 2000 from Feb. 15, 1913, to Apr. 1, 1915, at $4\frac{1}{2}$ %.
15. \$ 25 from May 1 to Dec. 15, at $4\frac{1}{2}$ %.
16. \$ 600 from May 1, 1913, to Oct. 31, 1914, at 5 %.
17. \$ 275 from Apr. 1, 1913, to July 1, 1914, at $4\frac{1}{2}$ %.
18. \$ 480 from Jan. 15, 1916, to Dec. 1, 1916, at $4\frac{1}{2}$ %.

Find by 60-day method the interest and amount of:

19. $350 for 60 da. at 6%.

20. $480 for 120 da. at 6%.

21. $85.60 for 90 da. at 6%.

22. $1000 for 6 da. at 6%.

23. $250 for 12 da. at 6%.

24. $680 for 30 da. at 6%.

25. $478 for 90 da. at 6%.

26. $340 for 66 da. at 6%.

27. $350 for 300 da. at 6%.

28. $278 for 240 da. at 4%.

29. $39.60 for 300 da. at $4\frac{1}{2}$%.

30. $82.94 for 126 da. at 3%.

31. $100 for 200 da. at 4%.

32. $98 from Oct. 1, 1913, to May 24, 1914, at 6%.

33. $207.85 from Dec. 12, 1914, to Sept. 5, 1915, at $3\frac{3}{4}$%.

Find the exact interest on:

34. $300 for 73 da. at 6%.

35. $300 for 150 da. at 6%.

36. $375 from June 1 to Dec. 20, at 5%.

37. $850 from May 1 to Dec. 31, at $4\frac{1}{2}$%.

38. $85 from June 1, 1913, to July 15, 1914, at $4\frac{1}{2}$%.

Indirect Problems in Simple Interest

Finding the Principal

Illustrative Example

1. What principal will amount to $595.20 in 2 yr. 200 da. at 4%?

$$\begin{array}{r} \$0.04 \\ \underline{\quad 2\tfrac{5}{9} \quad} \\ \$0.10\tfrac{2}{9} \\ \quad 1.00 \\ \underline{\$1.10\tfrac{2}{9})\$595.20} \\ \quad 9 \qquad\quad 9 \\ \underline{\$9.92)\$5356.80(540} \end{array}$$

Explanation:

$0.04 = the int. on $1 for 1 yr.

2 yr. 200 da., or $2\frac{5}{9}$ yr. = the time.

$\therefore 2\frac{5}{9} \times \$0.04$, or $\$0.10\frac{2}{9}$ = the interest on $1 for the time.

And $\$1.10\frac{2}{9}$ = the amount of $1 for the time.

$595.20 = the entire amount.

$\therefore \$595.20 \div \$1.10\frac{2}{9}$, or 540 = the number of dollars in the principal.

$\therefore \$540$ = the principal.

Exercise 117

1. What principal will produce $24 interest in 2 yr. at 6%?

2. What principal will produce $6 interest in 6 mo. at 4%?

3. What principal will produce $25 interest in 1 yr. 3 mo. at 4%?

4. What principal will amount to $336 in 3 yr. at 4%?

5. What principal will amount to $132 in 2 yr. 6 mo. at 4%?

6. What principal will earn $13.50 interest from Mar. 15 to Dec. 10, at 4%?

7. What principal will earn $15 interest from Apr. 1, 1913, to July 30, 1914, at 5%?

8. What principal will amount to $300 from June 1, 1912, to Dec. 31, 1913, at 6%?

9. What principal will amount to $1000 from May 1 to Dec. 2, at 6% exact interest?

283. The **present worth** of a debt due at some future time is such a sum as placed at interest will amount to the debt when due. The **true discount** is the difference between the debt and the present worth.

284. *To find the present worth of a debt is to find the principal.*

Thus, the illustrative example on page 312 might be stated thus: Find the *present worth* of $595.20 due 2 yr. 200 da. hence at 4%. The debt would then be $595.20, the present worth $540, and the true discount $595.20 − $540, or $55.20.

Finding the Rate

Illustrative Example

At what rate will $300 earn $24 in 2 yr. 60 da.?

Explanation:

.01 of $300, or $3 = the interest for 1 yr. at 1%.

2 yr. 60 da., or $2\frac{1}{6}$ yr. = the time.

$\therefore 2\frac{1}{6} \times \$3$, or $6.50 = the interest for the time at 1%.

$24 = the given interest.

$24 \div 6.50 = 3.692^{+}$.

$\therefore$ 3.69% = the required rate, correct to the nearest .01%.

$$
\begin{array}{r}
\$300 \\
.01 \\
\hline
\$3.00 \\
2\frac{1}{6} \\
\hline
\$6.50)\$24.00(3.692^{+}
\end{array}
$$

Exercise 118

At what rate will:

1. $320 earn $17.28 in 1 yr. 72 da.?

2. $1000 earn $138 in 2 yr. 315 da.?

3. $456 earn $4.56 from Aug. 1 to Oct. 12?

4. $100 amount to $104.60 from July 20, 1913, to Sept. 12, 1914?

SUGGESTION. $104.60 − $100, or $4.60 = the interest.

5. $425 amount to $446.25 from Jan. 1, 1914, to Apr. 1, 1915?

6. $400 amount to $416.80 from Feb. 8 to Dec. 20?

7. $500 amount to $508 at exact interest from Oct. 1 to Feb. 24?

8. Any sum double itself in 20 yr.?

SUGGESTION. 100 % of principal = the interest.
20 % of principal = the interest for 20 yr. at 1 %.

9. Any sum double itself in 15 yr.?

Finding the Time

Illustrative Example

In what time will $400 amount to $438.40 at 4%?

$400 $438.40
.04 400.00
$16.00) $38.400(2.4

Explanation:

.04 of $400, or $16 = interest for 1 yr.
$438.40 − $400, or $38.40 = the entire interest.
∴ 38.40 ÷ 16, or 2.4 = the number of years.
.4 yr. = .4 of 360 da., or 144 da.
∴ 2 yr. 144 da. = the required time.

Exercise 119

In what time will:

1. \$ 500 earn \$ 72 interest at $4\frac{1}{2}\%$?

2. \$ 1000 earn \$ 138 interest at $4\frac{4}{5}\%$?

3. \$ 200 amount to \$ 208 at 4% ?

4. \$ 425 amount to \$ 446.25 at 4% ?

5. \$ 250 amount to \$ 300 at 6% ?

6. \$ 200 amount to \$ 300 at $4\frac{1}{2}\%$?

7. On what date will \$ 200 have amounted to \$ 208.10, if loaned Jan. 1 at 6% ?

8. On what date will \$ 200 have amounted to \$ 203.50, if loaned Mar. 1 at $4\frac{1}{2}\%$?

9. In what time will any sum double itself at 4% ?

SUGGESTION. 100% of the principal = the interest.
4% of the principal = the interest for 1 yr. at 4%.

10. In what time will any sum double itself at 5% ?

11. In what time will any sum earn $\frac{1}{2}$ of itself at $4\frac{1}{2}\%$?

COMPOUND INTEREST

285. If interest is added to the principal at the end of each period as it becomes due, to form a new principal for the next period, the entire interest, that is, the difference between the final amount and the original principal, is the **compound interest** of the original principal.

286. The **compound amount** is the final amount when compound interest is reckoned.

287. Table of Compound Interest

AMOUNT OF $1 FOR EACH OF 20 PERIODS AT RATES FROM ¾% TO 6%

PERIODS	¾ PER CENT	1 PER CENT	1¼ PER CENT	1½ PER CENT	2 PER CENT
1	1.00750000	1.01000000	1.01250000	1.015000	1.020000
2	1.01505625	1.02010000	1.02515625	1.030225	1.040400
3	1.02266917	1.03030100	1.03797070	1.045678	1.061208
4	1.03033919	1.04060401	1.05094533	1.061364	1.082432
5	1.03806673	1.05101005	1.06408215	1.077284	1.104081
6	1.04585223	1.06152015	1.07738318	1.093443	1.126162
7	1.05369612	1.07213535	1.09085047	1.109845	1.148686
8	1.06159884	1.08285670	1.10448610	1.126493	1.171660
9	1.06956083	1.09368527	1.11829217	1.143390	1.195093
10	1.07758254	1.10462212	1.13227082	1.160541	1.218994
11	1.08566441	1.11566834	1.14642421	1.177949	1.243374
12	1.09380689	1.12682503	1.16075451	1.195618	1.268242
13	1.10301044	1.13809328	1.17526389	1.213552	1.293607
14	1.11027552	1.14947421	1.18995469	1.231756	1.319479
15	1.11860259	1.16096895	1.20482913	1.250232	1.345868
16	1.12699211	1.17257864	1.21988949	1.268985	1.372786
17	1.13544455	1.18430443	1.23513811	1.288020	1.400241
18	1.14396038	1.19614747	1.25047734	1.307341	1.428246
19	1.15254009	1.20810895	1.26610830	1.326951	1.456811
20	1.16118414	1.22019003	1.28193466	1.346855	1.485947

Table of Compound Interest — *Continued*

PERIODS	2½ PER CENT	3 PER CENT	3½ PER CENT	4 PER CENT	5 PER CENT	6 PER CENT
1	1.025000	1.030000	1.035000	1.040000	1.050000	1.060000
2	1.050625	1.060900	1.071225	1.081600	1.102500	1.123600
3	1.076891	1.092727	1.108718	1.124864	1.157625	1.191016
4	1.103813	1.125509	1.147523	1.169859	1.215506	1.262477
5	1.131408	1.159274	1.187686	1.216653	1.276282	1.338226
6	1.159693	1.194052	1.229255	1.265319	1.340096	1.418519
7	1.188686	1.229874	1.272279	1.315932	1.407100	1.503630
8	1.218403	1.266770	1.316809	1.368569	1.477454	1.593848
9	1.248863	1.304773	1.362897	1.423312	1.551328	1.689479
10	1.280085	1.343916	1.410599	1.480244	1.628895	1.790848
11	1.312087	1.384234	1.459970	1.539454	1.710339	1.898299
12	1.344889	1.425761	1.511069	1.601032	1.795856	2.012197
13	1.378511	1.468534	1.563956	1.665074	1.885649	2.132928
14	1.412774	1.512590	1.618695	1.731676	1.979932	2.260904
15	1.448298	1.557967	1.675349	1.800944	2.078928	2.396558
16	1.484506	1.604706	1.733986	1.872981	2.182875	2.540352
17	1.521618	1.652848	1.794676	1.947901	2.292018	2.692773
18	1.559659	1.702433	1.857489	2.025817	2.406619	2.854339
19	1.598650	1.753506	1.922501	2.106849	2.526950	3.025600
20	1.638616	1.806111	1.989789	2.191123	2.653298	3.207136

Exercise 120

[In solving the examples of this exercise use table, page 316.]

1. Find the compound amount and the compound interest of $600 for 4 yr. at 3%.

SUGGESTION. $1.125509 = the compound amount of $1 for 4 yr. at 3%.

2. Find the compound amount and the compound interest of $500 for 2 yr. 6 mo. at 6%, interest payable semiannually.

SUGGESTION. 2 yr. 6 mo. = 5 semiannual periods. The semiannual rate = 3%. Find by the table the compound amount of $1 for 5 periods at 3%.

3. Find the compound amount and the compound interest of $500 for 2 yr. at 5%, interest payable quarterly.

SUGGESTION. 2 yr. = 8 quarterly periods. The quarterly rate is $1\frac{1}{4}$%.

Find the compound amount and the compound interest of:

4. $400 for 3 yr. at 6%.
5. $300 for 4 yr. at 4%.
6. $250 for 5 yr. at $3\frac{1}{2}$%.
7. $3000 for 6 yr. at $2\frac{1}{2}$%.
8. $2000 for 3 yr. 6 mo. at 5%, payable semiannually.
9. $750 for 2 yr. 6 mo. at 5%, payable semiannually.
10. $2000 for 2 yr. 3 mo. at 8%, payable quarterly.

TAXES

288. A **tax** is money paid by the people for the expenses of the government.

289. **Local taxes** are those paid to counties, townships, cities, or towns, such as school taxes or road taxes.

290. **State taxes** are paid to a state for its expenses; but some states divide a part of the state taxes among the counties, townships, cities, and towns.

291. **National taxes** are paid to the treasurer of the United States at Washington to maintain the army and navy, and to pay pensions and other expenses of the national government.

292. A **poll tax** (head tax) is a low tax collected in some states from every male citizen over twenty-one years of age.

293. An **assessor** is an officer who assesses, that is, places a value upon property for taxation. This assessment, or tax value, is often less than the real value of the property.

294. A **tax rate** is usually a certain number of mills on each dollar of the assessed value of property.

295. A tax rate is sometimes expressed as a certain per cent of the assessed value of property, or as a certain amount on each $100 of its assessed valuation.

296. Most local and state taxes are paid by the owners of property at a rate or percentage on the value of their property. The tax rate is so fixed by the state or local officers as to raise the amount of money needed by the state or local governments.

297. National taxes are of two classes, **tariff** or **duties,** which are laid upon articles imported from other countries; and **internal revenue taxes,** which are levied upon a few things made in this country, usually upon alcoholic liquors and tobacco.

298. Duties on imports and internal revenue taxes are **indirect taxes,** because the importer or manufacturer who pays them adds that to the cost of the article imported or made, so that the people who buy and use these articles pay these taxes *indirectly.*

299. **Direct taxes** are taxes which cannot be shifted upon somebody else. Local taxes and state taxes are almost always direct taxes.

300. Duties are of two kinds: **specific duties,** which are those paid on the *quantity* of the imports; and **ad valorem duties,** which are those paid on the *value* of the imports.

301. Some goods are imported free of duty, some are subject to specific duty only, some to ad valorem duty only, and some to both kinds of duty.

Illustrative Example

In a town whose property is assessed at $350,000 and in which there are 420 persons who pay a poll tax of $1.50 each, $1855 is to be raised by taxation. What rate of taxation must be levied, and what tax must George Fox pay on property assessed at $7500?

Explanation:

$420 \times \$1.50$, or $\$630$ = the poll tax.
$\$1855 - \$630$, or $\$1225$ = the property tax.
$\$1225 \div 350{,}000$, or $\$0.0035$ = the tax on $\$1$.
$\therefore$ 3.5 mills = the tax rate.
$7500 \times \$0.0035$, or $\$26.25$ = George Fox's property tax.

Exercise 121

Find the tax on property the tax rate and valuation of which are as follows:

	Valuation	Tax rate		Valuation	Tax rate
1.	$1000	2 mills	2.	$5000	3 mills
3.	$2700	5 mills	4.	$7500	7 mills
5.	$2250	8 mills	6.	$8750	$4\frac{1}{2}$ mills
7.	$1400	12 mills	8.	$7000	$8\frac{1}{2}$ mills
9.	$8500	$12\frac{1}{2}$ mills	10.	$10,000	$1\frac{1}{2}$ mills

In the following find the tax rate expressed in mills on each dollar of valuation:

	Valuation	Tax		Valuation	Tax
11.	$1000	$1	12.	$10,000	$60
13.	$2000	$8	14.	$2500	$40
15.	$1500	$7.50	16.	$1400	$12.60
17.	$2400	$14.40	18.	$2200	$17.60
19.	$25,000	$40	20.	$5000	$7.50

Find the duty on the following:

	Imports	Duty
21.	$600 worth of cut glassware	60% ad valorem
22.	300 cu. ft. of marble	65¢ per cubic foot
23.	$250 worth of sponges	20% ad valorem
24.	4 T. (2240 lb.) maple sugar	4¢ per pound
25.	1200 bu. oats	15¢ per bushel
26.	A $5000 painting	15% ad valorem

27. Find the duty on 420 lb. of drugs valued at $1.20 per pound, on which there is a duty of $1\frac{1}{4}¢$ per pound and 10 % ad valorem.

28. Find the duty on 420 yd. of carpet, $\frac{3}{4}$ yd. wide and valued at $1.20 per yard, on which there is a duty of $10¢$ per square foot and 40 % ad valorem.

29. Find a farmer's total taxes for a year on 75 A. of land assessed at $20 per acre, if the rate of county tax levied is $2\frac{1}{2}$ mills, the rate of school tax $7\frac{1}{2}$ mills, and the rate of road tax 5 mills.

30. The assessed valuation of the property of a certain town is $480,000. If the school tax amounting to $2160 is to be raised, what must be the rate of taxation?

31. The assessed valuation of the property of a certain town is $420,000, upon which a tax of $3150 is to be raised; what must be the rate of taxation, and how much tax must B. F. Hess pay on property assessed at $7500?

32. When the tax rate is 12 mills, and a man pays a tax of $37.50 including $1.50 poll tax, what is the assessed value of his property?

33. A tax of $17,700 is to be raised in a county whose property is assessed at $3,650,000 and which has 850 polls. If the polls are assessed at $1.50 each, what must be the rate of taxation?

34. A tax collector paid over to the school authorities of a certain township $7095.20 school tax, after he had taken out his commission of 2 % for collecting; what amount of tax did he collect, and what was his commission?

35. Find the duty on an importation of goods from Paris, dutiable value 1850 francs, at 60 % ad valorem.

NOTE. All values of imported merchandise are reduced to U. S. money on the basis of the exact money equivalents given in the table under Section 244, before the duty is reckoned. Amounts less than 50 ¢ are rejected, and those equal to or greater than 50 ¢ are considered one dollar in the total sum upon which duty is charged.

36. Find the duty on a shipment of German toys, dutiable value 18,750 marks, at 55 % ad valorem.

37. Find the duty on 6 cwt. 3 qr. 20 lb. of wool, whose dutiable value is 30 cents per pound, and on which is charged a specific duty of $27\frac{1}{2}$ ¢ per pound and 40 % ad valorem.

NOTE. See table, page 489, note 2.

PROPERTY INSURANCE

302. Property insurance is a contract by which, for a consideration, one party agrees to indemnify another in case of loss.

303. Insuring property is called **taking a risk.** The party taking the risk, which is usually a company, is called the **insurer,** or **underwriter,** and the party whose property is insured is called the **insured.** Risks are usually taken for periods of one, two, three, four, or five years, or for an unlimited number of years, in which case the risk is called a **perpetual risk.**

304. The written contract between the insured and the insurance company is called the **policy.** The amount of

indemnity named in the **policy** is called the **face of the policy.** The amount charged for insuring property is called the **premium.**

305. The **annual rate of premium** is the number of cents charged for insuring property to the amount of one hundred dollars for one year.

306. Insurance rates are sometimes expressed in per cent. Rates for 2 yr., 3 yr., 4 yr., and 5 yr. are generally $1\frac{3}{4}$, $2\frac{1}{2}$, $3\frac{1}{4}$, and 4 times the annual rate.

307. Short rates are rates for less than a period, and are usually expressed as certain per cents of the periodic rates. Short rates on perpetual risks are usually 10 per cent of the premium paid on such risks.

308. A policy may be cancelled at any time at the request of the insured, in which case the company retains the customary short rate for the time the policy has been in force.

309. Property insurance is of various kinds, such as **fire insurance,** which is insurance against loss by fire; **marine insurance,** which is insurance against loss of property at sea; **hail insurance,** which is insurance against loss of property by hail.

Exercise 122

1. A barn was insured for \$ 2800 at 40¢ per \$ 100 for 1 year; find the premium.

2. A house was insured for \$ 2400 at 20¢ per \$ 100 for 1 year; find the premium.

3. A house was insured for \$ 2200 for 3 years, the rate for 3 years being $2\frac{1}{2}$ times the annual rate; what was the premium, if the annual rate was $12\frac{1}{2}$¢ per \$100?

4. A house was insured for $3200 for 4 years, the rate for 4 years being $3\frac{1}{4}$ times the annual rate; what was the premium if the annual rate was 16¢ per $100?

5. A company that charges annually 75¢ per $1000 for insuring brick houses should charge how much for insuring for a period of 5 years a brick house valued at $5000, the rate for 5 years being 4 times the annual rate?

6. A man had his library insured for each of 5 years at 40¢ per $100 per year; how much would he have saved had he had it insured at first for a period of 5 years, the insured value being $2000 and the rate for 5 years being 4 times the annual rate?

7. A man had a quantity of wheat insured for $1000 for 1 year, but at the end of 60 days he sold the wheat and cancelled the policy; how much should the company return, if the annual premium charged was 85¢ per $100 and the company retains as the short rate for 60 days 30% of the annual premium?

8. A company that is charging 16¢ per $100 insurance for 1 year is charging what rate per cent? What rate per $1000 is it charging?

9. If a building is insured for $4500 and the annual premium is $18, find the annual rate per $100.

10. A church is insured for $10,000 on the perpetual plan at $4 per $100; what premium was paid and how much would the insurance company retain if the policy should be cancelled, in which case the company would retain 10% of the premium paid?

LIFE INSURANCE

310. Life insurance is a contract whereby a life insurance company agrees to pay, in consideration of a certain payment or payments, a sum of money to a certain party called the *beneficiary.*

The beneficiary may be the insured, his estate, or any other party named in the policy.

Life insurance policies are payable at the death of the insured or at the expiration of a certain period.

311. There are three general kinds of life insurance policies issued, and several modifications of each; namely, *Ordinary Life, Limited Payment Life,* and *Endowment.*

312. Ordinary life policies provide for the payment of the insurance at the death of the insured, and require premiums to be paid during the entire life.

313. Limited payment policies provide for the payment of the insurance at the death of the insured or for the cancelling of the policy at the end of the insurance period in accordance with a plan or plans provided for in the policy, on condition that the insured outlives the insurance period.

314. Endowment policies provide for the payment of the insurance at the end of a stated period, should the insured live so long; or for the payment thereof at the death of the insured, if it happens within the endowment period. As in the "Limited Life," the premiums cease at the end of a stipulated period, usually in 10, 15, 20, 25, or 30 years.

315. Life insurance premiums are given in dollars and cents per $1000 insurance for one year, and are uniform throughout the insurance period.

The age of the insured is his age at his nearest birthday.

316. The following table shows the annual premiums charged by a leading company for $ 1000 insurance by the three plans in general use.

Age	Ordinary Life	Limited Payment Life			Endowment		
		10-Payment	15-Payment	20-Payment	20-Year	25-Year	30-Year
21	18.76	46.98	34.73	28.73	48.45	38.03	31.36
22	19.18	47.69	35.26	29.18	48.54	38.14	31.48
23	19.61	48.43	35.82	29.65	48.64	38.25	31.62
24	20.07	49.19	36.39	30.13	48.75	38.37	31.76
25	20.55	49.97	36.98	30.63	48.86	38.50	31.92
26	21.05	50.78	37.60	31.15	48.98	38.65	32.09
27	21.57	51.62	38.23	31.69	49.12	38.80	32.27
28	22.12	52.49	38.89	32.25	49.26	38.96	32.48
29	22.70	53.39	39.57	32.83	49.41	39.15	32.70
30	23.31	54.31	40.28	33.44	49.57	39.34	32.94
35	26.88	59.46	44.23	36.85	50.64	40.64	34.56
40	31.56	65.58	49.03	41.10	52.33	42.76	37.18
45	37.82	72.94	54.99	46.57	55.15	46.25	41.45
50	46.36	81.96	62.62	53.86	59.87	52.02	48.28

Exercise 123

[In answering questions in this exercise refer to table, Sec. 316.]

Find the annual premium on a $ 1000 policy, when the age of the insured and the kind of policy are as follows:

Age of Insured	Kind of Policy
1. 21	10-Payment Life
2. 28	20-Year Endowment
3. 30	Ordinary Life

Age of Insured	Kind of Policy
4. 40	15-Payment Life
5. 28	25-Year Endowment
6. 25	Ordinary Life
7. 30	20-Payment Life
8. 25	20-Payment Life
9. 22	30-Year Endowment

10. Find the annual premium on a $2000 ordinary life policy taken out when the insured was 30 years old.

11. Find the annual premium on a $5000 twenty-year endowment policy taken out when the insured was 29 years old.

12. Find the annual premium that a man must pay on a twenty-year endowment policy for $5000 taken out Dec. 1, 1912, if the insured was born July 15, 1882.

REVIEW

Exercise 124

1. When spices are bought at $1.20 per pound and sold at 15¢ per ounce, what is the gain per cent?

2. A retailer buys oilcloth at 75¢ per square yard, subject to a discount of $16\frac{2}{3}\%$ and 4%, and sells it at 80¢ per square yard; find his gain per cent.

3. An agent charged $12.40 for selling a consignment of poultry at a commission of 4%; how much should he return to the consignor after taking out his commission and paying storage amounting to $4.80?

4. A dealer marks goods at 50¢ per yard, but he throws off 10% and still makes a profit of 50%; find the cost of the goods per yard.

5. A tradesman's prices are 25% above cost; if he allows a discount of $2 on a bill of $20, find his per cent of profit.

6. A dealer marked an article at 12¢, which was 50% more than the cost; if he sold it at a discount of 2¢, what was his gain per cent?

7. When an article is bought at a discount of 25% from the marked price and sold at the marked price, what is the gain per cent?

8. A chafing dish was marked at $5, which was at an advance of 56¼% of the cost. If it was sold for $3.60, find the gain or loss per cent.

9. What inheritance tax must be paid in Michigan on $7500 which a man left to his wife and children, on which legacy there is a tax of 1%; and on $2000 left to a charitable institution, on which legacy there is a tax of 5%?

10. When shoes marked at $3.50 per pair are bought at a discount of 20% and sold at the marked price, what is the gain per cent?

11. When 40¢ interest is charged per month for the use of $100, what is the annual rate?

12. A bushel of clover seed (60 lb.) costing $9 was found to be 25% weed seed; what price was really paid per bushel for the pure clover seed?

13. If the weight of a dressed turkey is 75 % of its live weight, which is the more profitable to the farmer, to sell a 16-lb. turkey, live weight, at 16 ¢ per pound, or at 25 ¢ per pound dressed?

14. What rate per cent of tax is levied when $1.40 is levied to the $100?

15. If a piece of cloth 6 yd. long and 54 in. wide shrunk 16⅔ % in length and in width on being sponged, how many square yards did the piece then contain?

16. Find the duty on a painting whose dutiable value is 60,000 lire, at 15 % ad valorem.

17. What must the marked price of shirts be apiece that cost $7.20 per dozen, so that a dealer may allow a discount of 25 % and still make a profit of 25 %?

18. A piece of gold jewelry that is 18 carats fine is what per cent pure gold?

19. What per cent does a dealer gain on nails that he buys at $3.20 per keg (100 lb.) and retails at 5 ¢ per pound, if they lose 2 lb. in the retailing?

20. Find the exact interest on $250 from June 1 to December 15 at 4.5 %.

21. How much will a man pay in premiums who takes out at the age of 28 yr. a 20-year endowment policy for $5000, provided that he lives out the endowment period? (See table, page 326.)

22. A farmer calls his four cows Red, Spot, Bess, and Beauty. Red gives 22 lb. of milk daily which tests 4.2 %;

Spot gives 24 lb. of milk daily which tests 4.1 %; Bess gives 28 lb. of milk daily which tests 3.5 %; Beauty gives 30 lb. of milk daily which tests 3.0 %. With butter fat selling at 36¢ per pound, what is the income from each cow during the month of June? Which is the most profitable cow for the farmer to keep, provided that he feeds them equal amounts?

23. A barrel of 42 gal. of vinegar was bought for $4.20; what must it be retailed for per gallon to gain 20 %?

24. What is the simple interest on $200 from June 1, 1912, to Oct. 15, 1913, at 4.5 %?

25. When buttons are bought at 60¢ per gross and sold at 10¢ per dozen, what is the gain per cent?

26. What per cent does a merchant gain who buys coal at $6 per ton of 2240 lb. and retails it at $7.50 per ton of 2000 lb.?

27. A man borrowed $50 from a loan company in Pennsylvania March 1, and paid back $55 August 1; how much of this interest was usury? (See table, page 369.)

28. When goods are bought at a discount of 20 % and 10 % from the marked price and sold at 8 % from the marked price, what is the gain per cent?

29. I invest and sell at a loss of $37\frac{1}{2}$ %; I invest the proceeds and sell at a gain of 80 %; did I gain or lose, and what per cent?

30. If a mail clerk in distributing 12,000 pieces of mail matter makes 7 errors, what is the per cent of error, correct to the nearest .01 %?

31. How much fire tax should be paid on an insurance of $3250, when the rate is 20¢ per $100 of insurance?

32. The assessed valuation, in a recent year, of the taxable property of Denver, Col., was $132,909,000, which was 50% of its actual value. The tax rate levied for the year was $3.50 per $100 of assessed value. What amount of tax was levied, and what was the actual value of the taxable property?

33. Find the commission charged on a bill of goods the entire cost of which was $230, if the rate of commission was 10 % and freight charges amounted to $10.

34. A dealer gained $33\frac{1}{3}$ % when he sold bananas at 20¢ per dozen; what per cent did he gain on those that he sold at 2¢ each?

35. After deducting a commission of $\frac{3}{4}$ % and $25.80 for other expenses, an agent sent his principal $4500 as the net proceeds of a sale; find the commission charged.

36. When the duty on goods of a certain kind is 10¢ per square foot and 40 % ad valorem, find the duty on a piece of these goods 6 yd. long and 4 yd. wide and valued at $120.

37. A laborer saved $152.50 in three years; his savings the second year were 25 % more than those of the first, and those of the third, 25 % more than those of the second; how much did he save each year?

STOCKS AND BONDS

317. A **corporation** is a body doing business as a single person under a general law or a special charter.

318. The capital of a corporation is called **stock.** The stock of a corporation is divided into a number of equal parts called **shares.** The face or original value of a share is its **par value.**

319. The par value of a share of stock is usually $50 or $100. Unless a different par value is expressed, $100 is the par value used in the examples of this book.

A Certificate of Stock

320. A **certificate of stock** is a document specifying the number of shares to which the holder is entitled, and the par value of each.

321. The price at which stock sells in the market is the **market value.**

322. The market price of stock is usually quoted at a certain per cent of its par value, but sometimes the market price of a share is quoted.

Thus, on a certain day, Reading R.R. stock, par $ 50, was quoted in the New York market at 170¼, which means that it was sold on that day at 170¼ % of $ 50, or $ 85⅛ per share. The same stock selling at the same price was quoted in the Philadelphia market at 85⅛, meaning that it was sold at $ 85⅛ per share.

323. A **dividend** is that part of the profits of a corporation which is divided among the stockholders. It is usually a certain per cent of the par value of the stock.

Thus, if a 6 % dividend is declared on stock, par value $ 50, the owners of the stock are paid 6 % of $ 50, or $ 3, on each share owned.

324. **Preferred stock** is stock on which dividends, at a fixed rate, are payable before those on the **original** or **common stock** are paid.

325. A **bond** is a written obligation of a corporation or a government to pay a specified sum of money at a certain time, together with interest at regular intervals at a fixed rate per cent per annum.

326. Government bonds are of two classes, **coupon** and **registered.**

327. A **coupon bond** is a bond made payable to the bearer, and, when sold, passes by delivery without assignment. It has attached to it interest coupons payable to the bearer

STATE OF PENNSYLVANIA
COUNTY OF CHESTER
Borough of West Chester

GENERAL LOAN BOND

SERIES TWO

This is to Certify, That the Borough of West Chester, County of Chester, and State of Pennsylvania, is indebted to bearer, or if registered to the registered holder hereof, in the sum of Five Hundred Dollars, payable in lawful money of the United States, on the **First Day of January, A D. 1942,** at the office of the Treasurer of the Borough of West Chester, in said Borough, upon surrender hereof, with interest thereon at the rate of four per centum per annum, payable semi-annually, upon the first days of January and July, until said principal sum is paid at the Treasurer's office upon the presentation and surrender of the annexed coupon, free from all taxes levied or to be levied upon this bond under any law of the State of Pennsylvania, now enacted or hereafter to be enacted, and for the payment of the said principal and interest the faith, honor and credit of the said Borough of West Chester are solemnly pledged.

This obligation is one of a series, of one hundred and sixty obligations, of like tenor and date, of an authorized loan of Sixty Thousand Dollars. Bonds numbered from one to one hundred and ten inclusive, being for the sum of Five Hundred Dollars each, and bonds numbered from one hundred and eleven to one hundred and sixty inclusive, being for the sum of One Hundred Dollars each, issued in pursuance of an Ordinance duly enacted by the Council of said Borough and approved by the Burgess thereof on the 30th day of August, A.D. 1912, signifying the desire of the Borough of West Chester to increase its indebtedness by the sum of Sixty Thousand Dollars, and a Resolution of said Council likewise approved, passed on the 27th day of September, A.D. 1912, providing for the holding of an election at the time of the holding of the general election in said Borough on the Fifth day of November, A.D. 1912, for the purpose of obtaining the consent of the qualified electors of the Borough, to such increase of indebtedness, and another Resolution of said Council likewise duly approved, passed on the 27th day of November, A.D. 1912, after the holding of said election at which a majority of said electors consented to the increase of said indebtedness; said indebtedness being incurred for the purpose of providing funds for the improvement of the equipment of the fire department, for the extension and improvement of the water system, and for street improvements and the removal of garbage. All of said bonds being valid and binding obligations of the said Borough and in the issuing thereof all requirements of law have been complied with.

The said Borough of West Chester reserves the right to redeem all bonds of said issue, maturing at and after ten years from date at par, at any interest paying period, all accrued interest having been paid thereon, upon giving personal notice to the holder four weeks prior to the time of redemption, if the bond is registered, and if not registered, either by personal notice as aforesaid, or by an advertisement inserted once a week for four successive weeks, immediately prior to the date of said redemption, in a newspaper of general circulation published in said Borough, having respect to the lowest number in both denominations, in numerical order in making such redemption.

This bond shall pass by delivery, but any holder may have the principal thereof registered in the Office of the Treasurer of said Borough, and after registration no transfer except upon the books of the Borough shall be valid unless the last preceeding transfer shall have been restored, but no registry shall restrain the negotiability of the coupon by delivery merely.

IN WITNESS WHEREOF, **the said "BOROUGH OF WEST CHESTER, COUNTY OF CHESTER, PENNSYLVANIA," has caused these presents to be sealed with its corporate seal attested by its Secretary, and its Chief Burgess has hereunto set his hand this First day of January, A.D. 1913.**

Borough of West Chester, County of Chester, Pennsylvania.

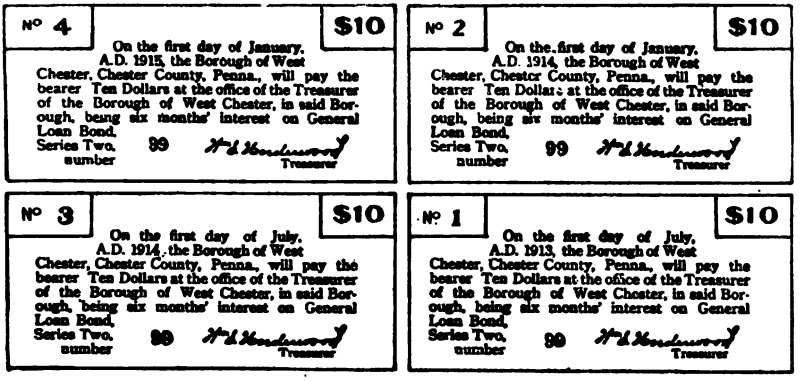
Clemmer E. Jefferis
Chief Burgess

Attest.

Wm S Underwood
Secretary

A Coupon Bond

at specified times. When interest is due, a coupon is cut off and presented to a bank for payment.


No 4
$10
On the first day of January, A.D. 1915, the Borough of West Chester, Chester County, Penna., will pay the bearer Ten Dollars at the office of the Treasurer of the Borough of West Chester, in said Borough, being six months' interest on General Loan Bond, Series Two, number 89
Treasurer
No 2
$10
On the first day of January, A.D. 1914, the Borough of West Chester, Chester County, Penna., will pay the bearer Ten Dollars at the office of the Treasurer of the Borough of West Chester, in said Borough, being six months' interest on General Loan Bond, Series Two, number 99
Treasurer
No 3
$10
On the first day of July, A.D. 1914, the Borough of West Chester, Chester County, Penna., will pay the bearer Ten Dollars at the office of the Treasurer of the Borough of West Chester, in said Borough, being six months' interest on General Loan Bond, Series Two, number 99
Treasurer
No 1
$10
On the first day of July, A.D. 1913, the Borough of West Chester, Chester County, Penna., will pay the bearer Ten Dollars at the office of the Treasurer of the Borough of West Chester, in said Borough, being six months' interest on General Loan Bond, Series Two, number 99
Treasurer

Interest Coupons

328. A **registered bond** is a bond issued without interest coupons, filled up in the name of the registered owner, made payable to him or his assigns, and registered in his name.

NOTE. Interest on registered bonds is sent to the registered owner or to his attorney as it becomes due.

329. Bonds may sell above or below their face value, and are usually quoted in the market at a certain per cent of their face value. Bonds are usually named from the rate of interest they pay and the year in which they are payable.

Thus, "U. S. 4s reg., 1925" means U. S. Government bonds registered, bearing 4 % interest and payable in 1925.

330. Brokerage is the commission charged by a broker for buying or selling stocks or bonds for another person. It is reckoned on the par value, and the usual rate is $\frac{1}{8}$ %.

331. Stock is usually bought and sold in a place called the stock exchange, which is merely an auction room where brokers having stocks to sell endeavor to dispose of them to other brokers having orders to buy such stocks. $\frac{1}{8}\%$ of the par value is usually charged by the broker selling the share and by the broker buying the share.

Illustrative Examples

1. Find the cost of 10 shares of Penn. R.R. stock ($50) at $61\frac{5}{8}$ a share, brokerage $\frac{1}{8}\%$.

Explanation:

$50 = the par value of 1 share.
$\frac{1}{8}\%$ of $50, or $0.0625 = the brokerage on 1 share.
$61.625 = the market value of 1 share.
$61.6875 = the entire cost of 1 share.
$\therefore$ 10 × $61.6875, or $616.88 = the entire cost of 10 shares.

2. Find the proceeds of 8 shares of Reading R.R. stock ($50) quoted at $171\frac{1}{4}\%$, brokerage $\frac{1}{8}\%$.

Explanation:

$50 = the par value of 1 share.
$171\frac{1}{4}\%$ of $50, or $85.625 = the market value of 1 share.
$\frac{1}{8}\%$ of $50, or $0.0625 = the brokerage on 1 share.
$85.5625 = the net proceeds of 1 share.
$\therefore$ 8 × $85.5625, or $684.50 = the net proceeds of 8 shares.

3. A sent his broker $6000 to be invested in Southern Pacific R.R. stock at $111\frac{1}{8}\%$, brokerage $\frac{1}{8}\%$. How many shares did this sum buy, and what was the surplus?

Explanation:

$100 = the par value of 1 share.
$\frac{1}{8}$ % of $100, or $0.125 = the brokerage on 1 share.
$111.125 = the market value of 1 share.
$\therefore$ $111.25 = the entire cost of 1 share.
$6000 = the amount A sent for investment.
$\therefore$ $6000 $\div$ $111.25, or 53 = the number of shares bought, with a surplus of $103.75.

4. Find the annual income on 25 shares of United Gas Improvement stock ($ 50), which pays a quarterly dividend of 2 %.

Explanation:

$50 = the par value of 1 share.
2 % of $50, or $1 = the quarterly dividend on 1 share.
$4 = the annual income on 1 share.
$\therefore$ 25 $\times$ $4, or $100 = the annual income on 25 shares.

5. What rate per cent of income from his investment does a man receive on 6 % stock, par value $ 50, if he purchased it when its market value was $ $56\frac{3}{4}$ per share, brokerage $\frac{1}{4}$ %? Answer correct to the nearest .01 per cent.

Explanation:

6 % of $50, or $3 = the income per share.
$\frac{1}{4}$ % of $50, or $0.125 = the brokerage per share.
$56.75 + $0.125, or $56.875 = the cost per share.
$3 = the income on $56.875 invested.
$0.56875 = the income per share if the rate were 1%.
3 $\div$ .56875 = 56875)300000.000(5.274 +.
284375
etc.
$\therefore$ 5.27 % = the required rate.

6. What must be the market value of 5 % stock in order that it may yield 4 % on the investment, brokerage $\frac{1}{8}$ % ?

Explanation:

\$100 = the par value of 1 share.
.05 × \$100, or \$5 = the annual dividend on 1 share.
4 % of the investment = the annual dividend.
∴ 4 % of the investment = \$5.
100 % of the investment = \$125.
\$0.125 = the brokerage on 1 share.
∴ \$125 − \$0.125, or \$124.875 = the market value of 1 share.

Exercise 125

1. Find the cost of 20 shares of Chesapeake & Ohio R.R. stock at $81\frac{1}{2}$ %, brokerage $\frac{1}{8}$ %.

2. Find the cost of 25 shares of Illinois Central R.R. stock at $127\frac{1}{4}$ %, brokerage $\frac{1}{8}$ %.

3. Find the cost of 8 shares of Lehigh Coal & Navigation Co. stock (\$ 50) at \$ $87\frac{1}{2}$ a share, brokerage $\frac{1}{4}$ %.

4. Find the cost of 200 shares of United States Steel stock at \$ 68 a share, brokerage $\frac{1}{8}$ %.

5. Find the cost of 120 shares of Lehigh Valley R.R. stock (\$ 50) at 175 %, brokerage $\frac{1}{8}$ %.

6. Find the proceeds from the sale of 200 shares of Canadian Pacific R.R. stock at $226\frac{1}{2}$ %, brokerage $\frac{1}{8}$ %.

7. Find the proceeds of 20 shares of Cambria Iron Co. stock (\$ 50) at \$ $44\frac{1}{4}$ per share, brokerage $\frac{1}{8}$ %.

8. Find the proceeds of 12 shares of Consolidated Coal stock at $106\frac{1}{4}$%, brokerage $\frac{1}{8}$ %.

9. Find the proceeds of 125 shares of mining stock, par \$ 1 per share, if its market value is \$ $10\frac{1}{2}$ per share and the rate of brokerage $\frac{1}{8}$ %.

10. Find the proceeds of 300 shares of F. W. Woolworth Co. stock (\$ 10) at $115\frac{1}{4}$ %, brokerage $\frac{1}{8}$ %.

11. Find the cost of five \$ 100 U. S. bonds at $101\frac{1}{2}$ %, brokerage $\frac{1}{8}$ %.

12. Find the cost of four \$ 50 U. S. bonds at $103\frac{1}{4}$ %, brokerage $\frac{1}{8}$ %.

13. Find the proceeds from the sale of \$ 10,000 Baldwin Locomotive Co. bonds at $102\frac{3}{4}$ %, brokerage $\frac{1}{8}$ %.

14. Find the proceeds from the sale of \$ 6000 Northern Pacific R.R. bonds at $98\frac{1}{2}$ %, brokerage $\frac{1}{8}$ %.

15. How many shares of bank stock at 112 % can be bought for \$ 1121.25, brokerage $\frac{1}{8}$ %?

16. How many shares of stock (\$ 50) at $109\frac{1}{2}$ % can be bought for \$ 3000, brokerage $\frac{1}{8}$ %, and what will be the surplus?

17. How many shares of stock (\$ 50) at \$ $81\frac{1}{2}$ per share can be bought for \$ 1750, brokerage $\frac{1}{4}$ %, and what will be the surplus?

18. How many shares of Penn. R.R. stock (\$ 50) at \$ $61\frac{5}{8}$ per share can be bought for \$ 1200, and what will be the surplus, brokerage $\frac{1}{8}$ %?

19. How many \$ 100 U. S. bonds at $113\frac{3}{4}$ %, brokerage $\frac{1}{8}$ %, can be bought for \$ 10,000, and what will be the surplus?

20. How many \$1000 U. S. bonds at $102\frac{1}{2}\%$ can be bought for \$12,315, brokerage $\frac{1}{8}\%$?

21. How many shares of stock (\$50) at $\$62\frac{1}{2}$ per share, brokerage $\frac{1}{4}\%$, must be sold to realize \$998?

22. I wish to sell sufficient U. S. 2s at $101\frac{1}{2}\%$, brokerage $\frac{1}{8}\%$, to pay a debt of \$3800; how many \$100 bonds must I sell, and what will be the surplus?

23. Find my annual income from 10 shares of Boston & Maine R.R. stock, the semiannual dividend being 6 %.

24. Find the annual income from 50 shares of Penn. R.R. stock (\$50), which pays a quarterly dividend of $1\frac{1}{2}\%$.

25. Find the annual income from 20 shares of Denver & Northwestern Railway Co. stock, which pays a quarterly dividend of 2 %.

26. Find the annual income from 25 shares of Utah Copper Co. stock (\$10), during a year in which it paid quarterly dividends of $7\frac{1}{2}\%$.

27. Find the annual income from 700 shares of mining stock, par \$1 per share, if it pays a quarterly dividend of 25 %.

28. Find the annual income from 20 shares of Standard Oil of N. J. stock, which pays a quarterly dividend of 5%.

29. When mining stock, par \$1 per share, pays 40% quarterly dividends, what income does a man receive each year from 40 shares of this stock?

30. How much is gained by buying 25 shares of Amalgamated Copper Co. stock at $60\frac{1}{2}\%$ and selling it at $92\frac{3}{4}\%$, brokerage $\frac{1}{8}\%$ on the sale and purchase?

31. What profit is made by buying 1300 shares of Chicago Great Western R.R. stock at 17% and selling it at $18\frac{1}{8}\%$, brokerage $\frac{1}{8}\%$ on the sale and purchase?

32. What profit is made by buying 20 shares of R.R. stock at $98\frac{1}{2}\%$ and selling it at $112\frac{1}{2}\%$, brokerage $\frac{1}{8}\%$ on the sale and purchase?

33. Find the annual income from \$2310 invested in Northern Pacific R.R. stock at $115\frac{3}{8}\%$, brokerage $\frac{1}{8}\%$, if this stock pays a quarterly dividend of $1\frac{3}{4}\%$.

34. Find the annual income from \$2584.50 invested in Adams Express Co. stock at $215\frac{1}{4}\%$, brokerage $\frac{1}{8}\%$, if this stock pays a quarterly dividend of 3%.

35. Find the annual income from six \$100 U. S. 4s, 1925.

36. Find the annual income from eight \$1000 U. S. 4s, 1925.

37. Find the annual income from six \$1000 Panama 2s, 1936.

38. Find the annual income from four \$1000 Reading R.R. Co. 4% bonds.

39. Find the annual income from eight \$1000 Baldwin Locomotive Co. 5% bonds.

40. How many shares of stock paying 5% dividends must be purchased that the annual income derived therefrom may be \$220?

41. How much must I pay for sufficient 8% stock ($50), market value $89½ per share, that my annual income may be $120, brokerage ¼%?

42. What must I pay for sufficient United Gas Improvement Co. stock ($50) that my income from it may be $240 per year, if I buy this stock when its market value is $89½ per share, brokerage ¼%, this stock paying a quarterly dividend of 2%?

43. If I purchase 6% stock at 124¼%, brokerage ⅛%, what rate of interest does my investment pay? (Answer to nearest hundredth per cent.)

44. If I purchase Penn. R.R. stock ($50) at $61½ per share, brokerage ⅛%, what rate of interest does my investment pay me, my annual income from each share of this stock being $3?

45. If I purchase 5% stock at 119⅞%, brokerage ⅛%, what rate of interest do I receive? (Answer to nearest .01%.)

46. What rate of interest does 4% stock ($50) pay an investor, if it is bought at 112½%, brokerage ⅛%? (Answer to nearest .01%.)

47. What must be the market value of 6% stock in order that it may yield 5% on the investment (which includes brokerage), if the rate of brokerage is ⅛%?

48. What must be the market value of 5% stock to yield an investor 4% interest on his investment, brokerage ⅛%?

NEGOTIABLE PAPER AND BANKING

332. A **bank** is an institution which cares for, lends, exchanges, and issues money.

333. National banks are authorized by the United States, and controlled by its laws.

334. State banks are authorized by the various states, and controlled by their respective state laws.

335. Savings banks are banks established to take care of people's savings. They will receive very small deposits, and they pay interest on these deposits, but usually at a low rate.

336. Trust companies are state institutions which settle estates, take care of the property of children, and execute other trusts. They also do a banking business.

337. Private banks are individuals or partners who do a banking business.

338. Deposits are money put into banks by people for safe keeping. Some banks pay a small rate of interest on part or all these deposits; others do not.

339. All banks lend most of their deposits, keeping in their safes only as much as the laws require and experience shows will be asked for. Banks make most of their profits by lending their deposits.

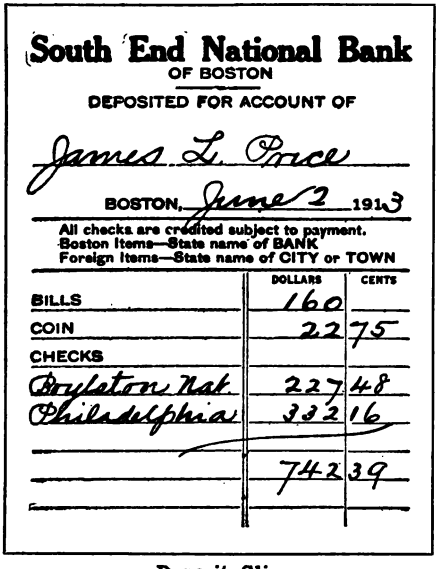
340. All banks receive deposits and lend money. Only national banks issue **bank notes**, and they must deposit in the United States Treasury bonds of the United States whose par value is equal to the notes they issue.

341. Besides national bank notes there are two other kinds of paper money in common use: **United States notes**, or **greenbacks**, which are issued by the United States; and gold and silver certificates, which are substitutes for **gold** and **silver** coin kept in the government vaults at Washington.

NOTE. Read carefully both sides of all these kinds of paper money.

OPENING AN ACCOUNT AT A BANK

342. To open an account at a bank, a person should fill out a deposit slip and deliver it, with the deposit, to the *receiving teller* or *cashier* of the bank.



South End National Bank
OF BOSTON

DEPOSITED FOR ACCOUNT OF

James L. Price

BOSTON, June 2 1913

All checks are credited subject to payment.
Boston Items—State name of BANK
Foreign Items—State name of CITY or TOWN

	DOLLARS	CENTS
BILLS	160	
COIN	22	75
CHECKS		
Boylston Nat	227	48
Philadelphia	332	16
	742	39

Deposit Slip

343. After opening an account at a bank, a depositor is given a bank book with the amount of his deposit credited therein, and also a blank check book.

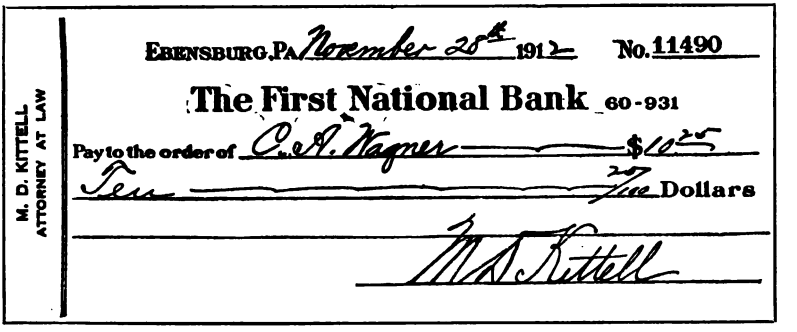
He is generally required to sign his name on a card or in a book of signatures.

CHECKING OUT MONEY

344. When a depositor wishes to use any or all of the money he has in a bank, he fills out a check for the amount desired.

345. A **check** is a written order by which a bank is directed by a depositor to pay out money belonging to him.

Check


M. D. KITTELL
ATTORNEY AT LAW
EBENSBURG, PA. November 28th 1912 No. 11490
The First National Bank 60-931
Pay to the order of C. A. Wagner $10.25
Ten 25/100 Dollars
M. D. Kittell

346. The **drawer**, or **maker**, of a check is the party who signs it. The **payee** is the party to whom the check is payable. The **face** of a check is the amount named therein.

NOTE. The holder of a check should present it for payment on the day he receives it or on the first business day following.

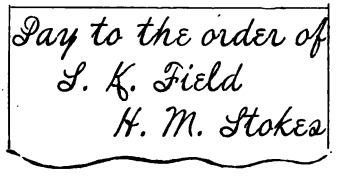
347. Checks are usually made payable *to the order of* the payee.

348. If a check is made payable to the order of the payee, it may be transferred by him to another. This is done by the payee's writing his own name across the back of the check, which process is called **indorsing** the check. The person receiving the check then becomes the holder, and may again transfer it by indorsement to another person, and so on.

Indorsement in Blank

1. An indorsement containing the name of the indorser only is called an **indorsement in blank.**

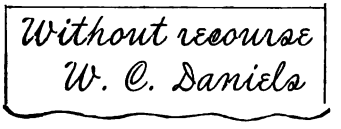
Indorsement in Full



2. An indorsement directing payment to be made to the order of a certain person is an **indorsement in full,** or a **special indorsement.**

3. An indorsement which limits the responsibility of the indorser is a **qualified indorsement.**

Qualified Indorsement



4. A check payable to *bearer,* or to a *payee* or *bearer,* is transferable without indorsement.

Exercise 126

In Examples 1–4 make out deposit slips for the following deposits to be made in your local bank and in your name:

1. Bills, $215; gold, $15; silver, $10.90; check on Boylston National Bank, Boston, $316.25.

2. Bills, $480; gold, $62.50; silver, $26.50; check on Third National Bank, New York, $98.75.

3. Bills, $365; gold, $25; silver, $20.60; check on Farmer's National Bank of Lancaster, Pa., $112.63.

4. Bills, $170; gold, $30; silver, $36.20; check on First National Bank of Bangor, Me., $65.

5. In the check on page 345 name the drawer, or maker; the payee. When is this check payable? Which of the parties named must be a depositor in the First National Bank of Ebensburg, Penn.? Which party named receives the sum mentioned in the face? Must he receive it direct from the bank named in the check? Which party will finally hold the check? Must it be indorsed before it is cashed? Who must first indorse it?

6. Suppose you have money deposited in the nearest bank; write the check which you would draw to pay G. M. Brown $20.

7. Suppose that P. J. Welsh draws a check in your favor for $10 on the Continental & Commercial Bank of Chicago, and you have W. P. Bolton cash the check for you; write the check and the indorsement it bears.

8. This form, called "Counter Check," is frequently used by depositors in drawing cash for their own use from their bank of deposit:

Copy the "Counter Check," page 347, and fill it out to draw $20 cash.

9. With a "Counter Check" draw $5 from your nearest bank, supposing that you have deposits therein.

10. A man had a balance in bank of $756.25 on May 1. Since that time he has drawn checks for $25.20, $18.75, and $9.85, and deposited $25, $50, and $75. What is his present balance?

11. A merchant had a balance in bank of $375.80 on June 30. Since that time he has drawn checks for $37.50, $18.75, $6.25, and $32.50, and has deposited $50, $18.90, and $13.20. What is his present balance?

SAVINGS BANK ACCOUNTS

349. Interest paid depositors in savings banks is compounded monthly, quarterly, or semiannually. Quarterly interest periods usually begin Jan. 1, Apr. 1, July 1, and Oct. 1; semiannual periods usually begin Jan. 1 and July 1.

350. Depositors in savings banks are given bank books which must be presented when deposits are made and when money is drawn out; they should also be presented at the end of each interest period.

351. Savings banks usually credit interest at the end of each interest period on the *smallest amount* on deposit during the entire period.

352. The practices prevailing among savings banks governing the length of the interest period, withdrawals, and deposits are so varied that no fixed rules can be laid down.

353. The following is an illustrative statement of deposits and withdrawals with interest compounded semiannually at 3 % per annum:

Date	Deposited		Drawn Out		Interest		Balance	
1911								
Nov. 13	400						400	
1912								
Jan. 1							400	
Feb. 10			60				340	
Apr. 26	25						365	
June 14	10						375	
July 1			15		5	10	365	10
Sept. 21			34				331	10
Nov. 27	40						371	10
Dec. 5			100				271	10
1913								
Jan. 1					4	07	275	17
May 7			10				265	17
July 1					3	98	269	15

Exercise 127

Arrange the following as in Section 353.

1. Deposits: Oct. 16, 1911, $200; March 2, 1912, $125; Nov. 6, 1912, $250. Withdrawals: Nov. 13, 1911, $100; Dec. 20, 1912, $225. Find the balance due Jan. 1, 1913, interest compounded semiannually, Jan. 1 and July 1, at 3 % per annum.

2. Deposits: Feb. 13, 1911, $300; Nov. 20, 1911, $100; Jan. 6, 1912, $275; Dec. 2, 1912, $50; Feb. 6,

1913, $25. Withdrawals: June 23, 1911, $75; Sept. 16, 1911, $150; Apr. 21, 1912, $100; June 11, 1912, $60. Find the balance due Jan. 1, 1914, interest compounded semiannually Jan. 1 and July 1, at 3% per annum.

3. Deposits: Nov. 30, 1911, $600; Apr. 20, 1912, $100; Aug. 10, 1912, $100; Oct. 3, 1912, $150; Jan. 25, 1913, $200. Withdrawals: Dec. 20, 1911, $200; Apr. 29, 1912, $50; Sept. 5, 1912, $300; June 20, 1913, $175. Find the balance due July 1, 1913, interest compounded quarterly Jan. 1, Apr. 1, July 1, and Oct. 1, at 3% per annum.

POSTAL SAVINGS SYSTEM

(This information is from a circular distributed from the Post Office Department.)

354. Object. The Postal Savings System is established for the purpose of providing facilities for depositing savings at interest with the security of the United States Government for repayment.

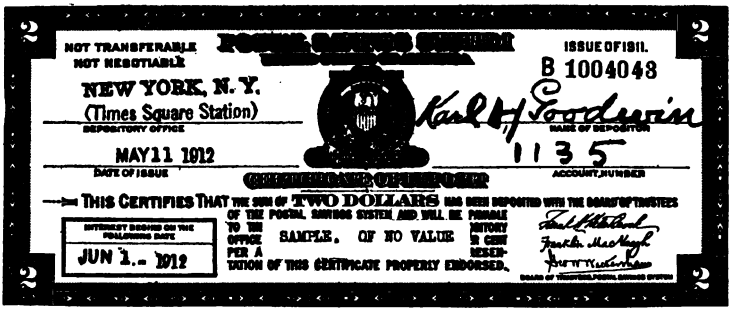
355. Who may Deposit. Accounts may be opened and deposits made by any person of the age of 10 years or over in his or her own name, and by a married woman in her own name and free from any interference or control by her husband. No person may open a postal savings account at any post office who is not a patron of that office. Deposits will be accepted only from individuals, and no account will be opened in the name of any corporation, association, society, firm, or partnership, or in the names of two or more persons jointly.

356. Service Free. The service of the Postal Savings System is free, and no charge or fee is collected or required in connection with the opening of an account or the withdrawal of money deposited.

357. How to open an Account. When a person applies to open an account, he must furnish the necessary information for the postmaster or his representative to fill out an application, which he will then be required to sign. If the applicant signs by mark, his signature must be witnessed by a disinterested person.

358. Deposits. Deposits are evidenced by postal savings certificates issued in fixed denominations of $1, $2, $5, $10, $20, $50, and $100, each bearing the name of the depositor, the number of his account, the date of issue, the name of the depository office, and the date on which interest begins. The postmaster or his representative will make out a duplicate of each certificate issued, which the depositor will be required to sign and which the postmaster will retain in his records.

Postal Savings Certificate



No account may be opened for less than $1, nor will fractions of a dollar be accepted for deposit.

No person is permitted to deposit more than $100 in any one calendar month nor to have a total balance to his credit at one time of more than $500, exclusive of accumulated interest.

Savings certificates cannot be transferred or negotiated and will be payable only to the person to whom issued.

On opening an account a depositor is supplied with an envelope in which he may keep his savings certificates. On this envelope is printed information for his guidance, and also a blank ledger record on which to keep an account of his deposits and withdrawals.

In case a savings certificate is lost or destroyed the depositor should notify the postmaster.

359. Savings Cards and Stamps. Amounts less than $1 may be saved for deposit by the purchase of 10-cent postal savings cards and adhesive 10-cent postal savings stamps. Each postal savings card contains blank spaces to which savings stamps may be affixed from time to time as purchased, and a postal savings card with nine 10-cent savings stamps thus affixed will be accepted as a deposit of $1 either in opening an account or in adding to an existing account.

Savings Card with One Postal Savings Stamp Affixed

U.S. POSTAL SAVINGS CARD
THIS CARD WITH NINE 10-CENT POSTAL SAVINGS STAMPS AFFIXED WILL BE ACCEPTED BY THE POSTMASTER AT ANY POSTAL SAVINGS DEPOSITORY OFFICE IN THE UNITED STATES AS A DEPOSIT OF ONE DOLLAR AND PLACED TO THE CREDIT OF THE DEPOSITOR, EITHER IN OPENING AN ACCOUNT OR IN ADDING TO AN EXISTING ACCOUNT.
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CENTS
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SAMPLE, OF NO VALUE

360. Interest. Interest will be allowed on all deposits at the rate of 2 per cent per annum, computed on each savings certificate separately, and payable annually. No interest will be paid on money which remains on deposit for a fraction of a year only.

Deposits will bear interest from the first day of the month next following that in which deposited.

Interest will continue to accrue on a savings certificate as long as it remains outstanding, certificates being valid until paid, without limitation as to time.

Compound interest is not allowed on an outstanding certificate, but a depositor may withdraw interest payable and include it in a new deposit, which will bear interest at the regular rate.

361. Withdrawals. A depositor may at any time withdraw the whole or any part of his deposits to his credit with any interest payable by surrendering savings certificates, properly indorsed, for the amount desired.

When a depositor desires to withdraw merely the interest payable on any certificate, instead of indorsing and surrendering the certificate as in case of full payment he will be required to give his receipt in duplicate for the amount of the interest paid.

362. Postal Savings Bonds. A depositor will be permitted to exchange the whole or any part of his deposits in sums of $20, $40, $60, $80, $100, or multiples of $100 up to and including $500, for United States registered or coupon bonds bearing interest at the rate of 2½ per cent per annum, payable semiannually, and redeemable at the pleasure of the United States after one year from date of issue, both principal and interest payable 20 years from such date in United States gold coin. Such exchange may be made under date of January 1 and July 1 of each year, provided such bonds are then available.

A depositor desiring to convert his savings deposits into bonds on January 1 and July 1 of any year must make application at least 15 days before either of the dates named to the postmaster.

Exercise 128

1. If I open a postal savings account and deposit $3, what certificates will be given me?

2. What will be given you at the post office if you deposit in postal savings $1.50?

3. How much money may one person deposit in one month in postal savings?

4. If I open a postal savings account Nov. 11, when does interest begin on my first deposit?

5. If I add a deposit of $25 to my postal savings account July 15 of the current year, when does interest begin thereon? What is the rate of interest? When may I draw the first year's interest on this deposit?

6. If I add a deposit of $50 to my postal savings account Feb. 20 of the current year, when does interest begin thereon? When may I draw my first year's interest on this deposit, and how much?

7. A savings certificate for $10, one for $20, and one for $50 were issued to me Nov. 20 of last year. On what date each year may I receive interest on these, and how much?

8. On Jan. 1 of the current year I exchange postal certificates to the amount of $400 for United States bonds. When may I draw the first interest on these bonds, and how much?

9. On July 1 of the current year I exchange postal certificates to the amount of $300 for United States bonds. When will I receive the first interest thereon, and how much?

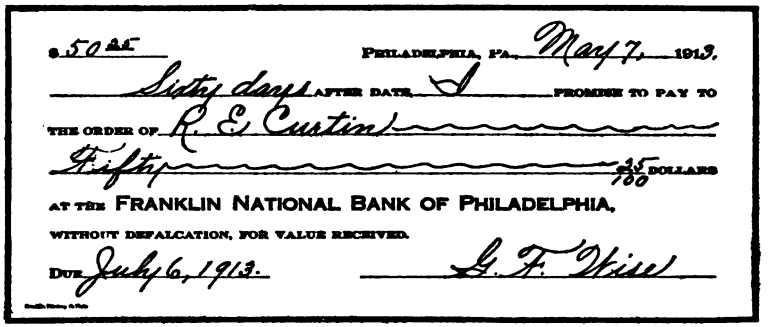
10. After having opened a postal savings account, what is the least amount that I may deposit? Explain how I may save 20¢ for deposit.

PROMISSORY NOTES

363. A **promissory note** is a written promise made by a party called the **maker**, to pay to a certain party, called the **payee**, or to his order, or to bearer, a specified sum of money at a certain time.

364. The **face** of a promissory note is the amount named in the note.

Individual Time Note


$ 50 25
PHILADELPHIA, PA. May 7, 1913.
Sixty days AFTER DATE I PROMISE TO PAY TO
THE ORDER OF R. E. Curtin
Fifty 25/100 DOLLARS
AT THE FRANKLIN NATIONAL BANK OF PHILADELPHIA,
WITHOUT DEFALCATION, FOR VALUE RECEIVED.
DUE July 6, 1913.
G. F. Wise

In the above note *G. F. Wise* is the *maker*, *R. E. Curtin* is the *payee*, and *$50.25* is the *face*.

365. If the maker of a promissory note is unable to write, he may sign it by making his mark and having it witnessed, after his name has been written. Thus, if G. F. Wise cannot write his own name, he may become the maker by having the note executed thus:

Witness: G. L. Brown. G. F. his X mark Wise.

366. An **individual note** is a note signed by one person.

367. A **joint and several note** is a note signed by two or more persons, each making himself responsible for the payment thereof.

If a joint and several note is written "I promise to pay," or "We or either of us promise to pay," and signed by two or more persons, each is responsible for the payment thereof; but if it is written "We jointly promise to pay," or "We promise to pay," and signed by two or more persons, they are jointly responsible for the payment thereof. This form of note is commonly used at public sales, where goods are sold on credit and settled for by notes payable at a bank at the end of the credit period. Satisfactory security is usually demanded of the purchaser, which is obtained by requiring the security to join with the purchaser in making the note. The security may write after his name "surety." The note is then a **principal and surety** note. This form of note may also be used by joint partners in a transaction when no security is required.

368. A **collateral note** is used when a person borrows money and deposits as security with the holder some valuable collateral, as government bonds, certificates of stock, or bonds of some reliable corporation. The holder is authorized to sell the collateral in case the maker fails to pay the note when it becomes due.

369. An **accommodation note** is chiefly used in borrowing money from a bank.

Accommodation Note

$100.00 Wilmington, Del. March 1, 1913.

Sixty days after date I promise to pay to the order of R. P. Miller at the Farmers' Bank at Wilmington One hundred Dollars without defalcation, value received.

CREDIT THE DRAWER
R. P. Miller

G. L. Smith.

370. Banks generally require from a borrower, besides his own name as the maker of a note, the name of some responsible party as security. This is usually obtained by making the note payable to the security and having him indorse it. The phrase "Credit the Drawer," subscribed to by the security, is not essential to the legality of an accommodation note, but it shows definitely the relation of the maker to the security.

Thus, if G. L. Smith desires to borrow one hundred dollars from the Farmers' National Bank and have R. P. Miller for his security, they may write the note as shown in Section 369, and R. P. Miller will indorse it. G. L. Smith may then present the note at bank and, after indorsing it, receive the face less the interest charged by the bank.

Important Facts concerning Promissory Notes

371. A note should be dated; that is, it should state the place *where*, and the time *when*, given.

372. A note may be made payable *on demand, a certain number of days after date, a certain number of months after date*, or upon *a certain date.*

1. The day on which a note becomes due is called the **day of maturity.**

2. A note payable on demand is due whenever the holder demands payment.

3. A note payable a certain number of days after date is due at the expiration of that number of days, not counting the date.

Thus, a note dated March 1 payable in 60 days is due April 30.

4. A note payable a certain number of months after date is due so many calendar months after date.

Thus, a note dated April 30 payable in 6 months is due October 30; a note dated July 31 payable in 3 months is due October 31.

5. In some states 3 days, called **days of grace,** are allowed for the payment of a note after the expiration of the time specified in the note.

373. A note may be made payable at a certain place.

1. If a note is made payable at a particular place, as at a certain bank, it should be presented at that place for payment, when due, during business hours.

2. When no place of payment is designated in a note, it should be presented for payment at the maker's place of business during business hours or at his residence.

374. A promissory note may be made payable to the order of the *payee,* to the *payee or bearer,* or to the *payee* only. It is indorsed or otherwise transferred in the same manner as is a check.

375. A note may be written so as to include the payment of interest.

1. When no rate is mentioned in an interest-bearing note, it bears interest at the legal rate of the state in which it is payable.

2. When a non-interest-bearing note is not paid when due, it will bear interest at the legal rate from the day it is due to the day it is paid.

3. Some interest-bearing notes have a condition stated in them that, if not paid when due, they will thereafter bear a higher rate of interest than the one named in the main note; when no such condition is stated, they will continue to bear interest at the original rate until paid.

Exercise 129

1. When is the note, page 355, due? Who pays this note when it is due? Where will the note be paid? Who holds the note after it has been paid?

2. Name the maker and the payee of the note, page 357. How long does this note run before it is due? Who is expected to pay the note when it is due? Who holds the note after it has been paid?

3. When is a note due that is dated December 31, 1914, payable in 90 days?

4. When is a note due that is dated December 31, 1913, payable in 2 months?

5. Write a promissory note for $20 with yourself as maker and George B. Smith as the payee. Date the note September 30 of the current year, make it payable in 60 days, and find when it is due.

6. If R. E. Curtin wishes to transfer the note on page 355 to W. J. Davis, in what ways may he write the indorsement?

7. Suppose you buy a horse from A. S. Harkness for $150 payable in 9 months. Write the joint and several note that you could give him in payment if B. J. French

signed the note with you. Date the note December 1 of the current year, make it payable at your nearest bank with interest at 5 %, and find what this note is worth at maturity. (Reckon interest for 9 mo.)

8. June 1 of the current year W. H. Jones accommodates you by becoming your security on a 60-day note to enable you to borrow $ 50 from your nearest bank; write the note.

9.

CLEVELAND, O., April 3, 1912.

Ninety days after date I promise to pay to the order of William F. Davis at

The First National Bank of Cleveland

Two hundred twenty ---------- Dollars

Value received, with interest at 4½ %.

L. J. DRIGGS.

Name the parties to the above note. When is it due? How much is due at maturity? Who pays the note when due? Who receives the note when paid?

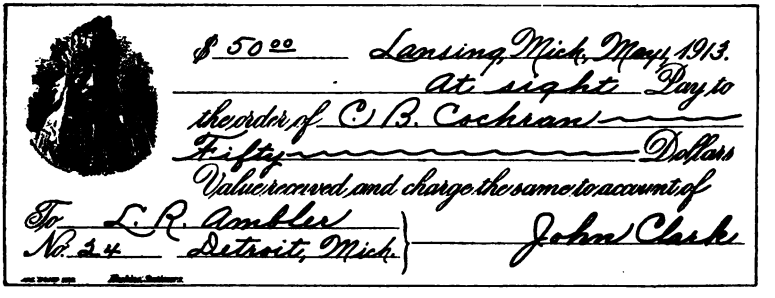
10. How much interest is due at maturity on a note for $ 500 dated Indianapolis, Ind., June 2, of the current year, and payable in 6 months with interest at the legal rate? (Reckon interest for 6 mo.)

11. When is a note due that is dated December 31, 1915, payable in 2 months?

DRAFTS

376. A **draft**, or **bill of exchange**, is a written order whereby one party directs another to pay a specified sum of money to the order of a certain party at a certain time.

Sight Draft



377. The **drawer** of a draft is the party who signs it and thereby orders its payment. The **drawee** is the party who is ordered to pay. The **payee** is the party to whom it is made payable. The **face** is the amount named in the draft.

Thus, in the above draft, *John Clark* is the *drawer*, *L. R. Ambler* the *drawee*, and *C. B. Cochran* the *payee*. *Fifty dollars* is the *face*.

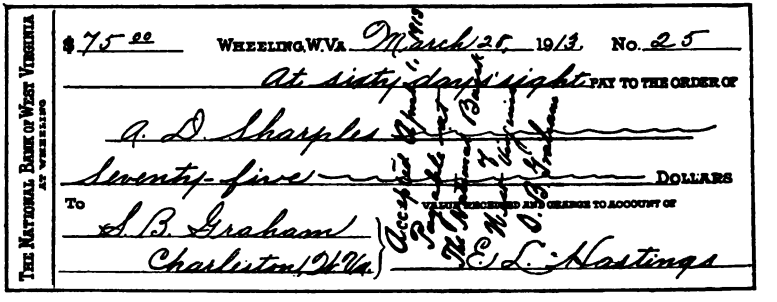
378. A sight draft is a draft that is payable whenever presented to the drawee for payment.

379. The purpose of the sight draft in Section 376 may be explained thus: John Clark, desiring to pay C. B. Cochran fifty dollars, orders L. R. Ambler, who owes him at least that amount, to pay the

same to C. B. Cochran, or his order, at sight, and sends the draft to C. B. Cochran, who may collect it from L. R. Ambler.

380. A **time draft** is a draft payable at a certain time after sight or after date.

Time Draft


THE NATIONAL BANK OF WEST VIRGINIA
AT WHEELING
$75 00
WHEELING, W. VA. March 25 1913 No. 25
At sixty days sight PAY TO THE ORDER OF
A. D. Sharples
Seventy-five DOLLARS
To
S. B. Graham
Charleston, W. Va.
VALUE RECEIVED AND CHARGE TO ACCOUNT OF
E. L. Hastings

381. All time drafts must be presented to the drawee for *acceptance.*

382. To accept a draft, the drawee writes across the face the word "accepted" or "seen," together with the date of acceptance and his name, unless the draft is payable "after date," in which case the date of acceptance may be omitted.

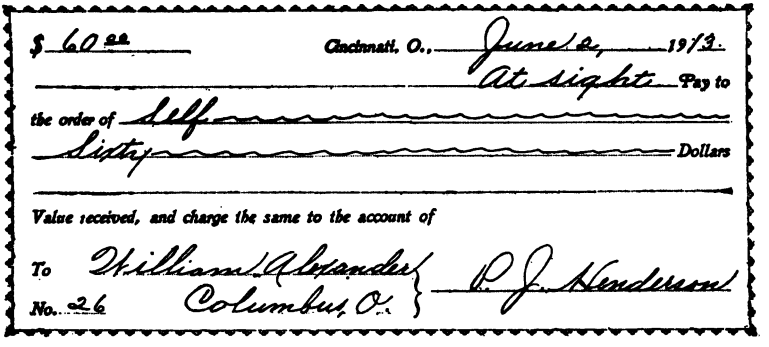
A time draft made payable a certain number of days after *sight* becomes due that number of days after acceptance; but when made payable at a certain time after date, it becomes due at the expiration of that time after date.

383. The purpose of the draft in Section 380 may be explained thus: E. L. Hastings desires to pay A. D. Sharples $75. He orders S. B. Graham, who owes him at least that sum, to pay the same to the order of A. D. Sharples, to whom the draft is sent. A. D.

Sharples presents it to S. B. Graham, who accepts it, as shown in the draft.

384. A draft, used merely for the purpose of collecting a debt, is sometimes called a **two-party** draft.

Two-Party Draft



385. The purpose of this two-party draft may be explained thus: P. J. Henderson wishes to collect a debt of $60 from William Alexander of Columbus, O. He makes himself the payee, then indorses the draft, and deposits it in a Cincinnati bank for collection. The Cincinnati bank forwards it to a bank in Columbus, and this bank collects it from William Alexander.

NOTE 1. P. J. Henderson might also collect the above sum by making the bank in which he is a depositor the payee.

NOTE 2. Two-party drafts may also be time drafts.

Bank Draft

386. A **bank draft** is a draft drawn by one bank upon another in which it has deposits. It is used mainly to transmit money from one place to another.

Bank Draft

387. The purpose of the draft in Section 386 may be explained thus: N. C. Cameron of West Chester, Penn., desiring to send $5 to C. E. Karlson of Cleveland, O., purchases the draft of the First National Bank of West Chester, Penn., indorses it "*pay to the order of C. E. Karlson, N. C. Cameron*" and sends it to C. E. Karlson, who then indorses it and presents it for payment to a bank in Cleveland, which bank cashes it and collects it from the Fourth National Bank of New York City.

Exercise 130

1. In the sight draft, page 361, who directs that payment be made? Who is ordered to pay? When is the draft due? Who is expected to pay it when due?

2. In the time draft, page 362, name the drawer; the drawee; the payee. When is the draft due? Who pays it when due? Who receives it when paid?

3. Write a sight draft for $ 5, making yourself drawer, E. W. Hayes drawee, and William Brown payee; date the draft July 15 of the current year.

4. Suppose Frank Jones owes you $20 and you owe George Robinson a like sum; write the sight draft that you could use both to collect from Frank Jones and to pay George Robinson.

5. Write a 30-day sight draft for $25, making yourself drawer, William Mack drawee, and N. C. Walton payee. Date the draft October 5 of the current year, and have it accepted October 7. When is the draft due? Who pays it when due? Who holds it when paid?

6. Suppose you owe G. R. French $50 and H. L. Beaver owes you at least that sum. Write the draft that you could draw on H. L. Beaver in favor of G. R. French for the sum you owe the latter; date the draft November 1 of the current year, making it payable 60 days after date. Write the acceptance and find when the draft is due.

7. Write the sight draft that you could draw on G. W. Boyd to collect a debt of $40. Explain the steps that you would take to collect the draft.

8. You wish to remit $20 to C. J. Carroll of Charleston, S.C. To do this you purchase a draft for this amount at your local bank, which has deposits in the Fourth National Bank, New York City. Write the draft and explain how you can use it to pay the debt.

PARTIAL PAYMENTS

388. When a note is paid in part, such payment is called a **partial payment**, and it is receipted on the back of the note. This acknowledgment is called an **indorsement.**

389. The following is the rule most generally used for determining the balance due on a note with indorsements:

United States Rule

The rule adopted by the Supreme Court of the United States, known as the "United States Rule of Partial Payments," has been adopted as the legal rule by nearly all the states.

Find the amount of the given principal to the time when the payment, or the sum of the payments, is equal to or exceeds the interest due; subtract from this amount the payment or the sum of the payments. Treat the remainder as a new principal, and proceed as before.

NOTE. New Hampshire, Vermont, and Connecticut have their own rules, which, being limited to these states, will not be considered here.

Illustrative Example

Find the balance due Dec. 15, 1913, on the following note:

$300.00 Camden, N. J. June 1 1912

Nine months after date I promise to

Pay to the order of J. J. Kane

Three hundred ——— Dollars

AT THE NATIONAL STATE BANK OF CAMDEN
CAMDEN, NEW JERSEY

without defalcation or discount for value received, interest at 6%.

M. J. Miller

Indorsements

Received on the within note.
1913, Mar. 1, $120
J. J. Kane.
1913, Sept. 15, $5
J. J. Kane.
1913, Nov. 1, $50
J. J. Kane.

Solution by United States Rule

Principal, June 1, 1912,	$ 300.00
Interest on principal from June 1, 1912, to Mar. 1, 1913 (273 da.),	13.65
Amount, Mar. 1, 1913,	$ 313.65
First payment,	120.00
New principal, Mar. 1, 1913,	$ 193.65
Interest on principal from Mar. 1, 1913, to Sept. 15, 1913 (198 da.), $ 6.39.	
(Interest due, Sept. 15, 1913, exceeds payment then made and no settlement is made.)	
Interest on principal from Mar. 1, 1913, to Nov. 1, 1913 (245 da.),	7.91
Amount, Nov. 1, 1913,	$ 201.56
Payments applied, Nov. 1, 1913 ($ 5 and $ 50),	55.00
New principal, Nov. 1, 1913,	$ 146.56
Interest on new principal from Nov. 1, 1913, to Dec. 15, 1913 (44 da.),	1.07
Amount due, Dec. 15, 1913,	$ 147.63

Exercise 131

Find the balance due on the date of settlement by the U. S. Rule:

1. On a note dated Albany, N.Y., Sept. 1, 1911, for $500 with interest at the legal rate, containing the following indorsements:

July 1, 1912, $50. Mar. 4, 1913, $50.
Aug. 31, 1912, $100. Aug. 31, 1913, $5.
Settled, Mar. 2, 1914.

2. On a note dated Harrisburg, Penn., June 1, 1910, for $1000 with interest at $4\frac{1}{2}\%$, containing the following indorsements:

Apr. 20, 1911, $200. Oct. 2, 1912, $50.
July 16, 1911, $100. Jan. 31, 1913, $200.
Settled, Apr. 1, 1913.

3. On a note dated Boston, Mass., Mar. 1, 1911, for $400 with interest at 4 %, containing the following indorsements:

Aug. 13, 1911, $200. Feb. 29, 1912, $100.
Oct. 13, 1912, $50.
Settled, Mar. 1, 1913.

4. On a note dated Richmond, Va., Aug. 1, 1911, for $600 with interest at $4\frac{1}{2}\%$, containing the following indorsements:

Nov. 20, 1911, $200. Dec. 15, 1912, $200.
May 31, 1912, $10. Jan. 18, 1913, $100.
Settled, Mar. 3, 1913.

States and Territories	Interest Laws				Statutes of Limitation		
	Legal Rate	Maximum Contract Rate	Grace	Payable	Judgments	Notes	Open Accounts
					Years	*Years*	*Years*
Alabama	8	8		S	20	6	3
Arkansas	6	10	G	P	10	5	3
Arizona	6	Any		S	5	4	3
California	7	Any		S	5	4	2
Colorado	8	Any		S	20	6	6
Connecticut	6	6		S			6
Delaware	6	6		P*	10	6	3
Dist. of Columbia	6	10		S	12	3	3
Florida	8	10		S	20	5	2
Georgia	7	8		P	7	6	4
Idaho	7	12		S	6	5	4
Illinois	5	7		S	20	10	5
Indiana	6	8		P	20	10	6
Iowa	6	8		S	20	10	5
Kansas	6	10		P	5	5	3
Kentucky	6	6		P	15	15	2
Louisiana	5	8		S	10	5	3
Maine	6	Any		S	20	6	6
Maryland	6	6		P	12	3	3
Massachusetts	6	Any		S	20	6	6
Michigan	5	7		S	6	6	6
Minnesota	7	10		P	10	6	6
Mississippi	6	10	G	P	7	6	3
Missouri	6	8		S	10	10	5
Montana	8	Any		S	10	8	5
Nebraska	7	10		S	5	5	4
Nevada	7	Any	G	P	6	4	4
New Hampshire	6	6		S	20	6	6
New Jersey	6	6		S	20	6	6
New Mexico	6	12		S	7	6	4
New York	6	6		S	20	6	6
North Carolina	6	6	G	S	10	3	3
North Dakota	7	12		S	10	6	6
Ohio	6	8		S	15	15	6
Oklahoma	7	12		S	5	5	3
Oregon	6	10		S	10	6	6
Pennsylvania	6	6		S	5	6	6
Rhode Island	6	Any		S	20	6	6
South Carolina	7	8	G	S	20	6	6
South Dakota	7	12	G	S	10	6	6
Tennessee	6	6		S	10	6	6
Texas	6	10	G	P	10	4	2
Utah	8	Any		S	8	6	4
Vermont	6	6		S	8	6	6
Virginia	6	6		S	20	5	2
Washington	6	12		S	6	6	3
West Virginia	6	6		S	10	10	3
Wisconsin	6	10		S	20	6	6
Wyoming	8	12		P	5	5	3

*Unless due on Sunday or Monday holiday, in New Castle Co.

INTEREST LAWS AND STATUTES OF LIMITATION

390. The table on page 369 gives for each state and territory of the United States and the District of Columbia the legal rate, the maximum contract rate, the statute of limitation, the time for payment of a note or draft falling due on Sunday or on a legal holiday, and whether or not grace is allowed.

The following notation is used in the table:

P represents previous business day; S, the succeeding business day; and G, three days of grace.

NOTE 1. By the statute of limitation is here meant the period of time from the date of the last acknowledgment of the debt, within which suit must be brought to collect a debt overdue.

NOTE 2. By maximum contract rate is meant the highest rate of interest that the law allows to be contracted for and collected.

BANK DISCOUNT

391. Bank discount is the amount deducted by a bank for paying a note or a draft before it is due. It is the simple interest on the face of the note or draft, generally from the day of discount to the day of maturity, which time is called the **term of discount**. The **proceeds** is the sum discounted less the discount.

NOTE 1. In Pennsylvania, Delaware, Maryland, Missouri, and the District of Columbia, *the day of discount* as well as *the day of maturity* is counted in reckoning the term of discount.

NOTE 2. Some banks occasionally discount interest-bearing notes. The discount is then reckoned on the amount of the note at maturity instead of on the face.

NOTE 3. The rate of discount charged by a bank is usually the legal rate of interest prevailing in the state.

Illustrative Examples

1. A note dated January 15, 1913, payable in 6 months, was discounted at a Pennsylvania bank February 2, 1913. Find the discount and the proceeds if the face of the note was $150.

Explanation:

6 months after January 15, 1913, or July 15, 1913, was the date of maturity.
February 2 was the date of discount.
$\therefore$ 164 days = the term of discount (Sec. 391, note 1).
6 %, the legal rate in Pennsylvania, is the rate of discount.
$\$0.027\frac{1}{3}$ = the bank discount on $1 for 164 days.
$150 \times \$0.027\frac{1}{3}$, or $4.10 = the required bank discount.
$150 − $4.10, or $145.90 = the proceeds.

2. Find the proceeds of a note of $250 dated Sept. 1, 1912, payable in 6 mo. with interest at 5 %, if this note was discounted at a New Hampshire bank Dec. 31, 1912.

Explanation:

$250.00 = the face of the note.
6.25 = the interest for 6 mo. at 5 %.
$256.25 = the amount of the note at maturity.
2.56 = the bank discount for 60 da. at 6 %.
$253.69 = the proceeds.

Exercise 132

Find the proceeds of the following notes:

	Date	Time	Face	Discounted
1.	June 1	4 mo.	$600	June 4, in New York
2.	May 15	90 da.	$200	May 16, in Maryland
3.	Mar. 12	60 da.	$250	Mar. 13, in Pennsylvania

	Date	Time	Face	Discounted
4.	Jan. 15	4 mo.	$225	Jan. 16, in Ohio
5.	Feb. 1	90 da.	$150	Feb. 1, in Michigan
6.	Feb. 1	4 mo.	$120	Feb. 5, in Idaho
7.	Oct. 20	90 da.	$500	Oct. 20, in Nebraska
8.	Sept. 1	3 mo.	$200	Sept. 1, in Massachusetts
9.	July 1	4 mo.	$318	July 1, in Maine
10.	Oct. 2	90 da.	$150	Oct. 9, in Indiana
11.	Sept. 1	3 mo.	$100	Sept. 15, in Oregon
12.	Sept. 30	90 da.	$155	Oct. 1, in West Virginia
13.	Nov. 1	3 mo.	$462	Nov. 2, in Washington
14.	Jan. 15	90 da.	$268	Jan. 16, in Vermont

Find the proceeds of the following notes:

	Face	Date	Time	Interest	Date of Discount	Rate of Discount
15.	$200	Feb. 1	6 mo.	5 %	June 1	6 %
16.	$350	June 1	6 mo.	4½ %	Oct. 15	6 %
17.	$500	Mar. 1	9 mo.	4½ %	June 15	6 %
18.	$275	Apr. 1	9 mo.	4 %	Nov. 1	6 %
19.	$250	Aug. 1	6 mo.	5 %	Jan. 2	6 %
20.	$218	Aug. 1	6 mo.	4½ %	Nov. 1	6 %

21. Find the proceeds of a note dated Mar. 10, payable in 60 da., if the face of the note was $600 and it was discounted Mar. 10 at a New Jersey bank.

22. Find the proceeds of a note of $268 dated May 2 and payable in 30 da., if it was discounted at an Illinois bank on May 10.

23. Find the proceeds of a note of $240 dated Sept. 19, 1912, and payable 6 mo. after date with interest at 5 %, if the note was discounted at a Pennsylvania bank Jan. 2, 1913.

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24. What should be the face of a note payable 60 da. after date to yield proceeds of $350, if discounted at a New York bank on the day the note was made?

SUGGESTION. $0.01 = the discount on $1 for 60 da.
$0.99 = the proceeds of $1 of the note.
$350 = the entire proceeds.
$350 ÷ .99 = the required face.

25. What should be the face of a note dated June 1, and payable in 60 days, to yield $100 proceeds, if discounted at a Pennsylvania bank on the date the note was made?

26. What should be the face of a 3 months' note dated Jan. 12 to yield proceeds of $300 when discounted at an Indiana bank on the date the note was made?

DOMESTIC EXCHANGE

392. **Exchange** is the process of making payments by drafts. Exchange between two places in the same country is **domestic exchange.**

393. When the merchants and banks of one city, as Chicago, owe those of another city, as New York, to such an extent that gold must be sent from Chicago to New York to settle the balances, drafts drawn on merchants and banks in New York by those of Chicago will sell at a **premium** in Chicago; and drafts drawn by merchants and banks in New York on those of Chicago will sell at a **discount** in New York.

394. Premiums and discounts on drafts are usually reckoned as a certain per cent of their face value.

Thus, if a sight draft of $200 is sold at a premium of $\frac{1}{8}$ %, each dollar of the face is sold for 1.00\frac{1}{8}$; but if sold at a discount of $\frac{1}{8}$ %, each dollar of the face is sold for 0.99\frac{7}{8}$.

395. When exchange is at a premium, a time draft is worth its face minus the bank discount for the time the draft has yet to run, plus the exchange premium. When exchange is at a discount, a time draft is worth its face, minus the bank discount for the time the draft has yet to run, minus the exchange discount.

Thus, the cost in New York of a $500 draft due in 60 days, exchange $\frac{1}{4}\%$ premium, may be found as follows:

$0.01 = the bank discount on $1 for 60 da. at the legal rate.
0.99 = the cost of $1 of the face, if exchange were at par.
0.0025 = the premium on $1 of the face.
$0.9925 = the cost of $1 of the face.
500 × $0.9925, or $496.25 = the cost of the draft.

REMARK. If exchange were at $\frac{1}{4}\%$ discount, the cost of each $1 of the face would be $0.9875.

Exercise 133

1. Find the cost of a sight draft for $250 when exchange is at a premium of $\frac{1}{2}\%$.

2. Find the cost of a sight draft for $318 when exchange is at a discount of $\frac{1}{2}\%$.

3. What is the cost of a 60 days' sight draft for $150 when exchange is at a discount of $\frac{3}{4}\%$?

4. What is the cost of a 30 days' sight draft for $275 when exchange is at a premium of $\frac{3}{8}\%$?

5. Find the proceeds of a draft for $220 payable 90 days after sight, if discounted at a Richmond bank on the day of acceptance, exchange being $\frac{5}{8}\%$ premium.

6. What is the cost in Cincinnati on June 1 of a draft for $200 payable 30 days after sight and accepted May 24, exchange being $\frac{1}{2}\%$ discount?

RATIO, PROPORTION, AND PARTNERSHIP

RATIO

396. The **ratio** of one number or amount to another is the result obtained by finding how many times the first, called the **antecedent**, contains the second, called the **consequent**, as a measure.

397. The **sign** of ratio is the colon (:).

Thus, the expression 12 yd. : 3 yd. denotes that the number of times that 12 yd. contains 3 yd. as a measure is to be found. 12 yd. is the antecedent and 3 yd. is the consequent. The expression is read, *the ratio of 12 yd. to 3 yd.*

REMARK. The expression 12 yd. : 3 yd. is called *a ratio*, and may also be read *12 yd. measured by 3 yd.*

398. The antecedent and the consequent of a ratio are called its **terms**.

399. It is evident from Section 37 that:

(1) Finding the ratio is simply finding the quotient in comparative division.

(2) The antecedent corresponds to the dividend and the consequent to the divisor.

(3) If one term of a ratio is an amount, the other must be a similar amount.

(4) The ratio is always a number.

400. A ratio may be written as a fraction whose numerator is the antecedent and whose denominator is the consequent.

401. The **inverse** of a ratio is the ratio resulting from an interchange of terms.

Thus, the inverse of $4 : 5$ is $5 : 4$.

402. It is evident that 12 bu. contains 3 bu. as many times as 12 contains 3; hence,

$$12 \text{ bu.} : 3 \text{ bu.} = 12 : 3, \text{ or } 12 \div 3. \quad [\S\ 37]$$

The ratio of two similar amounts equals the number of units in the antecedent divided by the number of units in the consequent.

403. From Section 89 it follows that:

Multiplying or dividing both terms of a ratio by the same number does not change the value of the ratio.

404. A ratio is in its *simplest form* when its terms are prime to each other.

Exercise 134

1. Find the ratio of $4\frac{1}{2}$ bu. to 2 bu.

Thus, $4\frac{1}{2}$ bu. : 2 bu. $= 4\frac{1}{2} \div 2$, or $2\frac{1}{4}$.

2. Find the ratio of 4 qt. to $2\frac{1}{2}$ gal.

Thus, 4 qt. : $2\frac{1}{2}$ gal. $= 4$ qt. : 10 qt.
$= 4 \div 10$, or $\frac{2}{5}$.

Find the ratio of:

3. $2 : 5$ 4. $2\frac{1}{2} : 5$ 5. $4 : 3\frac{3}{4}$

6. $3\frac{1}{2}$ yd. : 15 yd. 7. $\frac{2}{3}$ hr. : $1\frac{1}{4}$ hr. 8. $\$\frac{3}{4} : \$\frac{7}{8}$

Find the missing antecedent (a), consequent (c), or ratio (r):

9. $\frac{10}{4} = r$ 10. $\frac{a}{4} = 12$ 11. $\frac{15}{c} = 5$ 12. $\frac{\frac{3}{4}}{4} = r$

13. $\frac{a}{\frac{1}{2}} = 2$ 14. $\frac{3}{c} = \frac{1}{4}$ 15. $\frac{\frac{1}{2}}{c} = 4$ 16. $\frac{a}{5} = .25$

17. In a period of eight years recently, certain foodstuffs advanced in price as given below; state at sight the ratio of the first price to the second:

(1) Tomatoes per can from 8¢ to 12¢.
(2) Peas per can from 10¢ to 12¢.
(3) Corn per can from 6¢ to 10¢.
(4) Apricots per pound from 20¢ to 25¢.
(5) Lamb chops per pound from 20¢ to 28¢.

18. Find the ratio of a short ton to a long ton.

19. A cubic foot of water weighs 62.5 lb. and a cubic foot of platinum weighs 1345.625 lb.; find, correct to the nearest hundredth, the ratio of the weight of platinum to water.

20. The ratio of a cubic foot to a stricken bushel is about $\frac{4}{5}$. How many cubic feet are there in a bin that holds 160 stricken bushels?

PROPORTION

405. An expression of equality between two equal ratios is called a **proportion.**

Thus, $2 : 3 = 6 : 9$ is a proportion, and may be read, *the ratio of 2 to 3 equals the ratio of 6 to 9;* or, *2 measured by 3 equals 6 measured by 9.*

406. Since each of the two ratios in a proportion has two terms, a proportion has four terms; the first and fourth are called the **extremes,** the second and third the **means,** the first and third the **antecedents,** and the second and fourth the **consequents.**

407. The ratio of two similar amounts may equal the ratio of two other similar amounts; and the two ratios together form a proportion.

Thus, 12 yd. : 3 yd. = \$8 : \$2, since each ratio is 4.

408. From Section 402 we have:

$$12 \text{ yd.} : 3 \text{ yd.} = 12 : 3, \text{ and } \$8 : \$2 = 8 : 2;$$

therefore, the proportion 12 yd. : 3 yd. = \$8 : \$2 may be written,

$$12 : 3 = 8 : 2,$$

which is called a **numerical proportion;** that is, a proportion whose terms are all numbers.

409. The proportion $8 : 12 = 2 : 3$ may be written $\frac{8}{12} = \frac{2}{3}$; multiplying both members of the equation by 12×3, we have

$$\frac{\cancel{12} \times 3 \times 8}{\cancel{12}} = \frac{12 \times \cancel{3} \times 2}{\cancel{3}};$$

that is, $3 \times 8 = 12 \times 2$; therefore:

In any numerical proportion the product of the means equals the product of the extremes.

410. From $8 : 12 = 2 : 3$

we have $8 \times 3 = 2 \times 12$;

also, $8 = \frac{2 \times 12}{3}$, and $2 = \frac{8 \times 3}{12}$; therefore:

1. **In a numerical proportion one extreme equals the product of the means divided by the other extreme.**

2. **In a numerical proportion one mean equals the product of the extremes divided by the other mean.**

Exercise 135

In these examples x stands for the missing term; find the value of x:

1. $x : 8 = 4 : 16$
2. $9 : x = 12 : 8$
3. $32 : 16 = x : 2$
4. $20 : 25 = 100 : x$
5. $x : \frac{1}{2} = 6 : 12$
6. $\frac{3}{4} : x = 2 : \frac{2}{3}$
7. $x : 3\frac{1}{3} = 10\frac{1}{2} : 15$
8. $37\frac{1}{2} : 62\frac{1}{2} = 5 : x$
9. $\$x : \$5\frac{1}{2} = \$12 : \$18$
10. 10 yd. $: 3\frac{1}{3}$ yd. $= x : 2$
11. $\$x : \$87\frac{1}{2} = 6$ yd. $: 7$ yd.
12. 1 bu. $: 2\frac{1}{2}$ bu. $= \$x : \$10$

Types of Proportion Problems Considered

411. First Type. If 6 lb. of sugar cost 30¢, 12 lb. at the same rate would cost 60¢.

The statement may be abridged thus:

Weight	Cost
6 lb.	30¢
12 lb.	60¢

Observe:

(1) That when the weight is 6 lb., the corresponding cost is 30¢.

(2) That when the weight is 12 lb., the corresponding cost is 60¢.

(3) That $6 : 12 = 30 : 60$.

(4) That the ratio of the two values of weight equals the ratio of the corresponding values of cost.

(5) If any term of (3) were missing, it could be represented by x and found.

412. Two quantities are said to be directly proportional when the ratio of any two values of the one equals the ratio of the corresponding values of the other.

413. Second Type. If 6 lb. of sugar can be bought for a certain sum when sugar is 5¢ per pound, 3 lb. can be bought for the same sum when sugar is 10¢ per pound.

The statement may be abridged thus:

Weight	Price per Pound
6 lb.	5¢
3 lb.	10¢

Observe:

(1) That when the weight is 6 lb., the corresponding price per pound is 5¢.

(2) That when the weight is 3 lb., the corresponding price per pound is 10¢.

(3) That $6 : 3 =$ the inverse of $5 : 10$; that is, $6 : 3 = 10 : 5$.

(4) That the ratio of the two values of weight equals the inverse of the ratio of the corresponding values of the price per pound.

(5) If any term of (3) were missing, it could be represented by x and found.

414. Two quantities are inversely proportional when the ratio of any two values of the one equals the inverse of the ratio of the corresponding values of the other.

415. Test to determine to which type a problem of proportion belongs:

(1) If when the second quantity is increased the first is also increased, the first is directly proportional to the second.

(2) If when the second quantity is increased the first is decreased, the first is inversely proportional to the second.

Illustrative Examples

1. Find the cost of 20 yd. of cloth, if 5 yd. cost \$ 15.

Explanation:

Cost	Length
\$ x	20 yd.
\$ 15	5 yd.

The cost is *directly* proportional to the length; for if the length be increased, the cost will be increased; therefore:

$$x : 15 = 20 : 5.$$

$$x = \frac{15 \times 20}{5}, \text{ or } 60.$$

$$\therefore \$ 60 = \text{the cost.}$$

2. If a train running 40 mi. per hour runs a certain distance in 4 hr., in what time will it run the same distance if it runs 30 mi. per hour?

Explanation:

Time	Rate
x hr.	30 mi.
4 hr.	40 mi.

The time is inversely proportional to the rate; for if the rate be increased, the time will be decreased; therefore:

$$x : 4 = 40 : 30.$$

$$x = \frac{4 \times 40}{30}, \text{ or } 5\tfrac{1}{3}.$$

$\therefore 5\frac{1}{3}$ hr. = the time.

Exercise 136

1. In the following tell whether the first quantity named is directly or inversely proportional to the second:

(1) The cost of land and the quantity bought.

(2) The quantity of land bought for a certain price and the cost per acre.

(3) The principal on interest and the interest earned.

(4) The time required to go a certain distance and the speed.

(5) The time required to do work and the force.

(6) The quantity of work done in a day and the force working.

(7) The force required to do a piece of work and the time.

2. Steers should be fed 2.84 lb. of hay, or its equivalent, for each 100 lb. of their live weight. How much hay, or its equivalent, should be fed per day to a steer which weighs 1050 lb.?

3. If a 10-acre field of corn produces 600 bushels, what should a 6-acre field produce at the same rate?

4. When coal is retailing at \$0.38 per 100 lb., how much is that per ton of 2000 lb.?

5. When hay is selling for \$24 per ton, what should be charged for 1750 lb. of hay?

6. When hair for plastering is sold at the rate of $0.25 for 7 lb., how much can be bought at the same rate for $1.50?

7. When $2\frac{1}{2}$ in. on a certain scale represent 100 mi., what distance is represented on the same scale by 15 in.?

8. It is estimated that 100 lb. of hay possesses the same feeding value as 58 lb. of barley; how much barley has the same feeding value as 1 T. of hay?

9. If an arc of 30° measures 6 in., what should an arc of 100° on the same circle measure?

10. If $1.50 will buy a certain quantity of sugar when it sells for $0.05 per pound, what would the same quantity cost when sugar sells at $0.08 per pound?

11. If a train running 40 mi. an hour runs a certain distance in 5 hr., in what time will it run the same distance at the rate of 25 mi. per hour?

12. A certain sum will buy 100 lb. of sugar at $0.05 a pound; what weight will the same sum buy if the sugar is sold for 0.06\frac{1}{4}$ a pound?

13. An Edison incandescent lamp burning 4 hr. each night will last about 200 nights; how many nights would it last if it were burned 3 hr. each night?

14. In a plan of a farm 6.25 in. is made to represent a side 1.05 rd. long; what would represent a side 8.4 rd. long?

15. If 40 bu. of grain will last a herd of cows a week when each is fed 4 qt. per day, how long would the same quantity last them, if each were fed 5 qt. per day?

16. A farmer's crop of 240 bu. of oats weighs 30 lb. to the bushel; if he sells it to a miller who takes 32 lb. to the bushel, for how many bushels should the farmer be paid?

PROPORTIONAL PARTS

Illustrative Examples

1. Divide 1260 into parts proportional to 2, 3, and 4.

Explanation:

The number is to be divided into $2 + 3 + 4$, or 9 equal parts.
Then two of these parts, or $\frac{2}{9}$ of 1260, or $280 =$ the first part.
And 3 of these parts, or $\frac{3}{9}$ of 1260, or $420 =$ the second part.
And 4 of these parts, or $\frac{4}{9}$ of 1260, or $560 =$ the third part.
Check: $280 + 420 + 560 = 1260$.

2. Two numbers are to each other as 7 is to 2, and their difference is 75. What are the numbers?

Explanation:

Let l represent the larger and s the smaller.

Then $l : s = 7 : 2$.

$2\,l = 7\,s$.

$l = \frac{7}{2}\,s$.

$s = s$.

$l - s = \frac{5}{2}\,s$, or 75.

$\frac{1}{2}\,s = 15$.

$s = 30$.

$\frac{7}{2}\,s$, or $l = 105$.

Check: $105 - 30 = 75$.

Exercise 137

1. Divide $1250 into parts proportional to 3 and 2.

2. Divide 500 into parts proportional to 2, 3, and 5.

3. The sum of 2 numbers is 480, and the larger is to the smaller as 9 is to 7. What are the numbers?

4. The difference of two numbers is 50, and the larger is to the smaller as 6 is to 5. What are the numbers?

5. A and B together invested $225. A invested $7 as often as B invested $3. How much did each invest?

6. Divide 20 into parts proportional to $\frac{1}{2}$ and $\frac{1}{3}$.

SUGGESTION. 1st part : 2d part $= \frac{1}{2} : \frac{1}{3} = 3 : 2$.

7. Solder for lead consists of 1 part tin to $1\frac{1}{2}$ parts lead; how much of each of these substances is necessary to make 5 lb. of solder?

8. A, B, and C are charged $36 freight on a shipment of potatoes, of which A shipped 200 bu., B, 300 bu., and C, 500 bu.; how much should each pay?

9. Three men received $62.50 for husking a field of corn; the first husked 300 shocks, the second 450 shocks, and the third 500 shocks; how much should each receive?

10. After ducklings are 10 days old, a good feed for them consists of the following parts by weight:

3 parts bean	$1\frac{1}{2}$ parts beef scrap
1 part corn meal	$2\frac{1}{2}$ parts green food
1 part flour	

How many pounds of each ingredient are there in 360 lb. of this mixture?

PARTNERSHIP

416. A **partnership** is an association of individuals, called **partners**, for the transaction of business.

NOTE. A partnership is sometimes called a **firm**.

417. The **capital stock** is the amount which the partners invest in the business.

NOTE. The **assets** of a firm are all the property belonging to the firm. The **liabilities** are all the debts of the firm.

418. When the shares of the partners are invested for equal times, gains and losses are distributed among the partners in proportion to their respective shares.

419. When the shares of the partners are invested for unequal times, gains and losses are distributed among the partners in proportion to their respective shares and times, except when by special agreement some other plan is adopted.

Illustrative Examples

1. A, B, and C form a partnership, A investing \$ 600, B \$ 800, and C \$ 1000. Divide a gain of \$ 300 among them.

Explanation:

\$ 600 + \$ 800 + \$ 1000, or \$ 2400 = the capital stock.
$\frac{1}{4}$ of the capital stock = A's investment.
$\frac{1}{3}$ of the capital stock = B's investment.
$\frac{5}{12}$ of the capital stock = C's investment.
\$ 300 = the gain to be divided

The gain is distributed in proportion to the investments.

$\therefore$ $\frac{1}{4}$ of \$ 300, or \$ 75 = A's gain.
$\frac{1}{3}$ of \$ 300, or \$ 100 = B's gain.
$\frac{5}{12}$ of \$ 300, or \$ 125 = C's gain.

2. Three boys invested their savings in a news stand; A invested \$50 Jan. 1, B \$30 Mar. 1, and C \$20 Aug. 1. At the end of the year they had a profit of \$80 to divide among them; find each one's share.

Explanation:

A's capital was invested 12 mo., B's 10 mo., and C's 5 mo.
A's gain = the gain on \$50 for 12 mo., or the gain on \$600 for 1 mo.
B's gain = the gain on \$30 for 10 mo., or the gain on \$300 for 1 mo.
C's gain = the gain on \$20 for 5 mo., or the gain on \$100 for 1 mo.
The total gain = the gain on \$1000 for 1 mo.
The total gain = \$80.
$\frac{6}{10}$ of \$80, or \$48 = A's gain.
$\frac{3}{10}$ of \$80, or \$24 = B's gain.
$\frac{1}{10}$ of \$80, or \$8 = C's gain.

Exercise 138

1. Three men invested jointly \$3000 in a creamery. At the end of the first year their gains were \$1500. The first invested \$800, the second \$1000, and the third \$1200; find each one's share of the profit.

2. A and B in partnership gain in 1 year \$8560. A contributes \$8000 and B \$7000. A is to retain 25% of the profits for managing the business; how much of the balance should each receive?

3. A merchant failing in business has property to the value of \$3200. He owes A \$4500, B \$2500, and C \$2000; how much should each receive?

4. A, B, and C established a coal yard, A furnishing \$5000, and B \$4000. C's gain for the first year was

$198. Find C's capital and A's and B's gain, if the total capital was $18,900.

5. A and B formed a partnership, A putting in $600 for 18 months, and B $800 for a year. They gained $510; what was each partner's share of the gain?

6. A, B, and C formed a partnership. A invested $500 on Jan. 1, B $400 on Mar. 1, and C $300 on July 1. At the end of the year they had gained $555; what was each partner's share of the gain?

7. A, B, and C take the contract for building a public road. A furnishes 12 men for 60 days, B 20 men for 40 days, and C 10 men for 8 days. They receive for the work $4200, of which A is paid $200 for supervising the work; how much of the balance should each of the men receive?

SUGGESTION. The work of 12 men for 60 days equals the work of 720 men for 1 day.

8. A, B, and C contracted to do a piece of work. A sent 24 men for 45 da., B 40 men for 25 da., and C 35 men for 42 da. If the sum paid for doing the work was $10,650, find each partner's share.

9. A, B, and C invested jointly $3000. A's capital was invested 12 mo., and his gain was $144; B's capital was invested 8 mo., and his gain was $144; C's capital was invested 6 mo., and his gain was $180. Find the capital of each.

POWERS, SQUARE ROOT, AND MENSURATION

POWERS

420. The **square** of a number is the result of using the number twice as a factor.

Thus, the square of 2 equals 2×2, or 4; the square of $\frac{1}{2}$ equals $\frac{1}{2} \times \frac{1}{2}$, or $\frac{1}{4}$; the square of .3 equals $.3 \times .3$, or .09.

421. The **cube** of a number is the result of using the number three times as a factor.

Thus, the cube of .5 equals $.5 \times .5 \times .5$, or .125.

422. A small figure called an **exponent**, written to the right and above a number, denotes how many times the number is to be used as a factor.

Thus, 7^2, read *7 squared*, equals 7×7, or 49; and 7^3, read *7 cubed*, equals $7 \times 7 \times 7$, or 343.

Exercise 139

1. Name the square of each number from 1 to 12.
2. Name the square of each of the nine tens.
3. Name the square of each of the nine hundreds.
4. Name the cube of each number from 1 to 5 inclusive.
5. Name the square and also the cube of $\frac{1}{2}$; $\frac{1}{3}$; $\frac{1}{4}$; $\frac{1}{5}$; .2; .3; .5; $\frac{2}{3}$; $\frac{3}{2}$; .1; .01; .001.

Find the square and also the cube of each of the following:

6. 14	7. 37	8. 45	9. 123	10. 417
11. 841	12. $\frac{3}{4}$	13. $6\frac{1}{4}$	14. $\frac{7}{8}$	15. $87\frac{1}{2}$
16. .75	17. .461	18. .079	19. 4.97	20. .0025

SQUARE ROOT

423. The **square root** of a number is the number whose square is the given number.

Thus, the square root of 49 is 7, since $7^2 = 49$; the square root of $\frac{16}{25}$ is $\frac{4}{5}$, since $(\frac{4}{5})^2 = \frac{16}{25}$.

424. The symbol $\sqrt{\ }$, called the **radical sign,** written before a number, indicates that the square root of the number is to be taken.

Thus, $\sqrt{9}$, read *the square root of 9*, equals 3.

Exercise 140

1. Name the square root of: 25; 9; 16; 121; 36; 81; 49; 100; 144.

2. Name the square root of: $\frac{1}{4}$; $\frac{4}{9}$; $\frac{25}{16}$; $\frac{36}{25}$; .01; .25; .81; .64.

3. Find the square root of: .0004; .0009; .0016; .0025; .0036; .0049; .0064; .0081.

4. Find the square root of 441.

Thus, $441 = 3 \times 3 \times 7 \times 7,$
$= (3 \times 7) \times (3 \times 7),$
$= 21 \times 21;$
therefore, $\sqrt{441} = 21.$

Find the square root of:

5. 196 6. 225 7. 484 8. 1089

9. 1225 10. 1296 11. 2601 12. 3025

13. Extract the square root of 576.

SUGGESTION. $576 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3$.

Now take one of each two equal factors and multiply them together.

Extract the square root of:

14. 2025 15. 1521 16. 1764 17. 11025

18. Find the length of a square that contains 64 sq. ft.

SUGGESTION. Since the area of a square is the square of the number of units in its length, the number of units in its length is equal to the square root of its area.

Find the lengths of squares that contain:

19. 100 sq. rd. 20. 625 sq. yd. 21. .0025 sq. mi.

General Rule for extracting Square Root

425. The square root of numbers may be found by the following rule:

1. **Point off the number into periods of two figures each, beginning with the units.**

2. **Take the square root of the nearest square number below that expressed by the first period on the left for the first figure of the root.**

3. **Subtract the square of the first figure of the root from the first period of the number and annex the next period to the remainder for a dividend.**

4. Take twice the root found, as a trial divisor, and divide the dividend, exclusive of the right-hand figure, by it for the next term of the root.

5. Annex the last figure of the root now found to the trial divisor for the complete divisor.

6. Multiply the complete divisor by the last figure of the root now found and subtract the product from the remainder.

7. If the root is not fully found, repeat the above processes until all the periods of the number have been used.

8. Should a cipher occur in the root, annex a cipher to the trial divisor to form the next trial divisor, and bring down the next period.

Illustrative Examples

1. Extract the square root of 217156.

Explanation:

```
21'71'56 | 466
  16
86 | 571
     516
926 | 5556
      5556
```

Point off the number into periods of two figures each, beginning at units. The greatest number whose square is contained in 21 is 4, which is the first figure of the root. Square 4; subtract the result from 21, and annex 71, the next period, to the remainder, making the dividend 571. Double 4, the root found, for a trial divisor. Divide 571, exclusive of the right-hand figure, by 8 for the next figure of the root, which is found to be 6. Annex 6 to 8, making the complete divisor 86. Multiply 86 by 6, the last figure of the root found; subtract the product 516 from 571, and annex 56, the

last period, to the remainder, making the next dividend 5556. Double 46, the root already found, for a trial divisor. Divide 5556, exclusive of the right-hand figure, by 92 for the next figure of the root, which is found to be 6. Annex 6 to 92, making the complete divisor 926. Multiply 926 by 6, the last figure of the root found, and subtract the product from 5556; nothing remains. Therefore, the square root of 217156 is 466.

2. Extract the square root, correct to hundredths, of 142.3.

$$
\begin{array}{rl|l}
 & 1'42'.30'00'00 & 11.928 \\
 & 1 & \\
\hline
21 & 42 & \\
 & 21 & \\
\hline
229 & 2130 & \\
 & 2061 & \\
\hline
2382 & 6900 & \\
 & 4764 & \\
\hline
23848 & 213600 &
\end{array}
$$

SUGGESTION. Point off the number into periods of two figures each, rightward and leftward from the decimal point. The square root is found to be nearer 11.93 than 11.92; hence, the result is taken to be 11.93.

Exercise 141

Extract the square root of:

1. 1024	2. 7396	3. 524176	4. 498436
5. 2116	6. 9025	7. 258064	8. 577600
9. 3481	10. 14161	11. 318096	12. 643204

Extract the square root of:

13. 53.29	14. 7.7284	15. 5852.25	16. 65.2864

Extract the square root, to the nearest hundredth, of:

17. 32	18. 321	19. 3.42	20. .023

21. Extract the square root of $\frac{16}{25}$.

SUGGESTION. Since $\frac{4}{5} \times \frac{4}{5} = \frac{16}{25}$, the square root of $\frac{16}{25} = \frac{4}{5}$.

NOTE. If a fraction in its lowest terms is a perfect square, its terms are perfect squares. When the terms of a fraction in its lowest terms are not perfect squares, it is best to reduce it to a decimal before extracting the square root.

Extract the square root of:

22. $\frac{121}{144}$ 23. $\frac{256}{729}$ 24. $7\frac{1}{9}$ 25. $\frac{7}{8}$ to hundredths

MENSURATION OF PLANE SURFACES

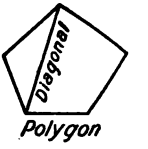
(Lines, angles, the square, and the rectangle have been treated; see Sections 174–179.)

General Definitions

426. Anything that has length and breadth without thickness is called **surface.**

427. A surface on which any number of straight lines can be drawn in different directions is called a **plane surface**, or a **plane.**

428. Any portion of a plane surface bounded by straight lines is called a **polygon.**



Polygon

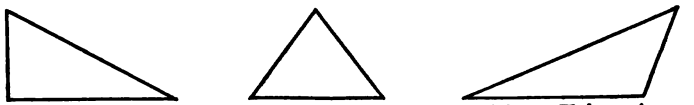
429. The points in which the sides of a polygon meet are the **vertices**, the sum of the sides is the **perimeter**, and a line joining two vertices, but not forming a side, is a **diagonal.**

430. Polygons are classified according to the number of their sides; some of the classes are *triangles*, *quadrilaterals*, *pentagons*, *hexagons*, and *octagons*.

431. (1) A **triangle** is a polygon of three sides.
(2) A **quadrilateral** is a polygon of four sides.
(3) A **pentagon** is a polygon of five sides.
(4) A **hexagon** is a polygon of six sides.
(5) An **octagon** is a polygon of eight sides.

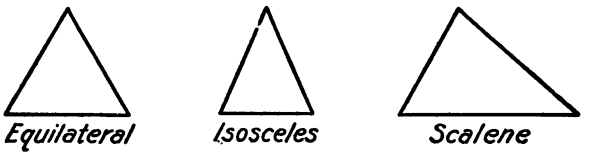
Triangles

432. Triangles are classified according to angles, into *right triangles*, *acute triangles*, and *obtuse triangles*.



433. (1) A **right triangle** is a triangle which has a right angle. The side opposite the right angle is called the **hypotenuse**.
(2) An **acute triangle** is a triangle whose angles are all acute.
(3) An **obtuse triangle** is a triangle which has an obtuse angle.

434. Triangles are classified according to sides, into *equilateral triangles*, *isosceles triangles*, and *scalene triangles*.

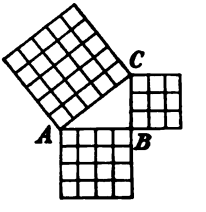


435. (1) An **equilateral triangle** is a triangle all of whose sides are equal.
(2) An **isosceles triangle** is a triangle two of whose sides are equal.
(3) A **scalene triangle** is a triangle no two of whose sides are equal.

436. The **base** of a triangle is the side upon which it is supposed to stand, and the **altitude** is the perpendicular drawn from the vertex opposite the base to the base, or the base extended.

Important Facts

1. *In a right triangle the square on the hypotenuse equals the sum of the squares on the other two sides.*



Thus, the square whose side is AC contains as many square units as the two squares whose sides are AB and BC.

2. *The sum of the angles of a triangle is 180°.*

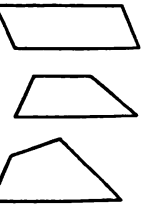
3. *The angles opposite the equal sides of an isosceles triangle are equal.*

4. *Each angle of an equilateral triangle is 60°.*

5. *If a right triangle has an angle of 30°, the side opposite this angle is one half the hypotenuse.*

Quadrilaterals

437. Quadrilaterals are divided into *parallelograms*, *trapezoids*, and *trapeziums*.



438. (1) A **parallelogram** is a quadrilateral whose opposite sides are parallel.

(2) A **trapezoid** is a quadrilateral two, and only two, of whose sides are parallel.

(3) A **trapezium** is a quadrilateral no two of whose sides are parallel.

439. The side on which a parallelogram is supposed to rest is called the **base**, and the perpendicular drawn from the side opposite the base to the base is called the **altitude.**

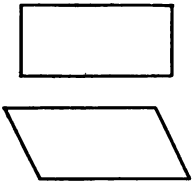
440. The parallel sides of a trapezoid are called the **bases,** and the perpendicular joining the bases is called the **altitude.**

Parallelograms

441. Parallelograms are divided into *rectangles* and *rhomboids.*

442. (1) A **rectangle** is a parallelogram whose angles are right angles. A rectangle whose sides are all equal is a **square.**

(2) A **rhomboid** is a parallelogram whose angles are not right angles. A rhomboid whose sides are all equal is a **rhombus.**



Important Facts

1. *The opposite sides of a parallelogram are equal.*
2. *The sum of any two adjacent angles of a parallelogram is 180°.*
3. *The diagonals of a parallelogram bisect each other.*
4. *A diagonal of a parallelogram divides the parallelogram into two equal triangles.*
5. *The diagonals of a rectangle are equal.*

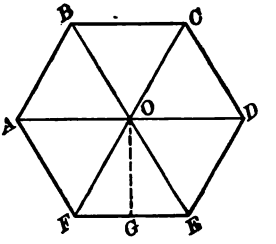
Regular Polygons

443. A **regular polygon** is a polygon whose sides and whose angles are equal.

444. The **center** of a regular polygon is a point within equally distant from the vertices; as the point *O.*

445. A **radius** of a regular polygon is a straight line drawn from the center to a vertex; as *OA*.

446. An **apothem** of a regular polygon is a perpendicular drawn from the center to a side; as *OG*.



A Regular Polygon

Important Facts

1. *The radii of a regular polygon are equal.*

2. *The apothems of a regular polygon are equal.*

3. *The radii of a regular polygon divide it into equal isosceles triangles.*

4. *The side of a regular hexagon equals the radius.*

5. *The apothem of a regular hexagon equals* $\frac{1}{2}\sqrt{3}$ *times the radius.*

Exercise 142

1. Cut from paper or cardboard figures like those shown in Section 430, and learn their names.

2. Cut from paper or cardboard triangles like those shown in Section 432, and learn their names.

3. Cut from cardboard triangles like those shown in Section 434, and learn their names.

4. If the base of a right triangle is 2 and its altitude is 1, what is the square of its hypotenuse?

5. Two angles of a triangle are 60° and 70°; what is the third angle?

6. The angle opposite one of the equal sides of an isosceles triangle is 50°; how many degrees are there in each of the other angles of the triangle?

7. One angle of an equilateral triangle is 60°; how many degrees are there in each of the other angles?

8. If one angle of a right triangle is 20°, how many degrees are there in each of the other angles?

9. One angle of a triangle is 100° and another is 30°; is the third angle acute, right, or obtuse?

10. Two angles of a triangle are 20° and 50°; is the third angle acute, right, or obtuse?

11. Two angles of a triangle are 30° and 60°; is the third angle acute, right, or obtuse?

12. Cut from paper or cardboard quadrilaterals like those shown in Section 438, and learn their names.

13. Cut from paper or cardboard parallelograms like those shown in Section 442, and learn their names.

14. One side of a parallelogram is 10 ft.; how long is the opposite side?

15. One diagonal of a rectangle is 6 ft.; how long is the other diagonal?

16. If one angle at the base of a right triangle is 30° and the hypotenuse is 12 ft., what is the altitude?

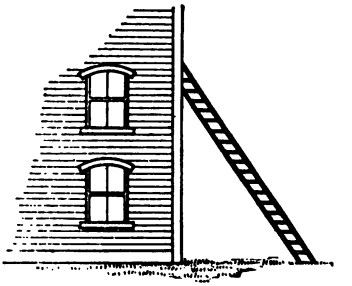
17. The base of a right triangle is 8 ft. and the altitude is 6 ft.; what is the hypotenuse?

SUGGESTION. $h = \sqrt{8^2 + 6^2}$, in which h stands for the number of units in the hypotenuse.

18. The hypotenuse of a right triangle is 29 ft. and the base is 21 ft.; what is the altitude?

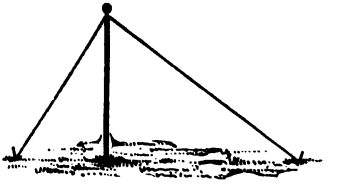
SUGGESTION. $a^2 + b^2 = h^2$, in which a, b, and h stand for the number of units in altitude, base, and hypotenuse, respectively; therefore, $a^2 = h^2 - b^2$ (§ 256).

19. The hypotenuse of a right triangle is 15 ft. and its altitude is 12 ft.; what is the base?



20. A ladder 20 ft. long is placed against a house so that the foot is 12 ft. from the house and on the same level with it; what is the distance from the top of the ladder to the ground?

21. A pole 52 ft. high stands on level ground; how long a rope will it take to reach from the top to a stake 39 ft. from the foot of the pole?



22. Two ropes, one 36 ft. long and the other 50 ft. long are fastened 30 ft. above the ground to a pole, and are stretched to reach two stakes on opposite sides of the pole; how far apart are the two stakes, the surrounding ground being level?

23. Find the length of the longest line that can be drawn on a floor 24 ft. long and 18 ft wide.

Areas

Finding the Area of a Parallelogram

447. If the shaded part of the parallelogram (Fig. 1) be cut off and placed as shown in Fig. 2, the parallelogram will be changed into a rectangle of the same base and altitude. The area of a rectangle is equal to the product of the number of units in the base and the number of units in the altitude; therefore:

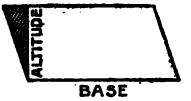


Fig. 1.

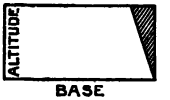


Fig 2.

448. The area of a parallelogram is equal to the product of the number of units in the base and the number of units in the altitude.

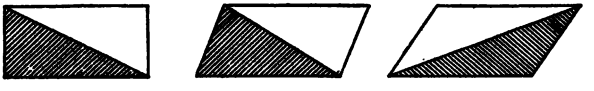
NOTE. The base and altitude must be stated in terms of the same unit.

449. If b represents the number of units in the base, a the number of units in the altitude, and S the area of a parallelogram,

$$S = a \times b.$$

Finding the Area of a Triangle

450. Each triangle in black is one half of a parallelogram of the same base and altitude, as may be seen by making figures like these, and by placing the shaded part of each upon the other part.



The area of a parallelogram is equal to the product of the number of units in the base and the number of units in the altitude; therefore:

451. The area of a triangle is equal to one half of the product of the number of units in the base and the number of units in the altitude.

452. If b represents the number of units in the base, a the number of units in the altitude, and S the area of a triangle,

$$S = \frac{a \times b}{2}.$$

453. To find the area of a triangle when its three sides are given, apply the following:

From half the sum of the three sides, subtract each side separately; multiply together the number of units in the half sum and in each of the three remainders, and extract the square root.

Illustrative Example

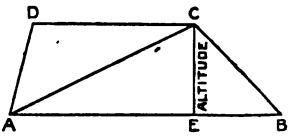
Let it be required to find the area of a triangle whose sides are 5 ft., 7 ft., and 8 ft. long.

$\frac{1}{2}$ of $(5+7+8)$, or $10 =$ the number of units in $\frac{1}{2}$ the perimeter.

$\therefore \sqrt{10 \times (10-5) \times (10-7) \times (10-8)}$, or $\sqrt{300}$, or $17.32 =$ the number of square feet in the triangle.

Finding the Area of a Trapezoid

454. Trapezoid $ABCD$ is divided into two triangles by the diagonal AC. The altitude of the trapezoid is the altitude of each triangle.



Area of triangle $CAB = \frac{1}{2}\ CE \times AB$.

Area of triangle $ADC = \frac{1}{2}\ CE \times DC$.

Area of trapezoid $ABCD = \frac{1}{2}\ CE \times (AB + DC)$.

455. The area of a trapezoid is equal to the product of half the number of units in the altitude and the number of units in the sum of the bases.

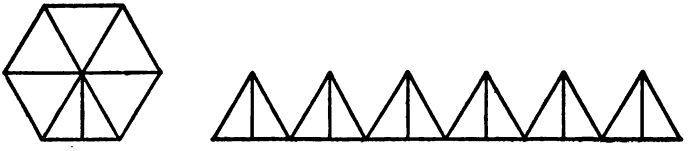
456. If B represents the number of units in the lower base, b the number of units in the upper base, a the number of units in the altitude, and S the area of a trapezoid,

$$S = \tfrac{1}{2}\, a(B + b).$$

Finding the Area of a Regular Polygon

457. A regular polygon may be divided into triangles, as shown here, each triangle having for its base a side of the polygon, and for its altitude the apothem of the polygon.

The sum of the bases of the triangles is the perimeter of the polygon.



The area of each triangle is equal to half the product of the apothem and a side of the polygon [§ 451]. The sum of the areas of the triangles is equal to half the product of the number of units in the apothem and the number of units in the sum of the sides; or:

458. The area of a regular polygon is equal to half the product of the number of units in the apothem and the number of units in the perimeter.

459. If a represents the number of units in the apothem, P the number of units in the perimeter, and S the area of a regular polygon,

$$S = \frac{a \times P}{2}.$$

Illustrative Examples

1. Find the area of a parallelogram whose base is 12 ft. and altitude 4 ft. 6 in.

Explanation:

12 ft. = the base.
4 ft. 6 in., or $4\frac{1}{2}$ ft. = the altitude.
$\therefore 4\frac{1}{2} \times 12$, or 54 = the number of square feet required.

Those who prefer to use formulæ may solve thus:
In a parallelogram, $S = a \times b$.
In the given parallelogram, $a = 4\frac{1}{2}$ and $b = 12$.
$\therefore S = 4\frac{1}{2} \times 12$, or 54.

2. Find the area of a triangle whose base is 5 rd. and altitude 20 yd.

Explanation:

5 rd., or $27\frac{1}{2}$ yd. = the base.
20 yd. = the altitude.
$\therefore \frac{20 \times 27\frac{1}{2}}{2}$, or 275 = the number of square yards required.

REMARK. Those who desire to do so may use formula of Section 452.

3. Find the area of a trapezoid whose bases are 15 ft. and 7 ft., and whose altitude is 5 ft.

Explanation:

15 ft. and 7 ft. = the bases.
5 ft. = the altitude.
$\therefore \frac{5}{2} \times (15 + 7)$, or 55 = the number of square feet required.

REMARK. Those who desire to do so may use formula of Section 456.

4. Find the area of a regular hexagon one of whose sides is 4 ft. and whose apothem is 3.46 ft.

Explanation:

6×4 ft., or 24 ft. = the perimeter.

3.46 ft. = the apothem.

$\therefore \frac{3.46 \times 24}{2}$, or 41.52 = the number of square feet required.

REMARK. Those who desire to do so may use formula of Section 459.

Exercise 143

Find the area of each of these parallelograms:

	Base	Altitude		Base	Altitude
1.	10 ft.	8 ft.	2.	9 ft.	5 ft.
3.	8 yd.	7 ft.	4.	3 rd.	4 yd.
5.	6 ft. 4 in.	9 ft.	6.	4 yd. 2 ft.	7 ft.

Find the area of each of these triangles:

	Base	Altitude		Base	Altitude
7.	12 ft.	7 ft.	8.	10 ft. 6 in.	20 ft.
9.	8 yd.	21 ft.	10.	4 rd. 2 ft.	12 ft.
11.	6 ch.	8 rd.	12.	4 ft. 9 in.	6 ft. 6 in.

Find the area of each of these trapezoids:

	Bases	Altitude		Bases	Altitude
13.	6 ft. and 4 ft.	10 ft.	14.	12 ft. and 3 yd.	10 ft.
15.	7 yd. and 6 ft.	9 ft.	16.	6 ft. and $4\frac{1}{2}$ ft.	2 ft.
17.	8 ch. and 3 ch.	6 ch.	18.	5 ft. and $6\frac{1}{2}$ ft.	2 yd.

In the following find the missing number:

Class of Polygon	Base	Altitude	Area
19. Parallelogram	8 yd.	6 ft.	( ) sq. ft.
20. Parallelogram	12 ft.	( ) ft.	72 sq. ft.
21. Parallelogram	( ) yd.	6 ft.	144 sq. ft.
22. Triangle	40 ft.	8 ft.	( ) sq. ft.
23. Triangle	10 yd.	( ) yd.	75 sq. yd.
24. Triangle	( ) rd.	30 rd.	150 sq. rd.
25. Parallelogram	8 ch.	5 ch.	( ) acres
26. Parallelogram	12 ch.	( ) ch.	24 acres
27. Triangle	( ) ch.	20 rd.	5 acres

28. Find the area of a regular hexagon one of whose sides is 8 ft., and whose apothem is 6.93 ft.

29. Find the area of a regular octagon one of whose sides is 10 ft., and whose apothem is 12.07 ft.

30. Find the area of a regular pentagon one of whose sides is 4 ft., and whose apothem is 2.75 ft.

31. Find the area of a triangle whose sides are 13 ft., 14 ft., and 15 ft.

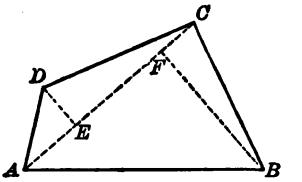
32. The base of a rectangle is 48 ft. and the altitude is 36 ft. What is the diagonal?

33. A field in the form of a rhomboid contains 9 A. 96 sq. rd., and the altitude is 32 rd. What is the base?

34. The perimeter of a rectangle is 112 ft. and the width is 24 ft. What is the area?

35. A square lot of land that contains 1 acre is how many feet long? Answer correct to the nearest tenth of a foot.

36. To find the area of trapezium $ABCD$, find the areas of the triangles of which it is composed. Diagonal AC is 34 ft. long and the perpendiculars DE and BF upon the diagonal are 4 ft. and 13 ft., respectively.



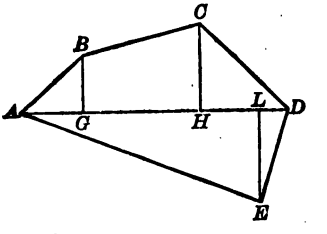
37. How many board feet are there in a board 16 ft. long, 18 in. wide at one end, and 12 in. wide at the other end?

38. The perimeter of a rectangle that is 80 rd. long is 200 rd. How many acres does it contain?

39. The length of a rectangle is to its width as 5 is to 4, and it contains 8 A. Find its length and its width in chains.

40. One side of a field in the form of a trapezoid measures 24 ch. 80 li., another side parallel to it measures 18 ch. 40 li., and the distance between these sides is 12 ch. 60 li. How many acres does the field contain?

41. Find the area of the polygon $ABCDE$ in which $AG=2$ ch., $GH=4$ ch., $HL=2$ ch., $LD=1$ ch., and the perpendiculars $BG=2$ ch., $CH=3$ ch., and $EL=3$ ch.



NOTE. The area of a polygon may be found by dividing it into right triangles and trapezoids, and taking the sum of their areas.

The Circle

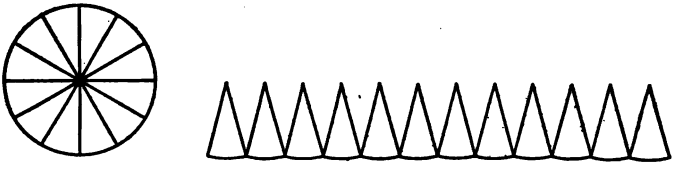
(For definition of terms and important facts see page 229.)

460. It is proved in geometry that the ratio of the circumference of a circle to the diameter is 3.1416, nearly. The Greek letter π (pi) stands for this ratio; then, if the circumference is represented by C, the diameter by D, and the radius by R,

$$(1)\ \frac{C}{D}=\pi.\quad (2)\ C=\pi D.\quad (3)\ C=2\pi R.$$

NOTE. Sometimes $\frac{22}{7}$ instead of 3.1416 is used to express the ratio of the circumference to the diameter.

461. A circle may be cut and separated, as here shown, into a series of figures that resemble triangles.



If we should call them triangles, each figure might then be regarded as having a curved side for the base and the radius of the circle for the altitude. The sum of the areas of the figures, or the area of the circle, would be the product of half the number of units in the radius and the number of units in the sum of the bases. It is shown in geometry that the area of the circle is equal to this product; therefore:

462. The area of a circle is equal to the product of half the number of units in the radius and the number of units in the circumference.

463. If C represents the number of units in the circumference, R the number of units in the radius, and S the area of the circle,

$$S = \tfrac{1}{2} R \times C.$$

Since $C = 2\pi R$,

$$S = \tfrac{1}{2} R \times 2\pi R.$$

Or, $S = \pi R^2$ (the area in terms of the radius)

Illustrative Examples

1. Find the area of a circle whose radius is 4 ft.

Explanation:

4 ft. = the radius.

In a circle, $S = \pi R^2$.

$S = 3.1416 \times 4^2$, or 50.2656.

That is, the circle contains 50.2656 sq. ft.

2. Find the area of a circle whose circumference is 31.416 ft.

Explanation:

31.416 ft. = the circumference.

$C = 2\pi R$.

$R = 31.416 \div 6.2832$, or 5.

$S = \pi R^2$.

$\therefore S = 3.1416 \times 5^2$, or 78.54.

That is, the circle contains 78.54 sq. ft.

Exercise 144

Find the areas of circles whose radii are as follows:

1. 6 in. 2. 4 yd. 3. 12 rd. 4. 2 ch. 5. 2 ft. 6 in.

Find the areas of circles whose diameters are as follows:

6. 10 yd. 7. 5 ch. 8. 6.4 ft. 9. 2.75 rd. 10. 5.9 ch.

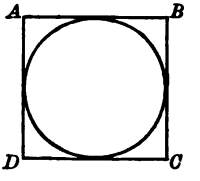
Find the areas of circles whose circumferences are as follows:

11. 18.8496 ft. 12. 31.416 rd. 13. 3.1416 ch.

14. 12.5664 in. 15. 628.32 yd. 16. 157.08 ft.

17. Find the perimeter and area of a square circumscribed about a circle whose diameter is 8 ft.

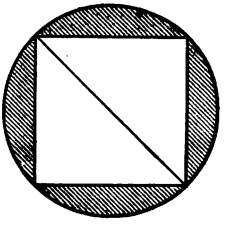
18. Find the difference between the circumference of a circle and the perimeter of a circumscribed square whose side is 12 ft.



A Circumscribed Square

19. Find the difference between the area of a circle and the area of a circumscribed square whose length is 10 ft.

20. A square courtyard, as large as possible, is made within a circle whose diameter is 66 ft., and the remainder of the circle is sodded. How many square feet are sodded? Answer correct to the nearest .1 ft.



SUGGESTION. The diameter of the circle is the diagonal of the square.

21. A cow is fastened by a rope 20 yards long to a stake in the center of a square lot containing 1600 sq. yd. Over how much land can she graze?

22. Find the area of the largest square that can be cut from a circular board whose diameter is 20 in.

MENSURATION OF SOLIDS

General Definitions

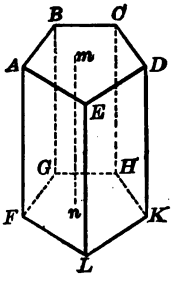
464. A **solid** is any bounded portion of space; as, a cube.

465. The **volume** of a solid is the number of cubic units it contains.

Right Prisms and Cylinders of Revolution

Definitions

466. A **right prism** is a solid bounded by two equal and parallel polygons, called **bases**, and by rectangles called **lateral faces.**



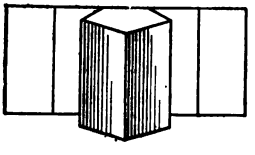
467. The sum of the areas of the lateral faces is called the **lateral area.**

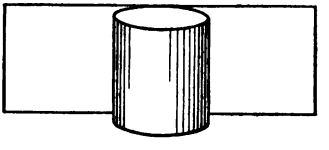
The perpendicular line joining the bases is the **altitude**; as, *mn*.

REMARK. For treatment of the cube and rectangular solid, which are classes of right prisms, see pp. 192–194. For partial treatment of the cylinder of revolution, see pp. 232–233. In this book right prisms and cylinders of revolution only are treated; they will be called merely prisms and cylinders.

Areas

468. The lateral surface of a prism or of a cylinder may be thought of as unrolled like a rectangular sheet of





paper, the perimeter of the base of the prism or the circumference of the base of the cylinder being the base of the rectangle, and the altitude of the prism or the cylinder, the altitude of the rectangle.

The area of a rectangle being the product of the number of units in the altitude and the number of units in the base, it follows that:

469. 1. **The lateral area of a prism is equal to the product of the number of units in the altitude and the number of units in the perimeter of the base.**

2. **The lateral area of a cylinder is equal to the product of the number of units in the altitude and the number of units in the circumference of the base.**

Volumes

470. The rectangular solid (Fig. 1), the prism (Fig. 2), and the cylinder (Fig. 3) have equal altitudes and the same number of square units in their bases.

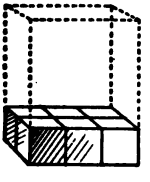


Fig. 1.

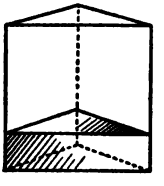


Fig. 2.

Fig. 3.

In Fig. 1 it is seen that a layer 1 unit high contains as many cubic units as there are square units in the base; hence for each layer one unit high in Figs. 2 and 3 there are as many cubic units as there are square units in the base. In each figure the number of cubic units is equal to the number of cubic units in a layer multiplied by the number of layers; it follows that:

471. The volume of a prism or of a cylinder is equal to the product of the number of units in the altitude and the number of square units in the base.

472. If S denotes the lateral area, V the volume, H the number of units in the altitude, P the number of units in the perimeter of the base, and B the number of square units in the base of a prism,

$$(1)\ S = P \times H, \qquad (2)\ V = B \times H.$$

If S denotes the lateral area, V the volume, H the number of units in the altitude, R the number of units in the radius of the base of a cylinder,

$$(3)\ S = 2\pi RH, \qquad (4)\ V = \pi R^2 H.$$

Exercise 145

1. Find the lateral area of a prism each side of whose square base is 6 ft. and whose altitude is 3 ft.

SUGGESTION. In a prism, $S = P \times H$. [§ 472, 1]
$\therefore S = 4 \times 6 \times 3.$

2. Find the lateral area of a cylinder whose altitude is 6 in. and whose radius is 4 in.

SUGGESTION. In a cylinder, $S = 2\pi RH$. [§ 472, 3]
$\therefore S = 2 \times 3.1416 \times 4 \times 6.$

3. Find the volume of a prism the area of whose base is 64.96 sq. ft. and whose altitude is 8 ft.

SUGGESTION. In a prism, $V = B \times H$. [§ 472, 2]
$\therefore V = 64.96 \times 8.$

4. Find the volume of a cylinder whose altitude is 5 in. and whose radius is 6 in.

SUGGESTION. In a cylinder, $V = \pi R^2 H$. [§ 472, 4]
$\therefore V = 3.1416 \times 6^2 \times 5.$

5. Find the lateral area of a prism whose base is a square 4 ft. long, and whose altitude is 6 ft.

6. Find the lateral area of a prism whose base is a regular hexagon 2 ft. on a side, if the altitude of the prism is 8 ft.

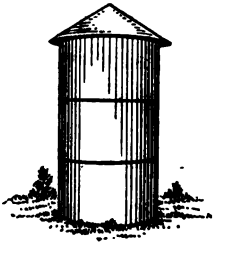
7. Find the lateral area of a cylinder whose radius is 4 in. and whose altitude is 1 ft.

8. Find the volume of a prism whose base is a square 8 ft. on a side, and whose altitude is 18 in.

9. What is the volume of a cylinder whose radius is 6 in., and whose altitude is 1 ft. 6 in.?

10. Find the number of square feet in the entire surface of a cube whose edge is 3 in.

11. Find, correct to the nearest gallon, the capacity of a tank in the form of a cylinder, the radius of whose base is 3 ft. and whose altitude is 6 ft.



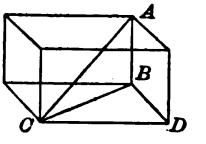
12. This figure represents a cylindrical silo 20 ft. deep and 10 ft. in diameter. How many cubic feet will it hold?

13. What is one side of the base of a square prism whose volume is 150 cu. ft. and whose altitude is 6 ft.?

14. Find the weight of a granite pillar of uniform size, whose base is a circle with a radius of 12 in. and whose height is 14 ft., if 1 cu. ft. of granite weighs $165\frac{5}{8}$ lb.

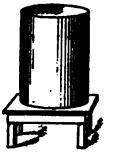
15. What is the distance from the upper corner of a room to the opposite lower corner, if the length of the room is 16 ft., the width 14 ft., and the height 12 ft.?

SUGGESTION. Let AC be the diagonal of the room; then $\overline{AC}^2 = \overline{AB}^2 + \overline{BC}^2$, and $\overline{BC}^2 = \overline{BD}^2 + \overline{CD}^2$; whence, $\overline{AC}^2 = \overline{AB}^2 + \overline{BD}^2 + \overline{CD}^2$.



16. What must be the depth of a bin 10 ft. long and 5 ft. wide to hold 12 tons of wheat, regarding 1 bu. as equal to $\frac{5}{4}$ of a cu. ft.?

17. This cylinder represents a tank 6 ft. in diameter and 8 ft. high. Find, correct to the nearest gallon, its capacity.

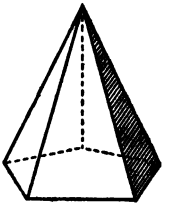


18. Find the altitude of a cylinder that contains 125.664 cu. ft., if the radius of its base is 2 ft.

Regular Pyramids and Cones of Revolution

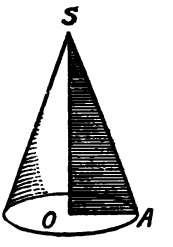
Definitions

473. A **regular pyramid** is a solid bounded by a regular polygon, called the **base**, and a number of equal isosceles triangles, called **lateral faces**, which meet in a point called the **vertex** of the pyramid.



474. The sum of the lateral faces is the **lateral area**, the perpendicular from the vertex to the base is the **altitude**, and the line drawn from the vertex to the middle point of a side of the base is the **slant height**.

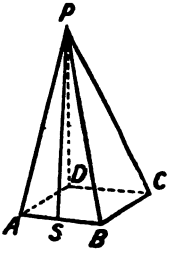
475. A **cone of revolution** is a solid formed by revolving a right triangle about one of its perpendicular sides as an axis.



476. The side about which the right triangle revolves is the **altitude**, the side perpendicular to it is the **radius of the base**, and the hypotenuse is the **slant height** of the cone.

REMARK. In this book regular pyramids and cones of revolution only are treated; they will be called merely pyramids and cones.

Areas



477. In the pyramid P–$ABCD$,

$$\text{Area } PAB = \tfrac{1}{2}\, PS \times AB.$$
$$\text{Area } PBC = \tfrac{1}{2}\, PS \times BC.$$
$$\text{Area } PCD = \tfrac{1}{2}\, PS \times CD.$$
$$\text{Area } PDA = \tfrac{1}{2}\, PS \times DA.$$
$$\text{Lateral area} = \tfrac{1}{2}\, PS \times (AB + BC + CD + DA).$$

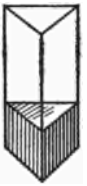
478. The lateral area of a pyramid is equal to the product of one half the number of units in the slant height and the number of units in the perimeter of the base.

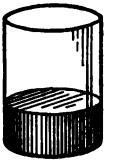
479. The lateral area of a cone is nearly equal to the lateral area of a pyramid whose base is a regular polygon of a great number of sides, whose slant height is the same as the slant height of the cone and the perimeter of whose base nearly equals the circumference of the base of the cone. Just as the lateral area of a pyramid is the product of one half of the number of units in the slant height and the number of units in the perimeter of the base, so it is proved in geometry that:

480. The lateral area of a cone is equal to the product of one half the number of units in the slant height and the number of units in the circumference of the base.

Volumes

481. By taking a hollow pyramid and a hollow prism of equal base and equal altitude, it may be shown, by measuring, that the pyramid holds one third as much as the prism. In the same way it may be shown that a cone holds one third as much as a cylinder of equal base and equal altitude; hence, from Section 471 it follows that:





482. The volume of a pyramid or of a cone is equal to the product of one third of the number of units in the altitude and the number of square units in the base.

483. If S denotes the lateral area, V the volume, H the number of units in the altitude, L the number of units in the slant height, P the number of units in the perimeter of the base, and B the number of square units in the base of a pyramid,

$$(1)\ S = \frac{P \times L}{2}, \qquad (2)\ V = \frac{B \times H}{3}.$$

484. If S denotes the lateral area, V the volume, H the number of units in the altitude, L the number of units in the slant height, R the number of units in the radius of the base of a cone,

$$(1)\ S = \pi RL, \qquad (2)\ V = \frac{\pi R^2 H}{3}.$$

Exercise 146

1. Find the lateral area of a pyramid whose base is a regular hexagon 8 ft. on a side, if the slant height of the pyramid is 10 ft.

SUGGESTION. $S = \frac{P \times L}{2}$. [§ 483]

$$\therefore S = \frac{6 \times 8 \times 10}{2}.$$

2. Find the lateral area of a cone the radius of whose base is 6 in. and whose slant height is 8 in.

SUGGESTION. $S = \pi RL$. [§ 484]

$$S = 3.1416 \times 6 \times 8.$$

3. Find the volume of a pyramid the area of whose base is 93.53 sq. yd. and whose altitude is 6 yd.

SUGGESTION. $V = \frac{B \times H}{3}$. [§ 483]

$$\therefore V = \frac{93.53 \times 6}{3}.$$

4. Find the volume of a cone the radius of whose base is 3 in. and whose altitude is 6 in.

SUGGESTION. $V = \frac{\pi R^2 H}{3}$. [§ 484]

$$\therefore V = \frac{3.1416 \times 3^2 \times 6}{3}.$$

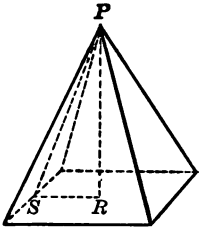
5. Find the lateral area of a pyramid whose base is a regular pentagon 10 in. on a side and whose slant height is 12 in.

6. Find the lateral area of a cone whose radius is 1 ft. and whose slant height is 18 in.

7. Find the volume of a pyramid whose base is a square 6 in. on a side and whose altitude is 9 in.

8. How many cubic inches will a conical vessel hold that is 9 in. high and the radius of whose base is 3 in.?

9. A side of a square pyramid is 6 ft. and the altitude (PR) is 4 ft. Find the length of the slant height PS.



SUGGESTION. PRS is a right triangle.

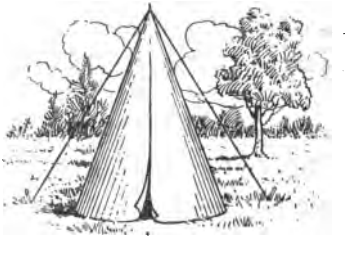
10. Find the lateral area of a pyramid whose slant height is 5 ft. and whose altitude is 3 ft., if its base is a square.

11. Find the volume of a pyramid whose base is a square 6 ft. on a side, if the slant height of the pyramid is 5 ft.

12. The slant height of a cone, the radius of whose base is 6 in., is 10 in. Find the altitude.

13. Find the lateral area and volume of a cone the radius of whose base is 8 in. and whose slant height is 10 in.

14. Find the volume of a cone the radius of whose base is 12 in. and whose slant height is 15 in.



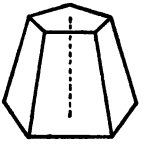
15. How many square yards of canvas are required to make a conical tent 18 ft. in diameter at the base and 12 ft. high?

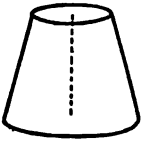
16. Find the lateral area and volume of a pyramid whose base is a square 24 ft. on a side, the altitude of the pyramid being 16 ft.

17. Find the lateral area and volume of a cone whose altitude is 15 ft. and the diameter of whose base is 40 ft.

Frustums

485. A frustum of a pyramid or of a cone is the part between the base and a plane parallel to the base.





REMARK. In this book frustums of regular pyramids and cones of revolution only are treated. They are called merely frustums.

486. The **lateral surface** of a frustum is the entire surface less the bases, and the area of the lateral surface is the **lateral area.** The line joining the centers of the bases is the **altitude,** and that part of the slant height of the pyramid or of the cone, included between the bases of the frustum, is the **slant height** of the frustum.

487. To find the lateral area of a frustum:

Multiply the number of units in the sum of the perimeters or circumferences of the bases by half the number of units in the slant height.

488. To find the volume of a frustum:

Multiply the number of square units in the lower base, plus the number of square units in the upper base, plus the square root of the product of these numbers, by one third of the number of units in the altitude.

Exercise 147

1. Find the lateral area of a frustum of a pyramid whose bases are squares, if each side of the upper base is 4 ft., each side of the lower base 12 ft., and the slant height of the frustum 5 ft.

2. Find the lateral area of a frustum of a cone the radii of whose bases are 3 ft. and 1 ft. and whose slant height is 6 ft.

3. Find the volume of the frustum of a pyramid whose bases are squares, if each side of the lower base is 3 in., each side of the upper base 2 in., and the altitude of the frustum 6 in.

4. Find the lateral area of the frustum of a pyramid whose bases are regular hexagons, if each side of the lower base is 10 in., each side of the upper base 5 in., and the slant height 8 in.

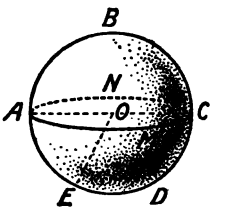
5. Find the volume of the frustum of a cone the radii of whose bases are 6 in. and 4 in. and whose altitude is 10 in.

6. How many cubic inches of water will a vessel hold which is 6 in. in diameter at the top, 4 in. in diameter at the bottom, and 9 in. deep?

7. How many square feet of zinc will be required to line the sides and bottom of an ash hopper 5 ft. square at the top and 3 ft. square at the bottom, if the slant height is 6 ft.?

8. How many quarts of milk, correct to the nearest hundredth, will a tin bucket hold whose top diameter is 14 in., bottom diameter 10 in., and depth 8 in.?

Spheres



489. A **sphere** is a solid bounded by a curved surface every point of which is equally distant from a point within called the center.

A **diameter** of a sphere is a straight line drawn from surface to surface through the center; as *AC*.

A **radius** of a sphere is a line drawn from the center to the surface. It is half a diameter; as *OC*.

A **circumference** of a sphere is a circumference of a circle whose radius is the radius of the sphere; as *AMCN*.

490. If S denotes the area of a surface, V the volume, and R the number of units in the radius of a sphere, then,

(1) $S = 4\pi R^2$, (2) $V = \frac{4}{3}\pi R^3$.

Exercise 148

1. Find the area of the surface of a sphere whose radius is 2 in.

SUGGESTION. $S = 4\pi R^2$. [§ 490, 1]
$\therefore S = 4 \times 3.1416 \times 4$.

2. Find the volume of a sphere whose radius is 3 in.

SUGGESTION. $V = \frac{4}{3}\pi R^3$. [§ 490, 2]
$\therefore V = \frac{4}{3} \times 3.1416 \times 27$.

Find the area of surfaces of spheres with the following radii:

3. 6 in. 4. 4 ft. 5. 2 rd. 6. 5 ch.
7. 3.2 in. 8. 4.5 ft. 9. 6.75 yd. 10. 3.4 ch.

Find the volumes of spheres with the following radii:

11. 1 in. 12. 5 ft. 13. 12 rd. 14. 7.5 ch.

15. 1.5 in. 16. 2.75 ch. 17. 12.5 rd. 18. .5 ft.

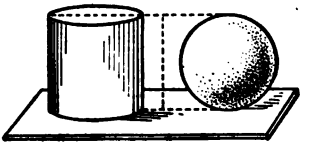
19. Find the area of the surface of a sphere whose circumference is 12.5664 ft.

20. Find the volume of a sphere whose circumference is 18.8496 yd.

21. What is the radius of a sphere the area of whose surface is 113.0976 sq. ft.?

22. Considering the earth to be a sphere whose radius is 4000 miles, find the area of the earth's surface.

23. The diameter of this sphere, the diameter of this cylinder, and the altitude of the cylinder are each 3 in. Find the volume of each, the area of the surface of the sphere, and the lateral surface of the cylinder. Compare the volumes and also the areas.



Similar Figures

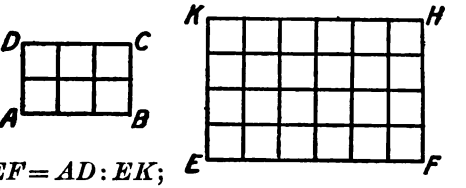
491. Similar figures are such as have the same shape.

Thus, all squares are similar, all circles are similar, all cubes are similar, and all spheres are similar.

492. In similar figures lines that are similarly placed are said to be **corresponding**, or **homologous**.

Thus, the radii of two circles are homologous lines, also the radii of two spheres.

493. In similar rectangles $ABCD$ and $EFHK$, AB and EF are homologous; also AD and EK.



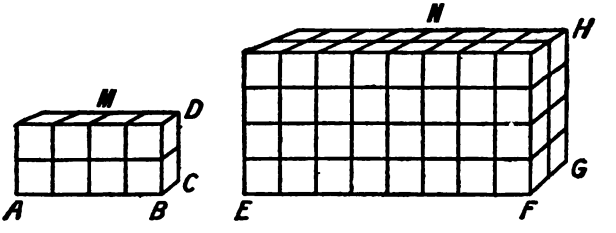
Observe that: (1) $AB : EF = AD : EK$;

(2) Area $ABCD$: Area $EFHK = \overline{AB}^2 : \overline{EF}^2 = \overline{BC}^2 : \overline{FH}^2$.

494. 1. **In similar surfaces homologous lines are proportional.**

2. **The areas of two similar surfaces are to each other as the squares of any two homologous lines.**

495. In similar rectangular solids M and N, AB and EF are homologous, BC and FG are homologous, CD and GH are homologous.



Observe that: (1) $AB : EF = BC : FG = CD : GH$;

(2) $M : N = \overline{AB}^3 : \overline{EF}^3 = \overline{BC}^3 : \overline{FG}^3 = \overline{CD}^3 : \overline{GH}^3$.

496. 1. **In similar solids homologous lines are proportional.**

2. **The volumes of two similar solids are to each other as the cubes of any two homologous lines.**

REMARK. $\overline{AB}^2 : \overline{EF}^2$ is understood to mean the ratio of the square of the *number* of units in AB to the square of the *number* of units in EF. Similarly, $\overline{AB}^3 : \overline{EF}^3$ denotes the ratio of the cube of the *number* of units in AB to the cube of the *number* of units in EF.

Exercise 149

1. The bases of two similar triangles are 6 ft. and 10 ft. and the altitude of the first is 2 ft. What is the homologous altitude of the second?

SUGGESTION. Let x be the number of feet in the required altitude.

2. The bases of two similar triangles are 3 ft. and 6 ft. If the area of the first is 24 sq. ft., what is the area of the second?

3. The radii of two circles are 4 ft. and 5 ft. and the circumference of the first is 12.5664 ft. What is the circumference of the second?

4. The area of an equilateral triangle one of whose sides is 5 ft. is 10.826 sq. ft. Find the area of an equilateral triangle one of whose sides is 10 ft.

5. What is the ratio between the area of two circles whose radii are 1 ft. and 2 ft.?

6. The altitudes of two similar cones are 4 ft. and 5 ft. If the volume of the first is 800 cu. ft., what is the volume of the second?

7. A farmer has two fields of the same shape, their lengths being 30 rd. and 20 rd. If the width of the second is 8 rd., what is the width of the first?

8. If a cone 3 in. high weighs 54 lb., what is the weight of a similar cone 4 in. high if made of the same material?

9. If a sphere whose radius is 1 in. weighs 10 lb., what is the weight of a sphere of the same material whose radius is 2 in.?

10. If 240 gallons of water will flow through a certain orifice 3 in. in diameter in a certain time, how much water will flow through an orifice 6 in. in diameter in the same time?

SUGGESTION. The amounts of water or other liquid that flow through two similar orifices are to each other as the squares of the homologous lines of the orifices.

11. If it requires 18 hours for a pipe 2 in. in diameter to empty a certain cistern, how long will it require a pipe 1 in. in diameter to empty the same cistern?

SUGGESTION. The time is inversely proportional to the squares of the diameters of the pipes.

REVIEW

Exercise 150

1. How many acres does a rectangular field contain that is 20 ch. long and 8 ch. wide?

2. How many acres does a field contain if it has the form of a right triangle whose base is 16 ch. and hypotenuse 20 ch.?

3. A square lawn is surrounded by a walk 8 ft. 3 in. wide; the walk and the lawn together contain 6.4 acres. Find the cost of paving the walk at $0.50 per square yard.

4. Find the volume of a prism whose base is a right triangle, the perpendicular sides of which are 20 ft. and 21 ft., if the altitude of the prism is 11 ft.

5. Find the lateral area of a prism whose base is a right triangle, the perpendicular sides of which are 3 ft. and 4 ft., if the altitude of the prism is 10 ft.

6. The length of a rectangle is to its width as 5 is to 4 and its area is 2 acres. Find its length and its width.

7. A has a rectangular lot whose frontage is 60 ft. and depth 120 ft.; on it he builds a house 48 ft. long and 42 ft. wide. The remainder of the lot is made into a yard. Find the area of the yard.

8. A rectangular glass 36 in. long and 20 in. wide is surrounded by a frame 4 in. wide. Find the area of the frame.

9. A rug 24 ft. long and 18 ft. wide is placed in the center of a floor 28 ft. long and 22 ft. wide. How many square feet of the floor remain uncovered?

10. If a rectangular piece of land 88 yd. long and 40 yd. wide produced 960 lb. of wheat, what was the yield per acre, wheat weighing 60 lb. to the bushel?

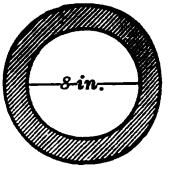
11. The area of the entire surface of a cube is 54 sq. in. What is its volume?

12. The bases of a trapezoid are 12 ft. and 10 ft. and the altitude is $8\frac{1}{2}$ ft. Find the area.

13. Find, correct to the nearest hundredth of a square foot, the area of a regular hexagon each side of which is 10 ft.

14. Find the total area of a cylinder if the altitude is 8 ft. and the radius of its base 4 ft. (Use $\pi = \frac{22}{7}$.)

15. How many square inches are there in the surface of a frame 1 in. wide around a clock dial 8 in. in diameter?



16. Trees are planted 40 ft. apart on both sides of a certain street for the distance of a mile. Find the number of trees.

17. For one inch of rainfall what weight of water falls on one square yard of ground, 1 cu. ft. of water weighing 62.5 lb.?

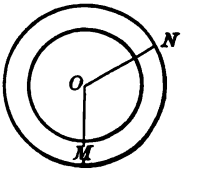
18. The diameters of the earth and the moon are as 1 to $\frac{3}{11}$. How many times as large as the moon is the earth?

19. Find the weight of a granite block 4 ft. long, 3 ft. wide, and 18 in. high, granite weighing 2.65 times as much as an equal bulk of water, which weighs 62.5 lb. per cubic foot.

20. A cubic foot of iron was formed into a bar 3 in. square. Find its length.

21. Find the cost, at 15 cents per square foot, of lining the sides and bottom of a cubical tank which is 6 ft. long.

22. Find the number of square feet in the walk surrounding the circle whose radius OM is 6 yd., if the distance ON from the center of the circle to the outer edge of the walk is 8 yd.



23. The length of the minute hand of the clock on City Hall, Philadelphia, is 10 ft. 8 in. How many feet does the point of this hand move in an hour?

24. A line 50 ft. long extends from the top of a center pole 40 ft. high to the edge of a circus ring. Find the area of the ring.

25. How many square feet of boards are required for the sides and ends of a barn 50 ft. long, 36 ft. wide, and 24 ft. high to the square, the ridgepole being 16 ft. higher than the square?

26. How many revolutions per minute are made by a locomotive wheel 16 ft. in circumference when the locomotive is running 45 miles per hour?

27. Regarding 1 cu. ft. as equal to $\frac{16}{25}$ of a heaped bushel, how many bushels of ashes, heaped measure, will a hopper hold whose upper base is a square 4 ft. long, and lower base a square 9 ft. long, the depth of the hopper being 6 ft.?

28. How many gallons will a cubical tank hold that is 4 ft. long?

29. Find the number of square feet in an equilateral triangle each of whose sides is 6 ft.

30. Estimating that a bushel is equal to $\frac{5}{4}$ of a cubic foot, how many bushels of wheat will a box hold that is 6 ft. long, 4 ft. wide, and 5 ft. deep?

31. Find, correct to the nearest square yard, the amount of rubber cloth required to make 100 footballs, spherical in form, the diameter of each to be 1 ft.

32. A tract of land 10 ch. long and 3.75 ch. wide was bought at $75 per acre. The purchaser had it inclosed by a fence which cost $0.60 per rod. Find the total cost.

33. If a pipe 2 in. in diameter will exhaust the water in a certain cistern in $1\frac{1}{2}$ hr., in what time will a pipe $\frac{1}{2}$ in. in diameter exhaust the water?

34. A farmer has a field in the form of a right triangle containing 2 A. 142 sq. rd. If one of the perpendicular sides is 242 yd., find the other.

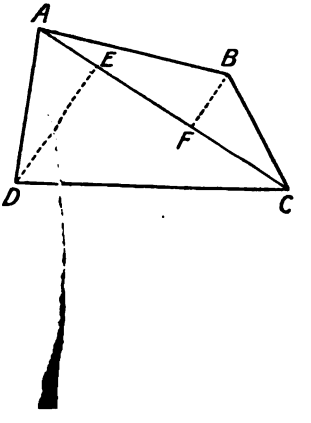
35. 1 cu. ft. of water equals how many quarts? Answer correct to the nearest .01 qt.

36. A cube whose volume is 1000 cu. in. was plated uniformly with 331 cu. in. of copper. Find the thickness of the plating.

37. Find the cost of 86 planks, each 16 ft. by 6 in. by 3 in., at $24 per M board feet.

38. Find in feet per minute the flow of water through a pipe 8 in. in diameter, if the discharge is 50 gal. per minute. Answer correct to the nearest .01 ft. (Use $\pi = \frac{22}{7}$.)

39. $ABCD$ is a field whose diagonal AC is 18 ch., and the perpendiculars DE and BF are 12 ch. and 6 ch. Find the number of acres in the field.



GENERAL REVIEW

1. A man has 150 quarter-acre lots. How many half-acre lots can he make of these?

2. 100,000 five-cent pieces are worth how many dollars? how many quarter dollars?

3. Find the selling price of 240,000 eggs at 25¢ per dozen.

4. Estimating a step to be a yard, how many steps wide must a patch of ground be made in order that it may contain an acre, if it is 88 steps long?

5. Tell which of the following numbers are (1) prime, (2) composite, (3) even, (4) odd; and give the reason for your answer: 18, 15, 23, 2, 1, 49, 81, 121, 93, 69.

6. Tell at sight which of the numbers 2, 3, 4, 5, and 9 will divide: 109; 111; 124; 819; 478; 215; 2340.

7. Sound travels 1090 ft. per second. Express this rate in miles per hour.

8. When a locomotive is running at the rate of 40 mi. per hour, how many revolutions does the driving wheel make per minute, if its circumference is 20 ft.?

9. Divide the difference between .5 and $.33\frac{1}{3}$ by their sum, and express the result as a fraction.

10. The quotient resulting from solving a problem in division is 48. What would be the quotient if:

(1) the dividend were multiplied by 2?
(2) the divisor were multiplied by 3?
(3) the divisor were divided by 3?
(4) the dividend were multiplied by 12?

11. Onions should yield about 400 bu. per acre. At this estimate, how much should a boy receive from a plat of land 110 yd. long and 11 yd. wide, planted with onions, if he sells the crop at $0.50 per bushel?.

12. Find the area of the entire surface of a cube whose volume is 1000 cu. in.

13. Show that a vessel 11 ft. long, 7 ft. wide, and 3 ft. deep will hold 1728 gal.

14. A street 33 ft. wide and $\frac{3}{4}$ mi. long is paved and curbed. The paving costs $1.75 per square yard and each line of curbing 25¢ per linear foot. Find the total cost.

15. A bushel of oats contains 10 oz. of nitrogen, 3 oz. of phosphoric acid, and 2 oz. of potash. How many pounds of each of these essential fertilizing substances were taken from the soil by a crop of 6 A. of oats that yielded 40 bu. per acre?

16. A man agreed to work for $20 a month and house rent free; he worked only 9 mo., but continued to use the house. The house rent was estimated at $100 per year. How much was due him at the end of the year?

17. How many grains of pure gold are there in a piece of jewelry 18 carats fine, if it weighs $\frac{3}{4}$ of an ounce?

18. My bill for 756 rails was $68.04. Find the price per hundred.

19. When lots 300 ft. deep are selling at $10 per front foot, what would be the selling price per acre at the same rate?

20. A bushel of shelled corn contains 14 oz. of nitrogen, 5 oz. of phosphoric acid, and 3 oz. of potash. How many pounds of each of these fertilizing substances are taken from each acre of the soil by a corn crop of 50 bu. per acre?

21. A farmer paid $70.10 for 5608 lb. of bran. Find the price per ton.

22. In a recent year there were shipped from Portland, Ore., to foreign countries 5,201,367 bu. of wheat, valued at $4,531,865. Find, correct to the nearest tenth of a cent, the average value per bushel.

23. Find the number of tons of rails, and the number of ties required for each mile of a single-track railroad, if each rail used is 30 ft. long and weighs 3000 lb., and there are 16 ties to each 30 ft. of track.

24. A good ration for cows consists of 10 bu. of corn and 10 bu. of oats ground together, and 1 T. of clover hay. This should last a cow 120 da. How much ground feed and how much hay does this allow the cow per day?

25. The perimeter of a rectangular field 4 ch. wide is 84 ch. How many acres does it contain?

26. In one year recently the farmer in the United States received $6,000,000,000 for his produce, while the consumer finally paid $13,000,000,000 for the same produce. The amount the consumer paid was how many times the amount the farmer received?

27. Count by $7\frac{1}{2}$'s to 120.

28. Simplify: $\frac{.875 \times \frac{3}{8} \times .16\frac{2}{3} \times .625 \times 2.5}{.375 \times \frac{7}{8} \times .33\frac{1}{3} \times .125 \times 5}$. (Cancel.)

29. What average price per pound does a farmer receive for his tobacco crop consisting of 2000 lb. of wrappers which he sells for $10\frac{1}{2}¢$ per pound and 500 lb. of fillers which he sells for $3¢$ per pound?

30. The distance between two places on a map is $1\frac{3}{8}$ in. If the actual distance between these places is 13.75 mi., the map is drawn on a scale of how many miles to the inch?

31. Assuming that one geographical mile equals 6086 ft., find, correct to the nearest thousandth, the number of statute miles in a league.

32. How many acres must be planted with corn to produce silage sufficient for a herd of 20 cows for 180 da., if each cow is to be fed 40 lb. of silage per day, and each acre produces 15 T. of silage?

33. A lot in one of our largest cities was recently sold for $1000 per front foot. The lot has a frontage of 20 ft. and a depth of 94 ft. What would be the price per acre at the same rate?

34. How many boards 12 ft. long are required to build a fence 40 rd. long and 4 boards high? If the boards are 6 in. wide and 1 in. thick, find their cost at $25 per 1000 board feet.

35. A herd of cows is to be fed 300 lb. of silage per day. How long will the silage produced from 2 A. keep this herd, if each acre planted with corn produced 15 T. of silage?

36. At what rate per cent is a man taxed who pays $2\frac{1}{2}$ mills on each $1 of the valuation of his property?

37. What per cent is gained on pencils bought at $1.80 per gross and retailed at 3¢ each?

38. An article was sold for $10 at a gain of $33\frac{1}{3}$ %. Had the cost been $0.50 more than it was, what would have been the gain per cent?

39. The enrollment of a certain school is 225 boys and 310 girls. What per cent of the pupils enrolled are boys and what per cent are girls? Answer correct to the nearest .01 %.

40. When lard is advanced from 12¢ per pound to 17¢, what is the per cent of advance?

41. An attorney collected a debt, retained a commission of $7\frac{1}{2}$ %, and remitted to his client $370. What commission did he charge?

42. When a laborer's wages are increased from $23\frac{1}{2}$¢ per hour to 25¢, what is the per cent of increase, correct to the nearest .1 %?

43. A merchant gains 20 % when he sells goods at wholesale. If his retail price is 20 % above his wholesale price, find his gain per cent when he sells at retail.

44. A merchant sold a couch for $15, which was at a discount of 25 % from the marked price. If his gain was 20 %, find what per cent the marked price was above cost.

45. A dealer gained 200 % by selling matches at the rate of 2 boxes for 3¢. What per cent would he have gained, if he had sold them at the rate of 3 boxes for 5¢?

46. A dealer gained 25 % when he sold lemons at 25¢ per dozen. What per cent did he gain on those that he sold at the rate of 3 for 10¢?

47. A dealer bought 50 bu. of white lime at 20¢ per bushel and paid $5 for having it hauled. At what price per peck must he retail it to make a profit of 60 %?

48. Two successive discounts of 20 % and $12\frac{1}{2}$ % on a bill of goods amount to $21. What is the bill?

49. If 35¢ interest is charged for $100 for 1 month, what rate per cent per annum is charged?

50. At what price must an article costing $0.80 be marked so that after deducting $16\frac{2}{3}$ % from the marked price a dealer may make a profit of $12\frac{1}{2}$ %?

51. Goods are marked to sell at a profit of 60 %, if sold at retail; the retail price is fixed at $33\frac{1}{3}$ % above the wholesale price. Find the gain per cent on the goods when sold at wholesale.

52. A invests 40 % of his money at 3.5 %, 35 % of it at 4 %, 10 % of it at 6 %, and the remainder at 5.5 %. What average rate of interest does he make on his money?

53. An article was marked at $ 4, but was sold at a discount of 20 % and 12½ %. Find the selling price.

54. A man charges $ 30 a month rent for property that costs him $ 5000. The outlay on the property for a certain year was: taxes of $ 1.50 per $ 100 on an assessed valuation of $ 3500; fire insurance 90¢ per $ 100 on a policy of $ 4000; repairs $ 31.50. What was the per cent of net income for that year?

55. The assessed valuation of the property of a certain town is $ 1,250,000. How many mills tax must be levied on each dollar to raise $ 5000?

56. At what price must an article costing $ 3.20 be marked so that after deducting 20 % from the marked price a profit of 25 % may be made?

57. Find, correct to the nearest cent, the interest on $ 1100 from Nov. 22 to Feb. 28 of the following year, if the rate is 5 %.

58. Find the gain on an article that sold for 30¢ at a profit of 500 %.

59. A dealer bought lemons at 20 ct. a dozen and twice as many at 30 ct. a dozen. If he mixed them and retailed them all at 40 ct. a dozen, find his gain per cent.

60. The publisher's price for a book is \$0.90, but he sells to retailers at a discount of $16\frac{2}{3}\%$. If the retailer sells the book at the publisher's price, what per cent does the retailer gain?

61. What should be the selling price per pound of a mixed spice formed of 8 lb. costing 50 ct. per pound, 8 lb. costing 40 ct. per pound, and 4 lb. costing 60 ct per pound, in order that the profit may be 25%?

62. A dealer sold 2 lots for \$1000 each, gaining 25% on the one and losing 20% on the other. Find his gain or loss.

63. A boy buys apples at the rate of 4 for 1¢ and twice as many at the rate of 2 for 1¢. If he sells them all at 1¢ each, find his gain per cent.

64. A piece of property was taxed \$33.60 at a tax rate of $\frac{3}{4}$ of one per cent. Find the assessed value of the property.

65. Green coffee loses 10% of its weight in roasting. At what price must the grocer sell the roasted coffee to gain 20%, if he paid 27¢ per pound for the green coffee?

66. If envelopes are bought at the rate of \$1 per 1000 and retailed at the rate of 2 for 1¢, what is the gain per cent?

67. If a manufacturer sells goods to a retailer at a profit of 20% and the retailer sells them to the consumer at a profit of 25%, what fractional part of the cost of the goods to the consumer is the cost of manufacturing them?

68. I sold coal at $6.25 per ton, thereby making a profit of 25%, and a total profit of $250. How many tons did I sell?

69. My agent purchased for me 154,014 lb. of corn at $0.60 per bushel (56 lb.). He prepaid freight amounting to 5¢ per hundredweight and charged me a commission of 2%. Find the total cost of the corn.

70. A farmer's herd of 20 cows averaged 8.75 lb. of milk daily for the month of November. The milk tested 3.6% butter fat, which he sold at 32¢ per pound. What was his income for the month?

71. A dealer bought a bunch of 72 bananas for $0.90. He retailed half of them at the rate of 2 for 5¢, a dozen of them at the rate of 3 for 5¢, and the remainder at 1¢ each. Find his gain per cent.

72. A tax of $22,315 is to be raised in a town whose property is assessed at $4,275,800 and 624 polls. If the polls are assessed at $1.50 each, what must be the rate of taxation?

73. Find the duty on a rug 16 ft. by 12 ft., whose dutiable value is $120 and on which there is charged a specific duty of 10¢ per square foot and 40% ad valorem.

74. A house and lot cost $2500 and $1850 respectively. The house was insured for $\frac{4}{5}$ of its value at $\frac{3}{5}$%. If the house was destroyed by fire within a year, and the lot was sold for $2350, find the gain or loss.

75. If a building is insured for $6000 and the annual premium is $19.20, find the annual rate per $100. What is the rate per $100 for 2 years? What is the rate per $100 for 3 years?

76. Find the annual premium that a man must pay on a 20-year endowment policy for $10,000 taken out April 1, 1913, if the insured was born December 20, 1887.

77. July 1, A borrowed $200 at $4\frac{1}{2}\%$, and when the principal had amounted to $203.75, he paid the debt. On what date did he make payment?

78. My agent purchased for me 1000 bu. of wheat at $0.80 per bushel. I sent him a check for $856, which paid the cost of the wheat, freight charges amounting to $40, and his commission. What rate of commission did he charge me?

79. What is the interest for 1 year on a $10 postal savings certificate, the rate of interest being 2%?

80. If the income from $100 invested in a certain kind of stock was $4.3846, estimated correct to the nearest tenth of a mill, find the investor's rate of income, estimated correct to:

(1) The nearest one per cent.
(2) The nearest tenth per cent.
(3) The nearest hundredth per cent.
(4) The nearest half per cent.
(5) The nearest quarter per cent.

81. When 6% stock pays 4% on the investment, what is its market value, if the rate of brokerage is $\frac{1}{8}\%$?

82. What rate of dividend does stock pay, if an investor realizes $4\frac{1}{2}\%$ on his investment, the stock costing him $\$ 120$ per share?

83. A sent his broker $\$ 2445$ to pay for 40 shares of P. R. R. stock (par value $\$ 50$), brokerage $\frac{1}{8}\%$. Find the market value of each share.

84. Find the profit on 20 shares of stock (par value $\$ 50$), bought at $\$ 58\frac{1}{4}$ per share and sold at $\$ 60\frac{1}{4}$, if two semiannual dividends of $2\frac{1}{2}\%$ each were received between the transactions, brokerage $\frac{1}{4}\%$ on the sale and the purchase.

85. What sum must be invested in stock at 199%, paying quarterly dividends of 2%, in order that the annual income may be $\$ 200$, brokerage $\frac{1}{8}\%$?

86. A laborer deposited $\$ 100$ in a savings bank at the beginning of a year, and withdrew principal and interest at the beginning of the next year. How much was he entitled to, if the savings bank paid 3% interest compounded quarterly?

87. What must be paid for sufficient P. R. R. stock (par value $\$ 50$) selling at $\$ 61$ per share, brokerage $\frac{1}{8}\%$, to yield the investor an annual income of $\$ 375$, if this stock pays a semiannual dividend of 3%?

88. When 6% stock (par value $\$ 50$) is bought at $\$ 71\frac{1}{8}$ per share, brokerage $\frac{1}{4}\%$, what rate of income, correct to the nearest $.01\%$, does it pay the investor?

89. When 6% stock pays 5% on the investment, what is the market value, if the rate of brokerage is $\frac{1}{4}\%$?

90. Type metal is made from antimony and lead mixed in the ratio of 1 part of the former to $4\frac{1}{2}$ parts of the latter. Find, correct to the nearest .01 of a pound, the amount of each substance in a ton of type metal.

91. How many posts set 12 ft. apart are necessary to fence a rectangular tract of land 8 rd. wide, containing 2 A.?

92. A rectangular piece of land containing $1\frac{1}{4}$ A. is 121 yd. long. Find its width.

93. A piece of land in the form of a trapezoid contains 4 A. If the length of one of the parallel sides is 118 rd. and of the other 10 rd., how far are they apart?

94. What must be the market value of 6% stock (par value \$50) in order that it may pay an investor 4% interest on his money, the rate of brokerage being $\frac{1}{4}$%?

95. The diameters of two cylindrical vessels of the same shape are 3 ft. and 4 ft. If the first holds 27 gal., how much does the second hold?

96. A and B entered into partnership. A invested \$500 for 10 mo. and gained \$50; B invested his capital for 6 mo. and gained \$60. What was B's capital?

97. A lady found that if she paid \$1.10 per yard for silk, she would lack \$1.20 of having enough to pay for the quantity desired, but that if she paid \$0.85 per yard, she would have \$1.80 remaining. How much money had she?

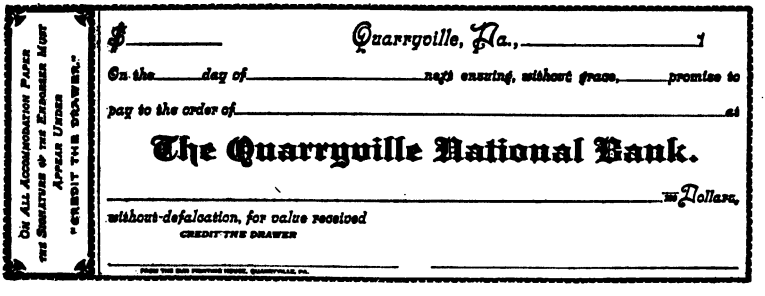
98. Through my broker I sold \$4000 U. S. 2s at $101\frac{1}{2}$% and invested the proceeds in railroad stock at

126¼%. The stock bought pays a quarterly dividend of 1¼%. Find the change in my annual income and the surplus from the sale of the bonds, brokerage ⅛% on the sale and the purchase.

99. The altitude of a prism is 12 ft. Its base is a right triangle with an hypotenuse 15 ft. long, and one side 12 ft. long. Find the lateral area and the volume of the prism.

100. A man at the age of 25 years took out an ordinary life policy for $1000; at the age of 30 years a twenty-payment life policy for another $1000; at the age of 35 a twenty-year endowment policy for another $1000; he died at the age of between 39 and 40 years. By how much did the amount of insurance exceed the sum of the premiums paid?

101. Copy and fill out the following note, dating it July 1 of the current year, and making it payable at the expiration of 60 days. Use appropriate names and explain the transaction that this note would require. Write such indorsements as are necessary before the Quarryville National Bank will cash the note:


$ Quarryville, Pa., 1
On the day of next ensuing, without grace, promise to
pay to the order of at
The Quarryville National Bank.
Dollars,
without defalcation, for value received
CREDIT THE DRAWER
DO ALL ACCOMMODATION PAPER THE SIGNATURE OF THE ENDORSER MUST APPEAR UNDER "CREDIT THE DRAWER."

102. Find the amount due on the following note Dec 31, 1913. When is the note due?

$150.00 Chicago, Ill. Feb 4, 1913.

Nine months after date for value received I promise to pay to the order of F. L. Fields

One hundred fifty Dollars,

with interest at the rate of 6 per cent. per annum after April 4, 1913.

Payable at Continental and Commercial National Bank

No. Due P. J. Garrett

103. Find the area of the regular hexagon whose side is 4 rd.

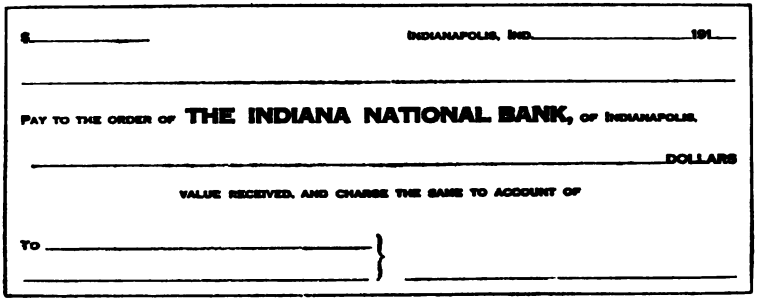
104. A tract of land in the form of a square containing 10 A. was divided into 4 equal square lots and fenced. At $1.50 per rod, find the cost of the fence necessary to inclose the 4 lots.

105. A and B are 180 yd. apart and walk toward each other, A walking 2 yd. while B walks 3 yd. How far will each walk before they meet?

106. A pole 10 ft. long casts a shadow 12 ft. long at the same time that a tree casts a shadow 60 ft. long. How high is the tree?

107. Write a promissory note dated Sept. 1 of the current year, payable in 60 da., with yourself as payee and H. P. Worth as maker. Properly indorse the note for discount and find the proceeds, if discounted at a bank in your state Sept. 1.

108. Copy and fill out the following draft, making yourself drawer, and W. C. Dunlap drawee. Date the draft May 15 of the current year and make it payable 60 days after sight, face $ 50 :


$
INDIANAPOLIS, IND. 191
PAY TO THE ORDER OF THE INDIANA NATIONAL BANK, OF INDIANAPOLIS
DOLLARS
VALUE RECEIVED, AND CHARGE THE SAME TO ACCOUNT OF
TO

After filling out a form like the above, write the acceptance. Explain which of the three gets the $ 50, and from whom. When is the draft due? Who holds the draft after it has been paid?

109. A Pittsburg firm drew a sight draft for $ 1000 on a New Orleans firm and had the draft cashed at a Pittsburg bank. How much was received for it, exchange being at $\frac{1}{2}\%$ discount?

110. A lawn 60 yd. long and 50 yd. wide is surrounded by a walk 6 ft. wide. Find the cost of graveling this walk at $ 0.12 per square yard.

111. A sphere whose radius is 6 in. weighs 324 oz. Find the weight of a sphere of the same material whose radius is 10 in.

112. Find the diagonal of a cube whose edge is 10 inches.

113. One man can build a fence in 9 days, working 10 hours a day; another can build it in $4\frac{1}{2}$ days, working 8 hours a day. In how many days can they build it together, working 6 hours a day?

114. Two thirds of my money equals four fifths of yours. If we put our money together, what part of the whole will you own?

115. A cylindrical tank 40 ft. high and 15 ft. in diameter is represented in a drawing as 15 in. high. What is the length of the diameter as represented in the drawing?

116. A painter received \$ 3 per day for his labor and paid \$ 5 per week for his board, that being his only expense; at the expiration of 10 wk. he had saved \$ 100. How many days was he idle?

117. A and B are 180 yd. apart and travel in the same direction. A travels 3 yd. while B travels 5 yd. How far will each travel before B overtakes A?

118. Three men invested jointly \$ 3000, and after a certain time, when the gains were to be divided, it was found that A was entitled to \$ 3 as often as B to \$ 4 and C to \$ 5. Find the investment of each.

119. Find the volume of a triangular pyramid the sides of whose base are 52 ft., 56 ft., and 60 ft., and whose altitude is 21 ft.

120. Find, correct to the nearest .01 of an acre, the area of a circle whose radius is 5 chains.

121. How many $\frac{1}{2}$-in. cubes can be placed in a box $4\frac{1}{2}$ in. long, $2\frac{1}{2}$ in. wide, and 2 in. high?

122. A farmer drills his corn in rows $2\frac{3}{4}$ ft. apart. How many rods are there in the width of a field that contains 160 rows, the outside ones being 1 yd. from the fence?

123. A high wind that has a velocity of 30 mi. per hour exerts a force of 4.428 lb. per square foot. What force would such a wind exert on a tight board fence 80 yd. long and 6 ft. high?

124. If it requires 1 lb. of barbed wire of a certain kind for each wire in a rod of fence, how many pounds of wire would be required to build a fence of 3 wires around a field, the sides of which measure 90 rd., 75 rd., 60 rd., and 52 rd.?

125. A year consists of 365 da. 5 hr. 48 min. 49.7 sec. How much time is lost in 4 yr. by counting 365 da. to the year?

126. Find the missing term in each of the following:

$$x : \tfrac{2}{3} = \tfrac{3}{4} : 1 \qquad 1 : \tfrac{1}{2} = x : 4$$
$$\tfrac{3}{4} : x = \tfrac{1}{4} : 2 \qquad \tfrac{2}{3} : \tfrac{3}{4} = \tfrac{4}{9} : x$$

127. Find $\sqrt{21 \times 42 \times 200}$ by first reducing the numbers to their prime factors.

128. How many cubic feet of masonry are necessary to build the breast of a dam, if the wall is 8 ft. thick at the base, 4 ft. thick at the top, 40 ft. long, and 8 ft. high?

129. The quantity of air taken into the lungs at each ordinary inspiration of an adult is on an average 30 cu. in. Assuming that he takes 17 inspirations per minute, how many cubic feet of air does he breathe in 24 hr.?

130. If $\frac{3}{4}$ of A's money equals $\frac{3}{5}$ of B's, then $\frac{3}{4}$ of B's is what part of A's?

131. Find the lateral area of the frustum of a cone the radii of whose bases are 10 ft., and 4 ft. respectively, and whose altitude is 8 ft.

132. Two men are to receive $48 for painting a barn. If they work together 6 days and then one of them finishes the work in 4 days more, how much should each receive?

133. What is the diameter of a wheel which makes 480 revolutions in going a mile?

134. One pipe can fill a tank in 20 hr. and another can fill it in 10 hr. The tank can be emptied by a spigot in 15 hr. If the tank is empty and the pipes and spigot set to flow at the same time, in how many hours will the tank be filled?

135. One geographic mile equals one minute of the earth's equatorial circumference, or about 6086 ft. Find the circumference of the earth in statute miles.

136. A woman deposited $200 in a savings bank at the beginning of the year and withdrew principal and interest at the end of 18 mo. How much was she entitled to, if the savings bank paid 3% interest compounded semi-annually?

137. If a ball 1 ft. in diameter weighs 24 lb., what is the weight of a ball $\frac{1}{2}$ ft. in diameter, made of the same material?

138. A barn valued at \$ 2400 was insured for $\frac{3}{4}$ of its value. Find the annual premium, if the rate charged was \$ 2.60 per \$ 1000 of insured value.

139. Six men agreed to do a piece of work in a certain time, but, two of them failing to come, the work was prolonged 5 days. How long would it have taken the 6 men to do the work?

140. Find the length of the diagonal of a rectangular solid 12 ft. long, 9 ft. wide, and 10 ft. high.

141. 1 cu. in. of steel weighs as much as 7.83 cu. in. of water. Find the weight of 1 cu. ft. of steel, water weighing 62.5 lb. per cubic foot.

142. Find the number of square feet in a square whose perimeter is 1 chain.

143. To hold silage sufficient for a herd of 10 cows for 6 mo., allowing each cow 40 lb. daily, a silo should have a diameter of 10 ft. and a height of 25 ft. How many cubic feet will this silo hold?

144. The area of a rectangle is 490 sq. ft., and its length is to its width as 5 is to 2. Find its perimeter.

145. Find the area of an equilateral triangle each of whose sides is 4 rods.

146. A field in the form of a rectangle contains 7 A. 80 sq. rd.; it is 40 rd. long. How many rods of fence are necessary to inclose it?

147. A dealer gained 15% when he sold bran at $1.15 per hundredweight. What per cent would he gain by selling it at $22.50 per ton?

148. A woman paid 45¢ for a dozen eggs that weighed 21 oz. At this rate what is a dozen eggs worth that weighs 28 oz.?

149. The four sides of a tract of land measure $108\frac{1}{3}$ rd., $116\frac{2}{3}$ rd., 75 rd., and 100 rd. The last two sides form a right angle. Find the number of acres in the tract.

150. Find the interest on $60 for 1 day at 6%.

151. A class of 150 pupils was examined in grammar, 20% of them being boys and the rest girls. If $16\frac{2}{3}$% of the boys and 20% of the girls failed, what per cent of the class passed?

152. At what price must I purchase 3% stock (par value $100) to secure the same rate of income that I obtain from 4% stock (par value $50) purchased at $59\frac{7}{8}$ per share, brokerage $\frac{1}{4}$% in each case?

153. What is the cost of a city square, exclusive of streets, at $1000 per acre, if the square is $\frac{1}{8}$ of a mile long and $\frac{1}{12}$ of a mile wide, measuring from the center of a 60 ft. street on each side?

154. A father willed $6600 to his two sons, whose ages are 11 years and 16 years, respectively, so that they may have equal sums on coming of age, if the money is loaned at 5%; how much did each receive?

155. A whitewash adopted by the United States Government is composed of 10 parts of freshly slaked lime to 1 part of hydraulic potash, and the whole is mixed with salt water. How much potash must be mixed with 2 bu. 2 pk. of lump lime, if the lump lime in slaking doubles its volume?

156. A school was open 20 da. during a certain month; 6 pupils attended every day, 8 attended 19 da., 4 attended 18 da., 4 attended 17 da., and 1 attended 8 da. Find the average daily attendance for that month.

157. Find the average cost per pound of 10 lb. of Japan tea costing 75¢ per pound, 40 lb. of Formosa costing 60¢ per pound, 80 lb. of Salada costing 60¢ per pound, and 90 lb. of Ceylon costing 70¢ per pound.

158. A man owns the northwest corner lot, rectangular in form, fronting 100 ft. on New Street, which runs east and west, and 80 ft. on State Street, which runs north and south. Along each street a pavement 10 ft. wide is taken from the lot. Back 20 ft. from each street the ground is staked out for a house fronting 40 ft. on New Street, the house to be 30 ft. wide.

(*a*) Make a drawing, scale $\frac{1}{2}$ in. to 10 ft., representing the outline of the pavements, the lot, and the cellar of the house.

(*b*) Find the number of square yards of the lot that remains after the pavement is made and the house built.

159. On what day of a leap year does the first half of it end?

160. The five-cent piece weighs 77.16 gr., and it is three fourths copper and one fourth nickel. How many ounces (Troy) of copper are used in making five dollars' worth of five-cent pieces?

161. The standard gold dollar weighs 25.8 grains and is .1 alloy. A ten-dollar gold piece contains sufficient pure gold to make how many grains of jewelry 18 carats fine?

162. If soil is cultivated to the depth of 4 in., how many cubic yards of tillable soil are there to the acre?

163. If $1\frac{1}{2}$ times A's money equals $2\frac{1}{2}$ times B's, which has the more money? How much has each, if the difference of their amounts is \$4?

164. What time, expressed in years, days, hours, and minutes, would be required to count 1,000,000,000, provided that there was no cessation in the counting from the time the counting was begun until it ended and that the counting was proceeded with at the rate of 60 per minute? Estimate 365 da. to the year.

165. A farmer sowed 20 bu. of oats, and the yield was 400 bu. What was the per cent of yield?

166. An article marked £6 5*s*. was bought at a discount of 20%. Find the cost.

167. A bought flour at \$4.10 a barrel, payable in 6 months. If he wishes to pay cash, what should he pay per barrel, money being worth 5%?

168. Show that the common interest at 6 % is equal to the principal multiplied by the number of days in the time and the product divided by 6000.

169. How many shares of stock at $90\frac{1}{2}$ % can be purchased with the proceeds of 60 bonds (face value $100) at $110\frac{1}{4}$ %, brokerage in each transaction $\frac{1}{8}$ %; and what will be the surplus?

170. Write a promissory note for $200, dated at your place of residence July 1 of the current year, payable in 60 days. Make John Williams the payee and yourself the maker. Properly indorse the note for discount and find the proceeds, if the note is discounted at a bank in your state July 7.

171. What is the difference in the amount of the premiums paid on a $5000 twenty-payment life policy taken at the age of 25, and a ten-payment life policy taken at the age of 35 for the same amount, if the insured lives to the end of the endowment periods?

172. The sum of two numbers is 200 and the ratio of the larger to the smaller is 7 to 3. What are the numbers?

173. A train can run from A to B in $5\frac{1}{2}$ hr., if it runs uniformly 30 mi. per hour; what should be its speed to make the run in $3\frac{3}{4}$ hr.?

174. A man bought sufficient fertilizer for 8 A. of land, allowing 240 lb. to the acre; he, however, sowed the amount of fertilizer bought on 10 A. How much did he sow to the acre?

175. A, B, and C invest \$2000, \$2500, and \$3000, respectively, and after being in business 6 years, dissolve partnership. If \$12,000 is realized from the sale of their joint property, how much of this should each receive?

176. A, B, and C invested jointly \$3000. A's capital was invested 12 mo., and his gain was \$144; B's capital was invested 8 mo., and his gain was \$144; C's capital was invested 6 mo., and his gain was \$180. Find the capital of each.

177. Six times the copper in a mixture equals seven times the zinc. Find the ratio of the copper to the zinc; of the copper to the mixture.

178. The quotient resulting from dividing a number successively by 2, 3, 5, and 7 is 3. The successive remainders are 1, 2, 4, and 4. Find the number divided.

179. A man divided \$4500 among his five children, two boys and three girls, giving to each boy twice as much as to each girl. How much did each receive?

180. 385 cubical blocks each 1 inch long can be built into a rectangular solid of what dimensions?

181. Divide \$67.50 among A, B, and C so that A's number of dollars shall equal B's number of quarters and C's number of dimes.

182. A dealer bought a bill of goods amounting to £5 6*s*. 3*d*. If £1 = \$4.86, what was the amount of the bill in U.S. money?

183. A passenger on a moving train observes that it takes him just 45 sec. to pass a freight train 1320 ft. long, standing on a side track. At what rate in miles per hour is his train running?

184. Simplify: $\dfrac{(1-\frac{1}{4})\times(1+\frac{1}{4})\div(1+\frac{1}{4})\times(1-\frac{1}{4})}{1\times\frac{1}{4}+1+\frac{1}{4}}$.

185. A, B, and C invested jointly \$3100. A's capital was invested 8 mo., and his gain was \$360; B's capital was invested 9 mo., and his gain was \$450; C's capital was invested 12 mo., and his gain was \$720. Find the capital of each.

186. Write all the numbers possible each with the figures 5, 6, 7. What two numbers of one figure each will divide each of the numbers?

187. A contractor has workmen enough to divide into groups of 12 men or 14 men, and in each case have one man left for overseer. How many workmen has he?

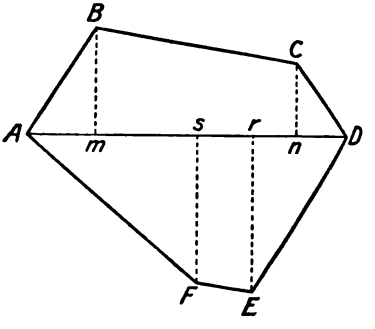
188. If one fourth of the sum a boy received for matches was gain, what was the gain per cent?

189. Of 225 weight tests of groceries purchased recently in one of our large cities, 158 showed a shortage in weight. What per cent of them were short in weight?

190. To a kitchen $18\frac{1}{2}$ ft. long and 16 ft. wide, an addition is built 12 ft. long and 16 ft. wide. What is the distance around the enlarged kitchen?

191. Find the number of acres in the field $ABCDEF$, having given the following measurements:

$AB = 15$ rd.
$DC = 10$ rd.
$DE = 20$ rd.
$AF = 25$ rd.
$Am = 9$ rd.
$As = 20$ rd.
$Ar = 26$ rd.
$An = 32$ rd.
$AD = 38$ rd.



Bm, Cn, Fs, and Er are perpendiculars drawn to the diagonal.

192. If a merchantman sailing $10\frac{1}{2}$ knots an hour is chased by a battleship sailing $13\frac{3}{4}$ knots an hour, how far ahead must the merchantman be to escape by four knots into a port which is 12 knots distant at the beginning of the race?

193. A piece of work was to be performed by 8 men in a certain time, but 3 of them failing to come, the work was prolonged 4 days; in what time would the work have been done had all come?

194. Assuming the radius of the earth to be 4000 mi., find the weight of a body 8000 mi. above the earth's surface if the body weighs 200 metric tons at the surface. [The weight of a body varies inversely as the square of its distance from the center of the earth.]

SUPPLEMENT

LATITUDE AND LONGITUDE

1. To locate places on the surface of the earth, a circumference, called the meridian of Greenwich, is supposed to be drawn through Greenwich, England, and the north and south poles of the earth. Halfway, or 90°, from either pole another circumference, called the **equator,** is supposed to be drawn east and west around the earth. These two circumferences divide each other into two equal parts of 180° each.

2. The **meridian** of a place is the half circumference supposed to be drawn from pole to pole through the place. It is divided by the equator into two equal parts of 90° each.

3. The meridian of Greenwich is called the **prime meridian,** and its opposite meridian, or the other half of the circumference of which the prime meridian is one half, is the 180th meridian.

4. The **latitude** of a place is its distance from the equator measured in degrees upon the meridian of the place, and is north or south according as the place is north or south of the equator.

5. The **longitude** of a place is the distance in degrees on the equator from the prime meridian to the meridian of the place, and is east or west according as the place is east or west of the prime meridian.

6. Parallels of latitude are circumferences extending east and west around the earth parallel to the equator.

7. The following truths are evident:

1. *All places on the equator have no latitude.*
2. *All places on the same parallel of latitude have the same latitude.*

3. *All places on the prime meridian have no longitude.*

4. *All places on the same meridian have the same longitude.*

5. *The greatest latitude that a place may have is 90°.*

6. *The greatest longitude that a place may have is 180°, being the distance from the prime meridian to the 180th meridian.*

Exercise 1

How far and in what direction by the shorter arc is:

1. 75° N. from 30° N.?
2. 5° N. from 70° N.?
3. 50° S. from 90° S.?
4. 50° S. from 40° S.?
5. 40° N. from 45° S.?
6. 90° S. from 20° N.?
7. 25° E. from 10° E.?
8. 25° E. from 90° E.?
9. 110° W. from 60° W.?
10. 25° W. from 65° W.?
11. 140° E. from 140° W.?
12. 15° W. from 55° E.?

13. The latitude of A is 60° 55′ N., and of B on the same meridian, 30° 20′ N. How far and in what direction is B from A?

14. The longitude of Portland, Me., is 70° 15′ 40″ W., and of St. Paul, Minn., 93° 5′ W. Find their difference of longitude.

15. The longitude of Vienna is 16° 23′ E., and of New York, 74° 3″ W. Find their difference in longitude.

16. The latitude of Portland, Me., is 43° 39′ 28″ N., and of Boston, 42° 21′ 28″ N. Find their difference of latitude.

17. The latitude of Savannah, Ga., is 32° 4′ 52″ N., and of Seattle, Wash., 47° 35′ 54″ N. Find their difference of latitude.

18–26. Find the difference in longitude between each city in the first table and each city in the second table.

Place	Longitude
London	9′ 17″ W.
Boston	71° 7′ 21″ W.
Manila	120° 58′ E.

Place	Longitude
Philadelphia	75° 9′ 5″ W.
Paris	2° 20′ 45″ E.
San Francisco	122° 24′ 15″ W.

LONGITUDE AND TIME

8. The earth rotates on its axis from west to east in 24 hours. This makes the sun appear to revolve about the earth from east to west once in the same time.

Since the sun appears to revolve through 360° in 24 hours, its apparent motion is 15° per hour, 15′ per minute, and 15″ per second.

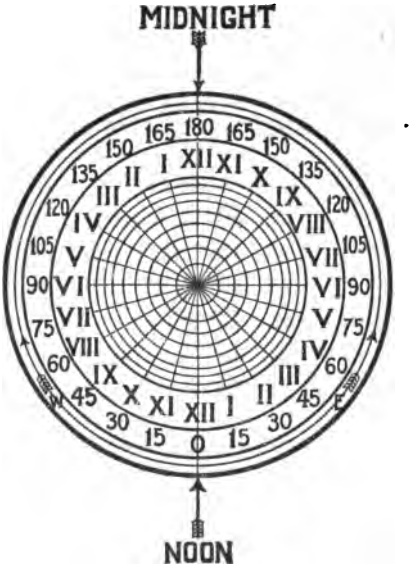
It is noon at any place when the sun is on the meridian of that place, and since the sun apparently moves 15° from east to west in 1 hour, it is then 1 P.M. 15° east of that place, 11 A.M. 15° west of it, 2 P.M. 30° east of it, 10 A.M. 30° west of it, and so on. From this we see that a difference of 15° in the longitude of two places makes a difference of 1 hour in their time. Similarly, a difference of 15′ in their longitude makes a difference of 1 minute in their time, and a difference of 15″ in their longitude makes a difference of 1 second in their time. Hence:

9. *$\frac{1}{15}$ of the number of degrees, minutes, and seconds in the difference of longitude of two places equals the number of hours, minutes, and seconds in the difference of their time.*

10. *15 times the number of hours, minutes, and seconds in the difference of time of two places equals the number of degrees, minutes, and seconds in the difference of their longitude.*

REMARK. It is evident from § 8 that places east of any given meridian have later time and those west of it earlier time than places on the meridian.

11. This figure shows the times on different given meridians when the time is noon on the meridian of Greenwich.



Exercise 2

(Refer to figure, Sec. 11, in answering the questions of this exercise; answer as many as possible orally.)

State how many degrees and in what direction by the shorter arc is:

1. 135° E. from 30° E.
2. 45° W. from 60° E.
3. 75° W. from 15° W.
4. 120° E. from 30° W.
5. 15° E. from 105° E.
6. 135° E. from 165° W.
7. 30° W. from 120° W.
8. 120° W. from 120° E.

State what meridian is:

9. 45° east of 75° E.
10. 30° east of 15° W.
11. 30° west of 60° W.
12. 60° west of 45° E.
13. 30° west of 120° E.
14. 90° west of 120° W.
15. 15° east of 90° W.
16. 60° east of 150° E.

Observe that:

If A and B stand for two cities, and A is west of B, A's time is 1 hour earlier (slower) than B's for every 15° that A is west of B.

If A and B stand for two cities, and B is east of A, B's time is 1 hour later (faster) than A's for every 15° that B is east of A.

What is the time at:

17. 90° E. when the time at 30° E. is 6 P.M.?
18. 150° W. when the time at 90° W. is 2 P.M.?
19. 30° E. when the time at 75° E. is 4 P.M.?
20. 15° W. when the time at 60° W. is 9 A.M.?
21. 45° W. when the time at 15° E. is 1 P.M.?
22. 60° E. when the time at 30° W. is noon?

On what meridian is the time:

23. 3 P.M. when the time at 60° W. is 6 P.M.?
24. 4 A.M. when the time at 90° W. is 2 A.M.?
25. 10 A.M. when the time at 60° E. is 8 A.M.?

Illustrative Examples

1. The longitude of Chicago is 87° 34′ 8″ W., and of Boston 71° 3′ 30″ W. When it is 10 A.M. at Boston, what is the time at Chicago?

87° 34′ 8″ W. is the longitude of Chicago.
71° 3′ 30″ W. is the longitude of Boston.
16° 30′ 38″ is the difference of longitude.

$\frac{1}{15}$ of the number of degrees, minutes, and seconds in the difference of longitude of two places equals the number of hours, minutes, and seconds in the difference of their time.

15)16 30 38
1 6 2.5

∴ 1 hr. 6 min. 2.5 sec. is the difference of time, correct to the nearest tenth of a second.

hr.	min.	sec.	
10	0	0	A.M. is the time at Boston.
1	6	2.5	is the difference of time.
8	53	57.5	A.M. is the time at Chicago.

The difference of time is subtracted from the time of Boston, since Chicago is west from Boston, and, therefore, has earlier time than Boston.

2. When it is 3 hr. 45 min. A.M. Wednesday at Paris, longitude 2° 20′ 30″ E., it is 9 hr. 11 min. 57.5 sec. P.M. Tuesday at Omaha. Find the longitude of Omaha.

da.	hr.	min.	sec.	
4	3	45	0	is the time at Paris.
3	21	11	57.5	is the time at Omaha.
	6	33	2.5	is the difference of time.

6 33 2.5
15
98 15 37.5

15 times the number of hours, minutes, and seconds in the difference of time of two places equals the number of degrees, minutes, and seconds in the difference of their time.

Since the time is later at Paris than at Omaha, Omaha must be west of Paris.

98° 15′ 37.5″ is the distance of Paris from Omaha.
2° 20′ 22 ″ is the distance of Paris east from the prime meridian.
95° 55′ 15.5″ is the distance of Omaha west from the prime meridian.

∴ 95° 55′ 15.5″ W. is the longitude of Omaha.

Exercise 3

1. The longitude of St. Louis is 90° 15′ 16″ W., and of New Haven, 72° 55′ 34″ W. When it is 6 P.M. at New Haven, what is the time at St. Louis?

2. The longitude of Philadelphia is 75° 9′ 5″ W., and of San Francisco 122° 26′ 15″ W. What is the time in Philadelphia when it is noon at San Francisco?

3. The longitude of Rome is 12° 27′ 14″ E., and of New York 74° 3″ W. When it is 11 P.M. Tuesday at New York, what time is it at Rome?

4. The time at Galveston, Texas, is 6 hr. 9 min. 9.9 sec. earlier than the time on the prime meridian. What is the longitude of Galveston, Texas?

5. The longitude of Jerusalem is 35° 32′ E., and of Edinburgh is 3° 12′ W. When it is 6 A.M. at Edinburgh, what is the time at Jerusalem?

6. When it is 1 P.M. at Oxford, England, it is 1 hr. 58 min. 45.3 sec. P.M. at Berlin. If the longitude of Oxford is 1° 15′ 6″ W., what is the longitude of Berlin?

Standard Time

12. For the convenience of business in general and of railroads especially, a system of **standard time** has been adopted; it was put in force in the United States on Sunday, Nov. 18, 1883. This system practically divides the country into four **belts** and adopts the local time of a meridian in each belt as the common time for all places in that belt.

13. The standard belts are:

(1) *Eastern*, whose standard time is the local time of the 75th meridian.

(2) *Central*, whose standard time is the local time of the 90th meridian.

(3) *Mountain*, whose standard time is the local time of the 105th meridian.

(4) *Pacific*, whose standard time is the local time of the 120th meridian.

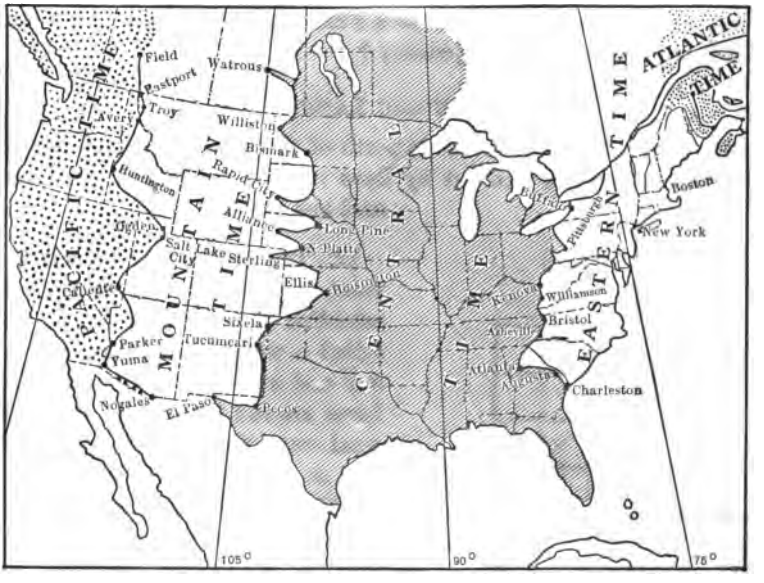
14. It is evident that within each of these belts the standard time is uniform, and that the time of each belt differs from that next to it by exactly one hour.

NOTE 1. It was intended that each standard meridian should govern the belt 7½ degrees on each side of it, but there have been some slight variations from this, due to the fact that railroad companies often change time at important railroad centers near a boundary when such centers are not located on the boundary.

NOTE 2. The Eastern belt includes all territory between the Atlantic and an irregular line drawn from Buffalo to Central Junction, Ga., the latter city being its southernmost point. The Central belt includes all the territory between this eastern line and another irregular line drawn from Bismarck, N.D., to El Paso, Tex. The Mountain belt includes all the territory between the last named line and nearly all the western boundaries of Idaho, Utah, and Arizona. The Pacific belt includes all the territory of the United States between the western boundary of the Mountain belt and the Pacific coast.

NOTE 3. It is evident that in any belt the local time of places east of the standard meridian is faster, and that of places west slower, than the standard time.

15. Map of Standard Time Belts



Exercise 4

1. When it is noon in the Eastern belt, what is the time in each of the other belts?

2. When it is noon by Mountain time, what is the time in each of the other belts?

3. Find the difference between the standard time of New York and that of Chicago, longitude 87° 34′ 8″ W.

4. In traveling from Philadelphia to San Francisco would a traveler find his watch too slow or too fast, and by how many hours?

5. How much slower is local, or sun, time than standard, or railroad, time at longitude 77° 30′ W.?

6. When it is noon at the 75th meridian west, is the local time of places that are east of the 75th meridian and in the Eastern belt later or earlier than noon?

7. A traveler that left San Francisco at noon Sunday, Pacific time, arrived in New York at noon Friday, Eastern time. How long did it require to make the journey?

PUBLIC LANDS

16. Most of the public lands north of the Ohio River and west of the Mississippi are laid out by lines running north and south and east and west.

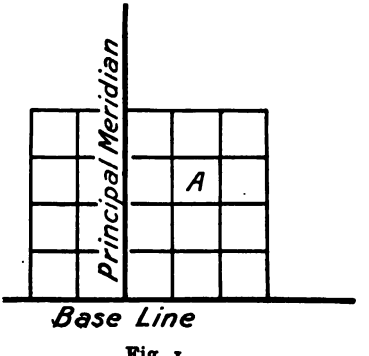


Fig. 1.

17. The method is to select a point in a tract and through it establish a north and south line called a **principal meridian**, and an east and west line called a **base line**. Lines are run parallel to the principal meridian and to the base line at intervals of 6 miles, thus dividing the tract into townships that are approximately 6 miles square. A series of townships ex-

tending north and south is called a **range**; a series extending east and west is called a **tier**. Ranges are numbered east and west from the principal meridian ; tiers are numbered north and south from the base line. A township is designated by the number of the tier and the number of the range in which it is located.

Thus, township marked A, Fig. 1, is designated T. 3 N., R. 2 E., read *township three north, range two east*, which means that A is in the third tier north of the base line and in the second range east of the principal meridian.

18. A township is divided into **sections** approximately* 1 mile square and numbered as shown in Fig. 2, beginning at the northeast corner.

6	5	4	3	2	1
7	8	9	10	11	12
18	17	16	15	14	13
19	20	21	22	23	24
30	29	28	27	26	25
31	32	33	34	35	36

Fig. 2.

19. A section is divided into quarter-sections, and the township, section, and quarter-section corners are permanently marked.

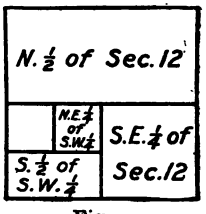


Fig. 3.

Quarter-sections may be subdivided, if desired, into half-quarter-sections and quarter-quarter-sections.

Exercise 5

1. How many square miles are there in a township ?

2. How many acres are there in a section? in a half-section? in a quarter-section? in a quarter-quarter-section?

In Examples 3–8 make drawing as in section 17 and locate with respect to the principal meridian and the base line:

3. T. 2 N., R. 3 W.
4. T. 2 N., R. 2 E.
5. T. 4 N., R. 1 W.
6. T. 1 N., R. 3 E.
7. T. 2 S., R. 2 W.
8. T. 3 S., R. 2 E.

* Townships and sections would be exact squares if it were not for the fact that the meridians converge; that is, all north and south lines meet at the North Pole.

9. Read N.W. $\frac{1}{4}$, Sec. 12, T. 3 N., R. 1 W.

10. Read S. $\frac{1}{2}$, Sec. 30, T. 2 N., R. 4 W.

11. Read W. $\frac{1}{2}$, Sec. 10, T. 2 S., R. 3 W.

Make drawing as in section 19, and locate each of the following tracts of land; give the number of acres in the tract:

12. S. $\frac{1}{2}$ of N.W. $\frac{1}{4}$.

13. W. $\frac{1}{2}$ of S.E. $\frac{1}{4}$.

14. S.E. $\frac{1}{4}$ of N.E. $\frac{1}{4}$.

15. S.W. $\frac{1}{4}$ of N.W. $\frac{1}{4}$.

METRIC SYSTEM

20. The **metric system** of weights and measures has been adopted by nearly all the countries of Europe and South America and by Mexico. Its use is permitted in the United States, and even required in certain work of the departments of the Government, such as the medical work of the Navy and the War Departments. It is the legal system of Porto Rico and the Philippines.

21. The fundamental unit of the metric system is the **meter** — the unit of *length*. From this the units of *capacity* (**liter**) and *weight* (**gram**) were derived. All other units are the decimal subdivisions or multiples of these. The three units named are simply related; thus:

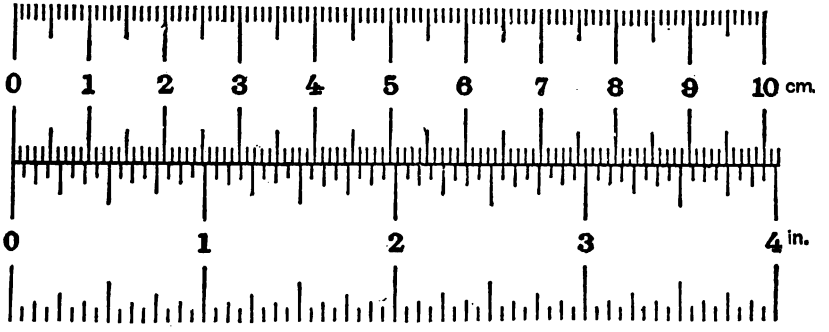
A liter is a cube .1 of a meter on an edge.

A gram is the weight of a cube of pure water .01 of a meter on an edge at the greatest density of water, 39.2° Fahrenheit.

22. The metric tables are formed by combining the words "meter," "gram," and "liter" with the six prefixes here shown:

Prefixes	Meaning		Units
milli-	.001		
centi-	.01		
deci-	.1	prefixed to	meter *for length* gram *for weight* liter *for capacity*
deka-	10		
hecto-	100		
kilo-	1000		

23. The primary unit of length is the **meter**; it is equal to 39.37 inches, nearly.



Comparison Scale: 10 Centimeters and 4 Inches.

24. **Table of Measures of Length**

10 *millimeters* (mm)	= 1 *centimeter* (cm)	= 0.3937 in.
10 centimeters	= 1 decimeter (dm)	
10 decimeters	= 1 *meter* (m)	= 39.37 in.
10 meters	= 1 dekameter (Dm)	
10 dekameters	= 1 hectometer (Hm)	
10 hectometers	= 1 *kilometer* (Km)	= 200 rd.
10 kilometers	= 1 myriameter (Mm)	

NOTE. Important units are printed in italic type.

25. A quantity expressed in different metric units may be expressed and read without reduction in any denomination desired, or in more than one denomination; just as we now express a sum of money made up of \$7000, \$25, 6 d., 5¢, 2 m., in one denomination, \$7025.652, which is sometimes read *7025.652 dollars*, and sometimes *seven thousand twenty-five dollars, sixty-five cents, two mills.*

Thus, $4^{Km}\ 6^{Dm}\ 5^{m}\ 4^{cm}\ 2^{mm}$ may be expressed 4065.042^{m}, and read *four thousand sixty-five and forty-two thousandths meters*, or *4 kilometers, 65 meters, 42 millimeters.*

26. The metric system is a *decimal system* and therefore reductions from one denomination in it to another may be made by simply moving the decimal point to the right or to the left.

Thus, $1825^{mm} = 182.5^{cm} = 18.25^{dm} = 1.825^{m}$,
and $0.638^{m} = 6.38^{dm} = 63.8^{cm} = 638^{mm}$.

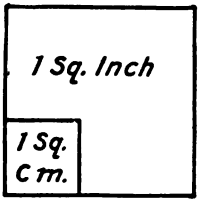
Exercise 6

Read:

1. 7^{Km} in hectometers; in dekameters; in meters.
2. 400^{mm} in centimeters; in decimeters; in meters.
3. 420^{mm} in centimeters; in decimeters; in meters.
4. 726^{mm} in centimeters; in decimeters; in meters.
5. 5^{Dm} in meters; in centimeters; in millimeters.
6. 5^{m} 6^{dm} in decimeters; in centimeters.
7. 4^{m} 3^{cm} in centimeters; in meters.
8. 2^{Dm} 6^{m} 5^{cm} in meters.
9. 4^{Km} 65^{m} 375^{mm} in meters.

Write:

10. 25^{Km} in meters.
11. 375^{mm} in meters.
12. 4356^{m} in kilometers.
13. $.075^{m}$ in millimeters.
14. 1425.62^{m} in kilometers, meters, and millimeters.



27. In square measure the units are squares whose sides are the units of length, as in our system. For land measure a square 100 meters long called an are (meaning "area") is the unit.

28. **Table of Measures of Surface**

100 *square millimeters* (qmm)	= 1 *square centimeter* (qcm)	= 0.155 *sq. in.*
100 square centimeters	= 1 square decimeter (qdm)	
100 square decimeters	= { 1 *square meter* (qm) ; 1 centare (ca) }	= 10.764 *sq. ft.*
100 centares	= 1 *are* (a)	= 119.6 *sq. yd.*
100 ares	= 1 hectare (Ha)	= 2.5 *A.*

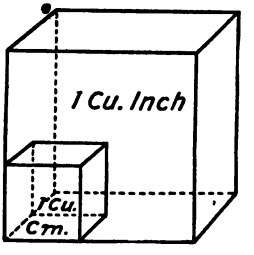
29. In square measure in the metric system 100 units of any denomination make one of the next higher denomination; therefore, reductions from one denomination to the next higher or lower may be made by moving the decimal point two places to the left or two places to the right.

Exercise 7

Express:

1. 4^{qm} in square decimeters; in square centimeters.
2. 5^{qdm} in square centimeters; in square millimeters.
3. 65^{qdm} in square meters.
4. 145^{qmm} in square meters.
5. $4^{qm}\ 6^{qdm}$ in square meters; in square decimeters.
6. $8^{qm}\ 6^{qmm}$ in square meters; in square millimeters.
7. 142^{a} in hectares.
8. 425^{qm} in ares.
9. $1^{a}\ 2^{ca}$ in square meters.
10. 75832.4^{qm} in hectares, ares, and centares.
11. 45.674^{Ha} in hectares, ares, and centares.
12. 456.78^{a} in hectares, ares, and centares.

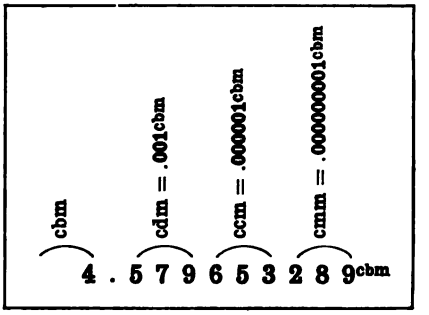
30. In cubic measure the units are cubes whose edges are the linear units. The **cubic meter** (sometimes called the **stere**, meaning "solid") is the unit of volume.



31. **Table of Measures of Volume**

1000 ***cubic millimeters*** (cmm)	= **1** ***cubic centimeter*** (ccm)	
1000 cubic centimeters	= **1** ***cubic decimeter*** (cdm)	
1000 cubic decimeters	= **1** ***cubic meter*** (cbm), or stere (s)	= **35.315 cu. ft.**

32. **Scale of Cubic Measure**



Exercise 8

Read and express:

1. 4.579^{cbm} in cubic decimeters.

Thus, 4.579 = 4579 thousandths.
$\therefore 4.579^{cbm} = 4579^{cdm}$.

2. 5.4^{cbm} in cubic decimeters; in cubic centimeters; in cubic millimeters.

3. 7.5^{ccm} in cubic meters; in cubic millimeters.

4. 146.750^{cbm} in cubic meters and cubic decimeters.

5. $1^{ccm}\ 10^{cmm}$ in cubic meters.

SUGGESTION. $1^{ccm}\ 10^{cmm} = .000001^{cbm} + .000000010^{cbm}$.

6. 16^{cdm} in cubic meters; in cubic centimeters; in cubic millimeters.

7. 250^{ccm} in cubic meters; in cubic decimeters; in cubic millimeters.

8. 4462.15^{cdm} in cubic meters, cubic decimeters, and cubic centimeters.

33. The primary unit of capacity is the *liter*, which is exactly one cubic decimeter; it is equivalent to 1.0567 liquid quarts or 0.908 dry quarts.

34. Table of Measures of Capacity

10 milliliters (ml)	= 1 centiliter (cl)	
10 centiliters	= 1 deciliter (dl)	
10 deciliters	= 1 *liter* (l)	= 0.908 qt. (dry); 1.0567 qt. (liq.)
10 liters	= 1 dekaliter (Dl)	
10 dekaliters	= 1 hectoliter (Hl)	= 2.8375 bu.; 26.417 gal.
10 hectoliters	= 1 kiloliter (Kl)	

Exercise 9

1. Read 8^{Kl} in hectoliters; in dekaliters; in liters.

SUGGESTION. See Sec. 26.

2. Read 275^{ml} in centiliters; in liters; in deciliters.
3. Express $3^{l}\ 2^{cl}\ 5^{ml}$ in liters.
4. Express $7^{dl}\ 8^{cl}\ 6^{ml}$ in liters; in milliliters.
5. Read 4000^{ml} in centiliters; in deciliters; in liters.
6. Read 380^{ml} in deciliters; in centiliters; in liters.
7. Express $2^{Kl}\ 5^{Hl}\ 3^{l}$ in liters.
8. Express $7^{Hl}\ 4^{Dl}\ 8^{l}$ in liters.
9. Express 4^{cdm} in liters.
10. How many liters will a cubical vessel hold if it is 2^{dm} long?
11. How many liters will a cubical vessel hold if it is 1^{m} long?

Measures of Weight

35. The primary unit of weight is the **gram**, which is the weight of one cubic centimeter of pure water at a temperature of 39.2 degrees Fahrenheit; it is equivalent to 15.432 grains.

Relative Size of Avoirdupois Ounce, 30-Gram, and Troy Ounce Weights.

36. Table of Measures of Weight

10 *milligrams* (mg)	= 1 *centigram* (cg)	
10 centigrams	= 1 decigram (dg)	
10 decigrams	= 1 *gram* (g)	= 15.432 gr.
10 grams	= 1 dekagram (Dg)	
10 dekagrams	= 1 hectogram (Hg)	
10 hectograms	= 1 *kilogram* (Kg)	= 2.2046 lb. (avoir.)
10 kilograms	= 1 myriagram (Mg)	
10 myriagrams	= 1 quintal (q)	
10 quintals	= 1 *Metric Ton* (MT)	= 2204.6 lb. (avoir.)

NOTE. $1^{mg} = 0.0154$ gr.

NOTE. It is evident that a liter of water weighs 1 kilogram and that 1000 liters weigh 1000 kilograms, or 1 metric ton.

Exercise 10

1. Read 7^{Kg} in hectograms; in grams.

SUGGESTION. See Sec. 26.

2. Read 2^{Kg} in grams; in centigrams; in milligrams
3. Read 2000^{mg} in centigrams; in decigrams; in grams.
4. Read 2.455^{g} in grams and milligrams.
5. Read 7200.005^{g} in kilograms, grams, and milligrams.
6. Express $2^{Kg}\ 7^{g}\ 3^{mg}$ in grams.
7. Express 5.630^{g} in grams and milligrams.
8. Express 7563.289^{g} in kilograms, grams, and milligrams.
9. Express $18^{Kg}\ 40^{g}\ 60^{mg}$ in grams.
10. Express 4278564^{g} as metric tons, kilograms, and grams.
11. Express $10^{Kg}\ 40^{g}\ 30^{mg}$ in grams.
12. Express $4^{MT}\ 2^{Kg}$ in grams.

Equivalents

37. Metric Units expressed in Terms of Common Units

Units	Equivalents	Approximate Equivalents
1 kilometer	0.62137 mile	220 rods
1 meter	39.37 inches	1.1 yards
1 centimeter	0.3937 inch	0.4 inch
1 hectare	2.471 acres	2.5 acres
1 square meter	1.1960 square yards	1.2 square yards
1 square centimeter	0.155 square inch	0.16 square inch
1 cubic meter	1.3079 cubic yards	1.3 cubic yards
1 cubic centimeter	0.061 cubic inch	0.06 cubic inch
1 hectoliter	2.83774 bushels	3 bushels
1 liter	0.9081 quart (dry)	0.9 quart (dry)
1 liter	1.05668 quarts (liquid)	1.06 quarts (liquid)
1 metric ton	2204.6 pounds	1 long ton
1 kilogram	2.20462 pounds (avoir.)	2.2 pounds
1 gram	15.4324 grains	15.4 grains

38. Common Units expressed in Terms of Metric Units

Units	Equivalents	Approximate Equivalents
1 inch	25.4001 millimeters	25 millimeters
1 foot	0.304801 meter	0.3 meter
1 yard	0.914402 meter	0.9 meter
1 mile	1.60935 kilometers	1.6 kilometers
1 square inch	6.4516 square centimeters	6.5 square centimeters
1 acre	0.4047 hectare	0.4 hectare
1 square yard	0.8361 square meter	0.8 square meter
1 cubic inch	16387.2 cubic millimeters	16 cubic centimeters
1 cubic yard	0.7646 cubic meter	0.75 cubic meter
1 gallon	3.78543 liters	3.75 liters
1 bushel	0.35239 hectoliter	0.35 hectoliter
1 ounce (troy)	31.10348 grams	31.1 grams
1 ounce (avoir.)	28.3495 grams	28.35 grams
1 pound (troy)	0.37324 kilogram	0.38 kilogram
1 pound (avoir.)	0.45359 kilogram	0.45 kilogram

39. The Metric System of Weights and Measures was first adopted in France in 1799. The system is almost universally used for scientific measurements. The following objections have been seriously urged against its general use, but as the public is becoming more and more familiar with it, these objections are disappearing.

It has been urged that:

1. The unit base, the meter, is too large to be clearly apprehended by the young and uninstructed.
2. The decimal division is too difficult, and is unsuited to practical purposes.
3. The unit of length should be some dimension of the human body.
4. It is inconvenient to think in one system and deal in another.
5. The decimal unit has failed when applied to the circle.
6. The adoption of the system would invalidate land titles.
7. The meter has never been correctly determined and never will be.

Some advantages are as follows:

1. Easiness of apprehension by pupils old enough to understand the decimal system of notation.
2. The facility with which the tables may be learned owing to the uniformity of scale.
3. The facility of reduction from one denomination to another of the same kind of quantity.
4. The simplicity of the relations of the units of surface, solids, capacity, and weight, to the units of length.
5. Its adoption would simplify international communication, since it is so generally used by other nations.

Exercise 11

(In changing from one system to another, use equivalents, page 473.)

1. Find, correct to the nearest hundredth, the number of millimeters in 2 inches.
2. Find the approximate number of kilometers in 5 miles.

3. Find the approximate number of hectares in a field containing 6 acres.

4. A cask containing 120 liters contains, approximately, how many liquid quarts?

5. Find the approximate number of bushels in 12 hectoliters.

6. 100 square meters of cloth are equal, approximately, to how many square yards?

7. 2000 pounds (avoir.) are equal, approximately, to how many kilograms?

8. A lot containing 4 hectares contains, approximately, how many acres?

9. 10 miles are equal, approximately, to how many kilometers?

10. A square foot is equal, approximately, to how many square centimeters?

11. A cubic centimeter of water weighs a gram. Show that a cubic decimeter, or a liter, of water weighs a kilogram, and that a cubic meter weighs a metric ton.

12. Find the weight of a liter of milk, if it is 1.03 times as heavy as water.

13. Find the weight of 2^{ccm} of silver, which is 10.47 times as heavy as water.

14. A piece of land 45^{m} long and 20.5^{m} wide contains how many ares?

15. How many cubic centimeters does a flask hold that has a capacity of 5^{cl}?

16. Find the weight of a milliliter of water.

17. How many hectoliters will a bin hold that is 4^{m} long, 2.5^{m} wide, and 1.75^{m} deep?

18. A certain flask holds 2.5^{Kg} of water; how many liters does it hold? how many cubic decimeters?

19. If a farmer sows $1^{Hl}\ 4^{Dl}$ of wheat on a hectare of land, how much seed would it require for a field 45^{Dm} long and 42^{Dm} wide?

20. How many steres of wood in a pile 15^{m} long, 1.25^{m} wide, and 1.25^{m} high?

21. Find in milligrams the weight of a centiliter of alcohol, which is .792 times as heavy as water.

22. If a field yielded on an average 15^{Hl} of wheat per hectare and each hectoliter of wheat yielded 55^{Kg} of flour, how many metric tons of flour did the field yield if it is 50.5^{Dm} long and 24^{Dm} wide?

23. The weight of the olive oil in a certain flask is $36^{g}\ 600^{mg}$. How many cubic centimeters of this oil are there, if it is .915 times as heavy as water?

24. A dealer bought a square piece of land 36^{Dm} long at \$ 200 per hectare. He divided it into lots containing 72^{a} each, which he sold at \$ 350 per lot. Find his profit.

25. A piece of steel bar 1^{m} long, 8^{cm} wide, and 5^{cm} thick weighs how many kilograms, steel being 7.81 times as heavy as water?

26. How many kiloliters of water will a tank hold that is 4.5^{m} long, 2.4^{m} wide, and 4^{m} deep? Find the weight of the water in metric tons.

27. A vessel weighs 4.8^{Kg} empty, and full of water 26.75^{Kg}. How many liters does it hold?

28. A certain liter jar weighs 700^{g}. What would it weigh when filled with alcohol, which is .792 times as heavy as water?

FOREIGN EXCHANGE

40. Foreign exchange is exchange between two places in different countries.

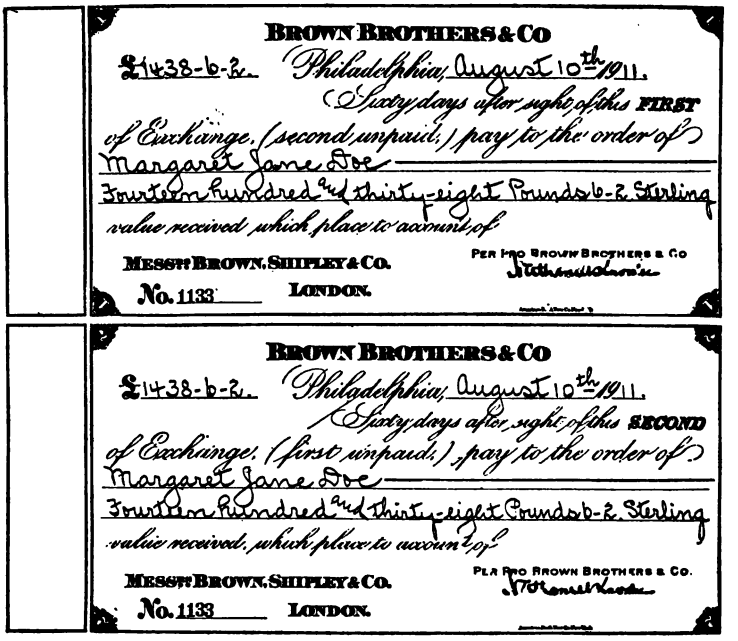
The face of a draft is usually expressed in the money of the country in which the foreign party lives.

Thus, the face of a draft on a party in England is written in *pounds*, *shillings*, and *pence;* on a party in France, in *francs;* on a party in Germany, in *marks;* and on a party in Holland, in *guilders*, or *florins*.

41. A **set of exchange** consists of two drafts of the same date and tenor drawn in one country upon a bank in another. The one draft is usually designated "**First of Exchange**," and the other "**Second of**

Exchange." The first is forwarded and the second retained to be used, if necessary. If one is paid, the other is void.

42. **Set of Foreign Exchange**



BROWN BROTHERS & CO

£1438-6-2. Philadelphia, August 10th 1911.

(Sixty days after sight of this FIRST

of Exchange, (second unpaid,) pay to the order of

Margaret Jane Doe

Fourteen Hundred and thirty-eight Pounds 6-2 Sterling

value received, which place to account of

MESSRS BROWN, SHIPLEY & CO. LONDON.

No. 1133

PER PRO BROWN BROTHERS & CO

BROWN BROTHERS & CO

£1438-6-2. Philadelphia, August 10th 1911.

(Sixty days after sight of this SECOND

of Exchange, (first unpaid,) pay to the order of

Margaret Jane Doe

Fourteen Hundred and thirty-eight Pounds 6-2 Sterling

value received, which place to account of

MESSRS BROWN, SHIPLEY & CO. LONDON.

No. 1133

PER PRO BROWN BROTHERS & CO.

43. The following quotations on 60 days' drafts on foreign countries were selling in New York at a certain date:

Sterling, 3 da., 4.88½	meaning	£1 = $4.88½.
Sterling, 60 da., 4.85	meaning	£1 = $4.85.
Francs, 3 da., 5.14⅛	meaning	5.14⅛ francs = $1.
Francs, 60 da., 5.16⅞	meaning	5.16⅞ francs = $1.
Reichsmarks, 3 da., 95⅜	meaning	4 marks = $0.95⅜.
Reichsmarks, 60 da., 94¼	meaning	4 marks = $0.94¼.
Guilders, 3 da., 40½	meaning	1 guilder = $0.40½.
Guilders, 60 da., 40¼	meaning	1 guilder = $0.40¼.

Exercise 12

1. Find the cost of a bill of exchange on London for £ 25 10s. at 4.87.

SUGGESTION. — 25.5 × $ 4.87 = the required cost.

2. Find the cost of a bill of exchange on Berlin for 6000 reichsmarks, at 95½.

SUGGESTION. — 6000 marks = 1500 × 4 marks.
1500 × $ 0.955 = the required cost.

3. Find the cost of a bill on Paris for 1000 francs, exchange quoted at 5.16¼.

SUGGESTION. — 5.1625 = the number of francs that can be purchased for $ 1.
1000 : 5.1625 = the required number of dollars.

4. What is the value in Philadelphia of a draft on London for £ 75, exchange 4.85?

5. What is the value in New York of a 60 da. draft for £ 225 on London, exchange 4.82½?

6. What is the value in Chicago of a 3 da. draft for 4500 francs on a Paris bank, exchange 5.14½?

7. What is the cost in Boston of a 60 da. draft for 2000 marks on a Berlin bank, exchange 95⅜?

8. What is the cost in Chicago of a 60 da. draft for 5000 francs on a Paris bank, exchange 5.16⅜?

9. What is the cost in San Francisco of a 3 da. draft on a Berlin bank for 4780 marks, exchange 95⅜?

10. What is the cost in Charleston of a 60 da. draft on an Amsterdam bank for 2000 guilders, exchange 40¼?

11. What is the cost in Washington of a 3 da. draft on an Amsterdam bank for 1000 guilders, exchange 40½?

12. How large a draft on London can be bought for $ 200 when exchange is 4.80?

COMPOUND PROPORTION

44. A **compound ratio** is an expression denoting the product of two or more simple ratios.

Thus, $\left\{\begin{matrix}1:2\\3:4\\5:6\end{matrix}\right\}$ is equivalent to $\frac{1}{2}\times\frac{3}{4}\times\frac{5}{6}$, and may be read, *1 measured by 2, times 3 measured by 4, times 5 measured by 6.*

45. A **compound proportion** is a proportion one or both of whose ratios are compound.

46. In a proportion whose second ratio is compound, the first extreme equals the product of all the terms in the means divided by the product of all the terms in the last extreme.

Thus, let $x:3=\left\{\begin{matrix}4:5\\6:7\\8:9\end{matrix}\right\}$

then, $\frac{x}{3}=\frac{4}{5}\times\frac{6}{7}\times\frac{8}{9}$.

Multiplying by $3\times5\times7\times9$, we have $\cancel{3}\times5\times7\times9\times\frac{x}{\cancel{3}}=3\times\cancel{5}\times\cancel{7}\times\cancel{9}\times\frac{4}{\cancel{5}}\times\frac{6}{\cancel{7}}\times\frac{8}{\cancel{9}}$;

or, $5\times7\times9\times x=3\times4\times6\times8$.

Illustrative Examples

1. If 20 horses, each consuming 12 quarts of oats daily, eat $262\frac{1}{2}$ bushels in 5 weeks, how many horses, each consuming 8 quarts daily, will consume 210 bushels in 10 weeks?

Let x = the number of horses required.

Amount of stock	*Daily allowance*	*Quantity*	*Time*
x horses	8 qt.	210 bu.	10 wk.
20 horses	12 qt.	$262\frac{1}{2}$ bu.	5 wk.

The amount of stock required is inversely proportional to the daily allowance of a horse, and to the length of time the horses are fed, and directly proportional to the quantity consumed.

$$x : 20 = \left\{ \begin{array}{c} 12 : 8 \\ 210 : 262\frac{1}{2} \\ 5 : 10 \end{array} \right\}$$

Hence, $x = \dfrac{20 \times 12 \times 210 \times 5}{8 \times 262\frac{1}{2} \times 10}$, or 12.

Exercise 13

1. If 21 bu. of oats are required to feed 8 horses 7 da., how many bushels would be required to feed 16 horses 3 da.?

2. If a train running at the rate of 30 mi. per hour runs 240 mi. in 8 hr., in how many hours would it run 360 mi., running at the rate of 45 mi. per hour?

3. A lot 121 yd. long and 80 yd. wide contains 2 A.; how wide must a lot 220 yd. long be to contain $3\frac{1}{2}$ A.?

4. A wheel 6 ft. in circumference makes 880 revolutions in going a mile; how many revolutions would a wheel 4 ft. in circumference make in going 3520 ft.?

5. If a man working 10 hr. per day can build a wall 24 ft. long, 10 ft. high, and two bricks thick, in 2 da., in how many days of 8 hr. each can 3 men build a wall 30 ft. long, 8 ft. high, and 3 bricks thick?

6. If a mow of hay 32 ft. long, 16 ft. wide, and 16 ft. high lasts 8 horses 20 wk., how many weeks will a mow of hay 28 ft. long, 20 ft. wide, and 12 ft. high last 5 horses?

7. If 2 lamps burning 4 hr. per night for 6 nights consume 5 qt. of oil, how many gallons would be required per year for the 24 lamps of a village, each burning 7 hr. per night, provided the lamps are all of the same size?

8. If 120 men working 8 hours per day can pave a street 1 mile long and 60 feet wide in 20 days, in how many days of 10 hours each can 8 men pave a street of the same kind 396 feet long and 40 feet wide?

TIMBER MEASURE

47. Squared timber is usually sold by the cubic foot. The number of cubic feet in a stick is determined by the following:

Multiply together the number of inches in the thickness, the number of inches in the width, and the number of feet in the length, and divide the product by 144.

Thus, the number of cubic feet in a stick of timber 8 in. thick, 10 in. wide, and 16 ft. long, is

$$\frac{8 \times 10 \times 16}{144}, \text{ or } 8\tfrac{8}{9}.$$

48. One half of the sum of the end diameters of a log, excluding the thickness of the bark, is called its **mean diameter.**

49. When a log to be squared is sold by the cubic foot, the number of cubic feet — squared timber — in it is found by the following:

Multiply four ninths of the square of the number of inches in the mean diameter by the number of feet in the length and divide the product by 144.

Thus, the number of cubic feet of squared timber in a log whose mean diameter is 16 in. and whose length is 18 ft., is

$$\frac{4}{9} \times (16)^2 \times 18, \text{ or } 14\tfrac{2}{9}.$$
$$144$$

NOTE 1. — In some localities the number of cubic feet — squared timber — in a log is found by the following:

Multiply half the square of the number of inches in the mean diameter by the number of feet in the length and divide the product by 144.

Thus, the number of cubic feet of squared timber in a log whose mean diameter is 18 in. and whose length is 20 ft., is

$$\frac{1}{2} \times (18)^2 \times 20, \text{ or } 22\tfrac{1}{2}.$$
$$144$$

NOTE 2. — If a log tapers suddenly at the butt, the diameter at the middle of the log is sometimes used instead of the mean diameter.

NOTE 3. — In a log of inferior quality, the diameter of the smaller end is sometimes used instead of the mean diameter.

50. To determine the number of board feet of square-edged inch boards that can be sawed from a log, the following rule, called **Doyle's Rule**, is generally used:

Subtract 4 from the number of inches in the diameter, square the result, divide by 16, and multiply the quotient by the number of feet in the length.

Thus, the number of board feet of square-edged inch boards that can be sawed from a log 30 ft. long and 24 in. in diameter is

$$\frac{(24-4)^2}{16} \times 30, \text{ or } 750.$$

NOTE. — By the *diameter* in the above rule is meant the mean diameter, the diameter at the middle of the log, or the diameter at the smaller end, depending on the agreement between the buyer and the seller.

Exercise 14

1. Find the number of cubic feet in a stick of timber 6 in. thick, 9 in. wide, and 24 ft. long.

2. Find the number of cubic feet in a stick of timber 22 in. thick, 24 in. wide, and 35 ft. long.

3. Find the number of cubic feet — squared timber — in a log 36 ft. long, if the diameters of the ends are 34 in. and 20 in.

4. How many cubic feet — squared timber — would be estimated in the log mentioned in example 3, if the rule stated in note 1, § 49, were applied.

5. Find the amount of square-edged inch boards that can be sawed from a log 16 in. in diameter and 18 ft. long.

6. Find the amount of square-edged inch boards that can be sawed from a log 12 in. in diameter and 20 ft. long.

7. Find the number of cubic feet — squared timber — in a log 40 ft. long, if the diameters of the ends are 60 in. and 45 in.

8. Find the amount of square-edged inch boards that can be sawed from a log 25 in. in diameter and 35 ft. long. (Answer to the nearest foot.)

Find the amount of square-edged inch boards that can be sawed from the following:

9. A log 20 in. in diameter and 20 ft. long.
10. A log 24 in. in diameter and 36 ft. long.
11. A log 36 in. in diameter and 30 ft. long.
12. A log 40 in. in diameter and 16 ft. long.

CUBE ROOT

51. The **cube root** of a given number is the number which cubed will produce the given number.

Thus, the cube root of 64 is 4, since, $4^3 = 64$; the cube root of $\frac{8}{27}$ is $\frac{2}{3}$, since $(\frac{2}{3})^3 = \frac{8}{27}$; the cube root of .125 is .5, since $(.5)^3 = .125$.

52. The symbol $\sqrt[3]{\ }$, written before a number, indicates that the cube root of the number is to be taken.

Thus, $\sqrt[3]{27}$, read *the cube root of 27*, equals 3.

Exercise 15

1. Name the cube root of: 8; 27; 125; 64; $\frac{1}{8}$; $\frac{1}{27}$.
2. Name the cube root of: 1; 1000; 1,000,000; .001; .000001.
3. Find the cube root of: .008; .027; .064; .125.
4. Find the cube root of: $\frac{8}{27}$; $\frac{27}{64}$; $\frac{8}{125}$; $\frac{27}{125}$; $\frac{64}{125}$.
5. Find the cube root of 216.

Thus, $216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3$
$= (2 \times 3) \times (2 \times 3) \times (2 \times 3)$
$= 6 \times 6 \times 6.$

Therefore, $\sqrt[3]{216} = 6.$

Find the cube root of:

6. 1728 7. 2744 8. 3375 9. 13,824
10. Find the length of a cube which contains 64 cu. in.

SUGGESTION. Since the number of cubic units in a cube is equal to the cube of the number of units in its length, then the number of units in its length is equal to the cube root of the number of cubic units in the cube.

Find the lengths of cubes that contain:

11. 125 cu. in. 12. 343 cu. in. 13. 729 cu. in.

General Rule for extracting Cube Root

53. The **cube root** of numbers may be found by the following rule:

1. *Point off the number into periods of three figures each, beginning with the units.*

2. *Take the cube root of the nearest cube number below that expressed by the first period on the left, for the first figure of the root.*

3. *Subtract the cube of the first figure of the root from the first period of the number, and annex the next period to the remainder for a dividend.*

4. *Take three times the square of the root found for a trial divisor, and divide the dividend, exclusive of the two right-hand figures, by it for the next figure of the root.*

5. *Take three times the root previously found multiplied by the last figure found for the first correction, and the square of the last figure found for the second correction.*

6. *Place the right-hand figure of the first correction one order to the right of the trial divisor, and the right-hand figure of the second correction one order to the right of the first correction, then take the sum of the trial divisor and the two corrections for the complete divisor.*

7. *Multiply the complete divisor by the last figure of the root now found, and subtract the product from the dividend.*

8. *If the root is not fully found, annex the next period of the number to the remainder for a dividend and again find the trial divisor, the next figure of the root, the corrections, and the complete divisor as before; and so continue till all the periods of the number have been used.*

9. *Should a cipher occur in the root, annex two ciphers to the trial divisor to form the next trial divisor, and annex another period to the dividend.*

Illustrative Examples

1. Extract the cube root of 45,118,016.

	45′118′016	356
	27	
$3 \times 3 \times 3 = 27$	18118	
$3 \times 3 \times 5 = 45$		
$5 \times 5 = 25$		
3175	15875	
$3 \times 35 \times 35 = 3675$	2243016	
$3 \times 35 \times 6 = 630$		
$6 \times 6 = 36$		
373836	2243016	

Explanation:

Point off the numbers into periods of three figures each, beginning at units. The greatest number whose cube is contained in 45 is 3, which is the first figure of the root. Cube 3; subtract the result from 45, and annex 118 to the remainder, making the dividend 18,118. Take 3 times the square of 3, or 27, for a trial divisor. Divide 18,118, exclusive of the two right-hand figures, by 27, making allowance for corrections to be added, for the next figure of the root, which is found to be 5. Take 3 times 3 times 5 for the first correction, and set it one place to the right of the trial divisor. Take the square of 5 for the second correction, and set it one figure to the right of the first correction. The sum of the trial divisor and the two corrections, 3175, is the complete divisor. Multiply 3175 by 5, the last figure of the root found, subtract the product 15,875 from 18,118, and annex 016 to the remainder, making the dividend 2,243,016. Take 3 times the square of 35 for the trial divisor. Find as before the next figure of the root 6, the first correction 630, the second correction 36, the complete divisor 373,836, the product 2,243,016, and the remainder 0. The cube root is 356.

2. Extract the cube root of 25, correct to the nearest hundredth.

	25.′000′000	2.924
	8	
$3 \times 2 \times 2 = 12$	17000	
$3 \times 2 \times 9 = 54$		
$9 \times 9 = 81$		
1821	16389	
$3 \times 29 \times 29 = 2523$	611000	
$3 \times 29 \times 2 = 174$		
$2 \times 2 = 4$		
254044	508088	
$3 \times 292 \times 292 = 255792$	102912000	
$3 \times 292 \times 4 = 3504$		
$4 \times 4 = 16$		
25614256	102457024	

Explanation:

Separate the number into periods of three figures each rightward and leftward from the decimal point. The cube root is found to be nearer to 2.92 than 2.93. Therefore 2.92 is the cube root of 25, correct to the nearest hundredth.

Exercise 16

Extract the cube root of:

1. 3375	2. 195,112	3. 17,576
4. 50,653	5. 421,875	6. 681,472
7. 1,442,897	8. 16,777,216	9. 28,094,464
10. 45,118,016	11. 131,096,512	12. 175,616,000

Extract the cube root, correct to the nearest hundredth, of:

13. 25	14. 126	15. 4.5
16. 3.75	17. 300	18. .005

19. Extract the cube root of $\frac{27}{64}$.

SUGGESTION. Since $\frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} = \frac{27}{64}$, the cube root of $\frac{27}{64} = \frac{3}{4}$.

NOTE. If a fraction in its lowest terms is a perfect cube, its terms are perfect cubes. If the terms of a fraction in its lowest terms are not perfect cubes, it is best to reduce the fraction to a decimal before extracting the cube root.

Extract the cube root of:

20. $\frac{64}{1728}$	21. $\frac{125}{1728}$	22. $12\frac{19}{27}$	23. $\frac{3}{5}$ to hundredths
24. $\frac{729}{1000}$	25. $\frac{9261}{10648}$	26. $5\frac{104}{125}$	27. $\frac{2}{7}$ to hundredths

REVIEW

Exercise 17

1. A traveler starts at longitude 75° 15′ W. and travels until his watch is 6 hours too slow. What is then his longitude?

2. Sea water weighs 1026 grams per liter. How many metric tons of sea water will a tank hold if it is 2 meters long, 1 meter 5 decimeters wide, and 2 meters deep?

3. The longitude of A is 2° E. When the time at A is 2 P.M., the time at B is 9.12 A.M. Find B's longitude.

4. One cubic centimeter of olive oil weighs 915 milligrams. Find the weight in grams of 15 liters 4 deciliters of olive oil.

5. If a kilogram of a certain substance is found to contain 2 grams 150 milligrams of adulterant, what is the per cent of adulterant in the substance?

6. If a traveler left San Francisco at noon Monday, Pacific time, and arrived at New York at noon Saturday, Eastern time, how long did it require him to make the journey?

7. In how many days of 10 hours each will 25 men husk 5000 shocks of corn, if 2 men in 8 days of 12 hours each husk 640 shocks?

8. Extract the square root of 2, correct to the nearest ten-thousandth.

9. Extract the cube root of 2, correct to the nearest thousandth.

10. Air weighs about 1.29 grams per liter. Find the weight in kilograms of the air in a room 10 meters long, 8 meters wide, 2.5 meters high.

11. Find the cost of a bill of exchange on London for £15 6*s.* at 4.8775.

12. The longitude of A is 60° W. When the time at A is 9 A.M., the time at B is 4.12 P.M. Find B's longitude.

13. Two spheres of the same material weigh 216 lb. and 8 lb., respectively. If the diameter of the first is 1 ft., what is the diameter of the second?

14. What is the value of a quarter-quarter-section of land at $ 40 per acre?

15. If 360 cart loads of earth were removed in excavating a cellar 36 ft. long, 30 ft. wide, and 9 ft. deep, how many cart loads would have to be removed in excavating 3000 ft. of canal 60 ft. wide and 10 ft. deep?

16. Find the amount of square-edged inch boards that can be sawed from a log 32 in. in diameter and 30 ft. long.

17. Find the diagonal of a cube whose edge is 6 in. Answer correct to the nearest .01 in.

18. If a hectoliter of soft anthracite coal weighs 145 kilograms, what sum will be obtained by selling 200 cubic meters of this coal at $ 5 a metric ton?

19. If a cylindrical vessel $18\frac{1}{2}$ in. in diameter and 8 in. deep holds a bushel, what must be the dimensions of a vessel of the same shape to hold a peck?

TABLES OF DENOMINATE AMOUNTS

(For Reference)

United States Money

10 mills = 1 cent
10 cents = 1 dime
10 dimes = 1 dollar
10 dollars = 1 eagle

Although the gold dollar is no longer coined, it is represented by 25.8 grains of standard gold .9 pure. The standard silver dollar consists of 412.5 grains of standard silver. All gold and silver coins are .9 pure metal and .1 copper.

Counting

12 units = 1 dozen
12 dozen = 1 gross
12 gross = 1 great gross
20 units = 1 score

Paper

24 sheets = 1 quire
20 quires = 1 ream
2 reams = 1 bundle
5 bundles = 1 bale

Dry Measure

2 pints = 1 quart
2 quarts = 1 small measure
8 quarts = 1 peck
4 pecks = 1 bushel stricken measure
5 pecks = 1 heaped bushel, nearly

The heaped bushel, which contains about 5 pecks, or $\frac{4}{5}$ of a stricken bushel, is used in measuring corn in the ear, potatoes, apples, lime, turnips, and various other substances.

A stricken bushel equals about $\frac{4}{5}$ of a cubic foot, and a cubic foot equals about $\frac{4}{5}$ of a stricken bushel.

A heaped bushel is about $\frac{2}{1}\frac{5}{6}$ of a cubic foot, and a cubic foot is about $\frac{1}{2}\frac{1}{5}$ of a heaped bushel.

Two bushels of corn in the ear are considered the equivalent of one bushel of shelled corn.

The standard bushel is a cylindrical vessel whose inner diameter is 18.5 in. and depth 8 in.; it contains 2150.42 cu. in.

Liquid Measure

4 gills = 1 pint
2 pints = 1 quart
4 quarts = 1 gallon
31½ gallons = 1 barrel
2 barrels = 1 hogshead

Nearly all kinds of liquids are sold in the United States on the basis of 231 cu. in. to the gallon; but ale and beer are sold on the basis of 282 cu. in. to the gallon.

The barrel (31½ gal.) and the hogshead (63 gal.) in the above table represent *measures*. In practice a barrel holds from 28 gal. to 43 gal., and a hogshead from 54 gal. to 63 gal.

In mixing liquid medicines druggists use **Apothecaries' Fluid Measure**, which is as follows:

60 minims (♏) = 1 fluid dram (ƒ ʒ)
8 fluid drams = 1 fluid ounce (ƒ ʒ)
16 fluid ounces = 1 pint (O.)
8 pints = 1 gallon (Cong.)

A minim is about a drop.

An ounce bottle holds a fluid ounce, or $\frac{1}{16}$ of a pint; a 2-ounce bottle, $\frac{1}{8}$ of a pint; a 3-ounce bottle, $\frac{3}{16}$ of a pint; and so on.

Troy Weight

24 grains = 1 pennyweight
20 pennyweights = 1 ounce
12 ounces = 1 pound

NOTE. The unit of weight for gems and precious stones is the carat, which equals 200 milligrams, or about 3.08 troy grains. The term *carat* is also used to denote the purity, or fineness, of gold, 24 carats denoting pure gold.

The troy pound contains 5760 grains, which is also the unit of apothecaries' weight.

Medicines are sold in prescriptions by apothecaries' weight, which is as follows:

1 pound (℔) = 12 ounces (℥) = 96 drachms (ʒ) = 288 scruples (℈) = 5760 (gr.)

Avoirdupois Weight

16 ounces (oz.) = 1 pound (lb.)
25 pounds, 28 pounds = 1 quarter (qr.)
4 quarters = 1 hundredweight (cwt.)
20 hundredweight = 1 Ton (T.)

NOTE 1. 1 lb. avoir. = 7000 troy grains.

NOTE 2. The long hundredweight, 112 pounds, and the long ton, 2240 pounds, are used in custom houses and in some wholesale and retail transactions.

Weights of a Bushel of Grain, Seed, and Other Produce

Commodities	Lb.	Commodities	Lb.
Barley	48	Clover seed	60
Beans	60	Oats	32
Buckwheat	48	Peas	60
Bran	20	Rye	56
Corn (shelled)	56	Timothy seed	45
Corn (on cob)	70	Wheat	60

In Maryland and Pennsylvania the legal weight of a bushel of potatoes is 56 lb.; in the other states it is 60 lb.

Linear Measure

12 inches = 1 foot
3 feet = 1 yard
5½ yards = 1 rod
320 rods = 1 statute mile

The **furlong** (40 rd.) is now seldom used.

Other Linear Units

(1) A **fathom** = 6 feet, used at sea to measure cables, cordage, and the depth of the sea.

(2) A **hand** = 4 inches, used in measuring a horse's height.

(3) A **geographic mile**, or **knot** = 6086 ft. = the length of one minute of the equatorial circumference of the earth.

(4) A **league** (England and U. S.) = 3 geographic miles, used to measure distances at sea.

(5) A **pace** = 2½ feet in ordinary walking, but 3.3 feet, or ⅕ of a rod, in stepping distances.

(6) 7.92 inches = 1 link.

(7) 100 links = 1 chain.

Land Measure

10,000 square links (sq. li.) = 1 square chain
16 square rods (sq. rd.) = 1 square chain
10 square chains = 1 acre
160 square rods = 1 acre
640 acres = 1 square mile

A square rod is sometimes called a **perch**.

Square Measure

144 square inches (sq. in.) = 1 square foot
9 square feet = 1 square yard
30¼ square yards = 1 square rod

Cubic Measure

1728 cubic inches = 1 cubic foot
27 cubic feet = 1 cubic yard

1 cubic yard of earth is called a **load**.

A pile of wood 8 ft. long, 4 ft. wide, and 4 ft. high is called a **cord**.

24¾ cu. ft. of masonry is called a **perch**.

Angles and Arcs

60 seconds = 1 minute
60 minutes = 1 degree
90 degrees = 1 right angle
90 degrees = 1 quadrant
360 degrees = 1 circumference

Weights of Everyday Things

A barrel of flour weighs	196 lb.
A barrel of salt weighs	280 lb.
A barrel of pork, beef, or fish weighs	200 lb.
A bushel of coarse salt weighs	56 lb.
A keg of nails weighs	100 lb.
A firkin of butter weighs	56 lb.
A keg of powder weighs	25 lb.
A bushel of Portland cement weighs	96 lb.
A bushel of loose lime weighs	70 lb.
A cubic foot of water weighs	62½ lb.
400 cu. ft. of hay in the mow weighs	1 ton

Time

60 seconds	= 1 minute
60 minutes	= 1 hour
24 hours	= 1 day
7 days	= 1 week
52 weeks	= 1 year (nearly)
365 days	= 1 common year
366 days	= 1 leap year
100 years	= 1 century

The exact length of the year is 365 days 5 hours 48 minutes 49.7 seconds. But, as it is not convenient to have other than a whole number of days in a year, it has been decreed that:

Every year whose number is not divisible by 4 shall consist of 365 days, called a **common year.**

Every year whose number is divisible by 4 shall consist of 366 days, called a **leap year,** except when it is a centennial year.

A centennial year whose number is divisible by 400 shall be a leap year, and when not so divisible, a common year.

Foreign Money

4 farthings	= 1 penny
12 pence	= 1 shilling
20 shillings	= 1 pound

1 guinea = 21*s.* 1 crown = 5*s.*
1 florin = 2*s.* £1 = $4.8665

NOTE. In Canada the units of money are the same as in the United States.

The standard unit of value in France is the **franc**; its value in U. S. money is $0.193, or $1 = about 5.18 francs.

The standard unit of value in Germany is the **reichsmark,** or the **mark**; its value in U. S. money is $0.2385, or 4 marks equals $0.945.

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